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Frequency-adaptive pole placement for parallelized proportional, integral and resonant control systems

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Introduction

Motivation and problem statement

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