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Port-Hamiltonian based modeling of nonlinear synchronous machines without electrical excitation

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Abstract

This paper introduces the first generic and fully nonlinear port-Hamiltonian (pH) based modeling approach for synchronous machines (SMs) without electrical excitation. The proposed approach incorporates nonlinear flux linkages and nonlinear machine torque, both depending on stator currents and mechanical angle, and therefore covers magnetic saturation, cross-coupling and cogging torque effects. Iron losses in stator and rotor are neglected. The paper briefly recapitulates port-Hamiltonian systems and conventional high-fidelity nonlinear modeling of synchronous machines. Three different port-Hamiltonian based models of increasing generality are then derived. The proposed models lay the foundation for the application of energy shaping, damping injection, and interconnection and damping assignment passivity based control (IDA-PBC) to electrical drive systems.

Motivation, pH systems & SM dynamics

Motivation & research gap

Port-Hamiltonian (pH) systems provide a unified, energy-based framework for modeling, analysis and control of physical systems: instead of focusing on differential equations, they emphasize power exchange between (sub)systems, energy storage, dissipation and interconnection. This paradigm enabled major control advances (energy/power shaping, damping injection, passivity-based control (PBC), und IDA-PBC) by "putting energy back in control" [1].

The gap: *All* pH models of synchronous machines (SMs) published so far – for permanent-magnet SMs, SMs with electrical excitation, or induction machines – impose **affine flux linkages with constant (apparent) inductances and constant permanent-magnet (PM) flux linkages**. Magnetic saturation, cross-coupling and rotor-angle dependency (cogging torque) are neglected. A pH representation for *realistic*, fully nonlinear SMs does *not yet* exist.

Contribution: This paper closes this gap with **three pH-based models** of increasing generality for SMs w/o electrical excitation (SPMSMs, IPMSMs, RSMs), laying the foundation for energy shaping, damping injection and IDA-PBC of realistic electrical drives.

Port-Hamiltonian systems

Input-state-output pH systems take the form

$$\left.\begin{aligned} \tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} &= [\boldsymbol{S}(\boldsymbol{x})-\boldsymbol{D}(\boldsymbol{x})]\tfrac{\mathrm{d}H(\boldsymbol{x})}{\mathrm{d}\boldsymbol{x}}+\boldsymbol{G}(\boldsymbol{x})\boldsymbol{u}\\ \boldsymbol{y} &= \boldsymbol{G}(\boldsymbol{x})^\top\tfrac{\mathrm{d}H(\boldsymbol{x})}{\mathrm{d}\boldsymbol{x}}\end{aligned}\right\} \tag{1}$$

with state $\boldsymbol{x}$, input $\boldsymbol{u}$, output $\boldsymbol{y}$, energy(-like) Hamiltonian $H(\boldsymbol{x})$, skew-symmetric interconnection matrix $\boldsymbol{S}(\boldsymbol{x})=-\boldsymbol{S}(\boldsymbol{x})^\top$ and symmetric, positive semi-definite dissipation matrix $\boldsymbol{D}(\boldsymbol{x})\geq 0$. Since $\tfrac{\mathrm{d}}{\mathrm{d}t}H(\boldsymbol{x})\leq\boldsymbol{y}^\top\boldsymbol{u}$, system (1) is passive with supply rate $\boldsymbol{y}^\top\boldsymbol{u}$ – the cornerstone property exploited by pH-based control.

Nonlinear SM dynamics – why constant inductances mislead

Neglecting iron losses, the SM dynamics with flux linkages $\boldsymbol{\psi}_\mathrm{s}^{dq}=\boldsymbol{\psi}_\mathrm{s}^{dq}(\boldsymbol{i}_\mathrm{s}^{dq},\phi_\mathrm{m})$ as states are

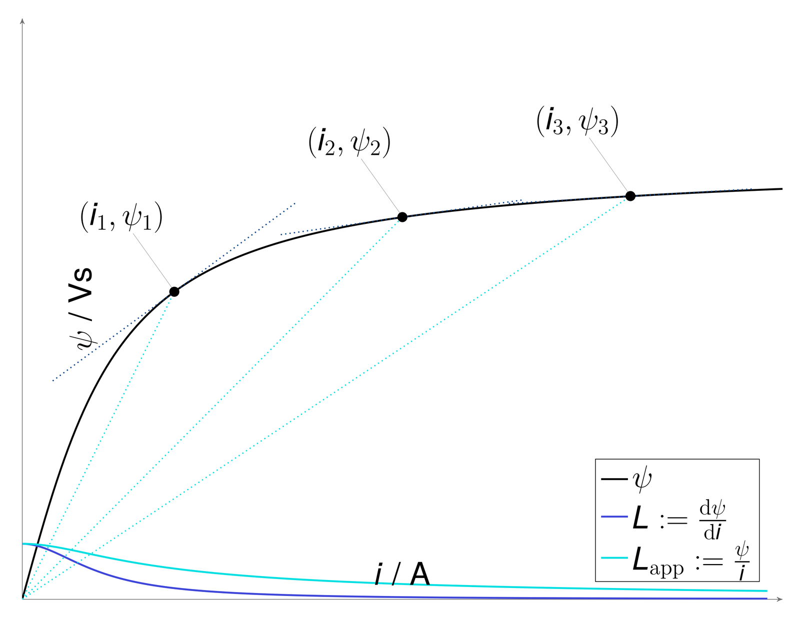
$$\tfrac{\mathrm{d}}{\mathrm{d}t}\underbrace{\begin{pmatrix}\boldsymbol{\psi}_\mathrm{s}^{dq}\\ \omega_\mathrm{m}\\ \phi_\mathrm{m}\end{pmatrix}}_{=:\boldsymbol{x}_\psi}=\begin{pmatrix}\boldsymbol{u}_\mathrm{s}^{dq}-\boldsymbol{R}_\mathrm{s}^{dq}\boldsymbol{i}_\mathrm{s}^{dq}-n_\mathrm{p}\omega_\mathrm{m}\boldsymbol{J}\boldsymbol{\psi}_\mathrm{s}^{dq}\\ \tfrac{1}{\Theta_\mathrm{m}}(m_\mathrm{m}+m_\mathrm{l}-\nu_\mathrm{m}\omega_\mathrm{m})\\ \omega_\mathrm{m}\end{pmatrix}, \tag{2}$$

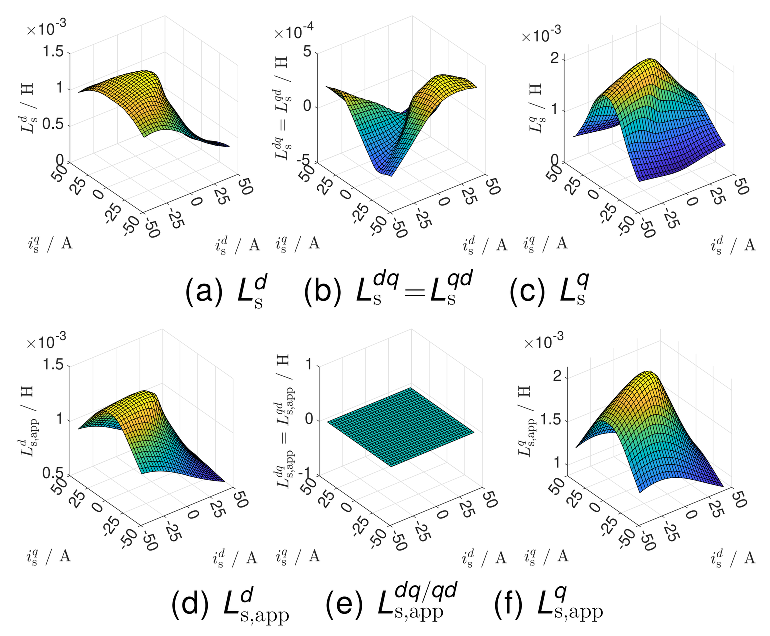
with $\kappa=\sqrt{2/3}$ (power-invariant Clarke transformation), stator voltages $\boldsymbol{u}_\mathrm{s}^{dq}$ and currents $\boldsymbol{i}_\mathrm{s}^{dq}$, resistance matrix $\boldsymbol{R}_\mathrm{s}^{dq}$, pole pairs n_p, inertia Θ_m, friction ν_m and machine torque

$$m_\mathrm{m}=\underbrace{n_\mathrm{p}(\boldsymbol{i}_\mathrm{s}^{dq})^\top\boldsymbol{J}\boldsymbol{\psi}_\mathrm{s}^{dq}}_{\text{average torque}}\quad\underbrace{-\tfrac{\partial E_\mathrm{mag}(\boldsymbol{\psi}_\mathrm{s}^{dq},\phi_\mathrm{m})}{\partial\phi_\mathrm{m}}}_{=:m_\mathrm{m,co}\ \text{(cogging torque)}} \tag{3}$$

derived from the magnetic energy $E_\mathrm{mag}(\boldsymbol{\psi}_\mathrm{s}^{dq},\phi_\mathrm{m}):=\int_{\boldsymbol{0}_2}^{\boldsymbol{\psi}_\mathrm{s}^{dq}}\boldsymbol{i}_\mathrm{s}^{dq}(\boldsymbol{\psi},\phi_\mathrm{m})^\top\,\mathrm{d}\boldsymbol{\psi}$ via the reciprocity law. Only if $\boldsymbol{\psi}_\mathrm{s}^{dq}$ depends nonlinearly on $\boldsymbol{i}_\mathrm{s}^{dq}$ and ϕ_m, the cogging torque $m_\mathrm{m,co}\neq 0$.

Apparent vs. differential inductance: These two are *not* identical for nonlinear $\psi(i)$: $L_\mathrm{app}=\psi/i$ (secant) vs. $L=\mathrm{d}\psi\,/\,\mathrm{d}i$ (tangent). Mixing them up is a common but misleading simplification.





(a) L^d (b) $L_\mathrm{s}^{dq}=L_\mathrm{s}^{qd}$ (c) L^q

(d) $L_\mathrm{s,app}^{d}$ (e) $L_\mathrm{s,app}^{dq/qd}$ (f) $L_\mathrm{s,app}^{q}$

Selected subfigures from Fig. 1 of the manuscript: differential inductances (a)–(c) and apparent inductances (d)–(f) of a nonlinear 3.8 kW IPMSM, alongside the apparent-vs-differential distinction (left).

Three port-Hamiltonian based SM models

Overview

Model	Flux linkage $\boldsymbol{\psi}_\mathrm{s}^{dq}$	Captures
1)	affine, constant $\boldsymbol{L}_\mathrm{s,app}^{dq}$, constant $\boldsymbol{\psi}_\mathrm{pm}^{dq}$	baseline (almost all literature)
2)	affine, constant $\boldsymbol{L}_\mathrm{s,app}^{dq}$, $\boldsymbol{\psi}_\mathrm{pm}^{dq}(\phi_\mathrm{m})$	torque ripple / cogging
3)	fully nonlinear $\boldsymbol{\psi}_\mathrm{s}^{dq}(\boldsymbol{i}_\mathrm{s}^{dq},\phi_\mathrm{m})$	saturation, cross-coupling, cogging

1) Affine SM, constant inductances & constant PM flux

States, Hamiltonian and its gradient are given by

$$\boldsymbol{x}_1:=\begin{pmatrix}\boldsymbol{L}_\mathrm{s,app}^{dq}\boldsymbol{i}_\mathrm{s}^{dq}\\ \Theta_\mathrm{m}\omega_\mathrm{m}\end{pmatrix},\quad H_1(\boldsymbol{x}_1):=\tfrac{1}{2}(\boldsymbol{i}_\mathrm{s}^{dq})^\top\boldsymbol{L}_\mathrm{s,app}^{dq}\boldsymbol{i}_\mathrm{s}^{dq}+\tfrac{1}{2}\tfrac{(\Theta_\mathrm{m}\omega_\mathrm{m})^2}{\Theta_\mathrm{m}},\quad \frac{\mathrm{d}H_1(\boldsymbol{x}_1)}{\mathrm{d}\boldsymbol{x}_1^\top}=\begin{pmatrix}\boldsymbol{i}_\mathrm{s}^{dq}\\ \omega_\mathrm{m}\end{pmatrix}.$$

For $\boldsymbol{S}_1(\boldsymbol{x}_1):=\begin{bmatrix}\boldsymbol{O}_{2\times2} & -n_\mathrm{p}\boldsymbol{J}(\boldsymbol{L}_\mathrm{s,app}^{dq}\boldsymbol{i}_\mathrm{s}^{dq}+\boldsymbol{\psi}_\mathrm{pm}^{dq})\\ n_\mathrm{p}(\boldsymbol{L}_\mathrm{s,app}^{dq}\boldsymbol{i}_\mathrm{s}^{dq}+\boldsymbol{\psi}_\mathrm{pm}^{dq})^\top\boldsymbol{J}^\top & 0\end{bmatrix}$, $\boldsymbol{D}_1:=\begin{bmatrix}\boldsymbol{R}_\mathrm{s}^{dq} & 0\\ 0 & \nu_\mathrm{m}\end{bmatrix}\geq 0$ and $\boldsymbol{G}_1:=\boldsymbol{I}_3$, the SM reduces to the pH system

$$\left.\begin{aligned} \tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x}_1 &= [\boldsymbol{S}_1(\boldsymbol{x}_1)-\boldsymbol{D}_1]\tfrac{\mathrm{d}H_1(\boldsymbol{x}_1)}{\mathrm{d}\boldsymbol{x}_1}+\boldsymbol{G}_1\boldsymbol{u}_1\\ \boldsymbol{y}_1 &= \boldsymbol{G}_1^\top\tfrac{\mathrm{d}H_1(\boldsymbol{x}_1)}{\mathrm{d}\boldsymbol{x}_1},\end{aligned}\right\} \tag{4}$$

with $\boldsymbol{u}_1:=(\boldsymbol{u}_\mathrm{s}^{dq},m_\mathrm{l})^\top$, $\boldsymbol{y}_1:=(\boldsymbol{i}_\mathrm{s}^{dq},\omega_\mathrm{m})^\top$ – **this matches almost all existing pH models of SMs in the literature**, but neglects magnetic saturation and angle dependency entirely.

2) Affine SM with angle-dependent PM flux linkage (1^{st} main result)

State $\boldsymbol{x}_2:=(\boldsymbol{\psi}_\mathrm{s}^{dq},\Theta_\mathrm{m}\omega_\mathrm{m},\phi_\mathrm{m})^\top$ and Hamiltonian $H_2(\boldsymbol{x}_2):=\tfrac{1}{2}(\boldsymbol{\psi}_\mathrm{s}^{dq})^\top(\boldsymbol{L}_\mathrm{s,app}^{dq})^{-1}(\boldsymbol{\psi}_\mathrm{s}^{dq}-2\boldsymbol{\psi}_\mathrm{pm}^{dq})+\tfrac{1}{2}\tfrac{(\Theta_\mathrm{m}\omega_\mathrm{m})^2}{\Theta_\mathrm{m}}$ now yield an *extra* state ϕ_m coupled through the cogging torque $m_\mathrm{m,co}:=(\boldsymbol{\psi}_\mathrm{pm}^{dq})^\top(\boldsymbol{L}_\mathrm{s,app}^{dq})^{-1}\tfrac{\partial\boldsymbol{\psi}_\mathrm{pm}^{dq}}{\partial\phi_\mathrm{m}}$:

$$\left.\begin{aligned} \tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x}_2 &= [\boldsymbol{S}_2(\boldsymbol{x}_2)-\boldsymbol{D}_2]\tfrac{\mathrm{d}H_2(\boldsymbol{x}_2)}{\mathrm{d}\boldsymbol{x}_2}+\boldsymbol{G}_2\boldsymbol{u}_2\\ \boldsymbol{y}_2 &= \boldsymbol{G}_2^\top\tfrac{\mathrm{d}H_2(\boldsymbol{x}_2)}{\mathrm{d}\boldsymbol{x}_2}=\boldsymbol{y}_1,\end{aligned}\right\} \tag{5}$$

with $\boldsymbol{S}_2(\boldsymbol{x}_2):=\begin{bmatrix}\boldsymbol{O}_{2\times2} & -n_\mathrm{p}\boldsymbol{J}\boldsymbol{\psi}_\mathrm{s}^{dq} & 0\\ n_\mathrm{p}(\boldsymbol{\psi}_\mathrm{s}^{dq})^\top\boldsymbol{J}^\top & 0 & -1\\ 0 & 1 & 0\end{bmatrix}=-\boldsymbol{S}_2(\boldsymbol{x}_2)^\top$, $\boldsymbol{D}_2:=\begin{bmatrix}\boldsymbol{R}_\mathrm{s}^{dq} & 0 & 0\\ 0 & \nu_\mathrm{m} & 0\\ 0 & 0 & 0\end{bmatrix}\geq 0$ and $\boldsymbol{G}_2:=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\end{bmatrix}$ **– now torque ripples from angle-dependent PM flux are captured**, but PM flux linkages are still simplified (no current dependency) and magnetic saturation and cross-coupling are still neglected.

3) Fully nonlinear SM (2^{nd} main result – key contribution)

Same state $\boldsymbol{x}_3:=\boldsymbol{x}_2$, but with the *most general* Hamiltonian

$$H_3(\boldsymbol{x}_3):=\underbrace{E_\mathrm{mag}(\boldsymbol{\psi}_\mathrm{s}^{dq},\phi_\mathrm{m})+\tfrac{1}{2}\tfrac{(\Theta_\mathrm{m}\omega_\mathrm{m})^2}{\Theta_\mathrm{m}}}_{=:E_\mathrm{int}(\boldsymbol{\psi}_\mathrm{s}^{dq},\phi_\mathrm{m},\omega_\mathrm{m})}, \tag{6}$$

so that $\tfrac{\mathrm{d}H_3(\boldsymbol{x}_3)}{\mathrm{d}\boldsymbol{x}_3}=(\boldsymbol{i}_\mathrm{s}^{dq},\omega_\mathrm{m},\tfrac{\partial E_\mathrm{mag}}{\partial\phi_\mathrm{m}})^\top$, with *identical* $\boldsymbol{S}_3:=\boldsymbol{S}_2$, $\boldsymbol{D}_3:=\boldsymbol{D}_2$, $\boldsymbol{G}_3:=\boldsymbol{G}_2$ as in Model 2), gives

$$\left.\begin{aligned} \tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x}_3 &= [\boldsymbol{S}_3(\boldsymbol{x}_3)-\boldsymbol{D}_3]\tfrac{\mathrm{d}H_3(\boldsymbol{x}_3)}{\mathrm{d}\boldsymbol{x}_3}+\boldsymbol{G}_3\boldsymbol{u}_3\\ \boldsymbol{y}_3 &= \boldsymbol{G}_3^\top\tfrac{\mathrm{d}H_3(\boldsymbol{x}_3)}{\mathrm{d}\boldsymbol{x}_3}=\boldsymbol{y}_2.\end{aligned}\right\} \tag{7}$$

Only (7) correctly represents realistic SMs with current- and angle-dependent flux linkages as in the IPMSM example on the left – magnetic saturation, cross-coupling and cogging torque are all captured through the single nonlinear function $E_\mathrm{mag}(\boldsymbol{\psi}_\mathrm{s}^{dq},\phi_\mathrm{m})$.

Conclusion & References

Three pH-based models of increasing generality were derived for nonlinear SMs w/o electrical excitation. Model A reproduces the (constant-inductance) state of the art; Model B adds angle-dependent PM flux and torque ripple; Model C is fully nonlinear and the *first* pH representation covering magnetic saturation, cross-coupling and cogging torque. These results enable energy shaping, damping injection and IDA-PBC for realistic electrical drives.

Next steps: (i) pH-based control design (ES, IDA-PBC, PCHD); (ii) extension to SMs with electrical excitation/damper windings and to induction machines; (iii) numerical/experimental validation on IPMSM/RSM benchmarks; (iv) inclusion of iron losses.

[1] Romeo Ortega, Arjan J. van der Schaft andIven Mareels, and Bernhard Maschke. Putting energy back in control. *IEEE Control Systems*, 21(2):18–33, April 2001.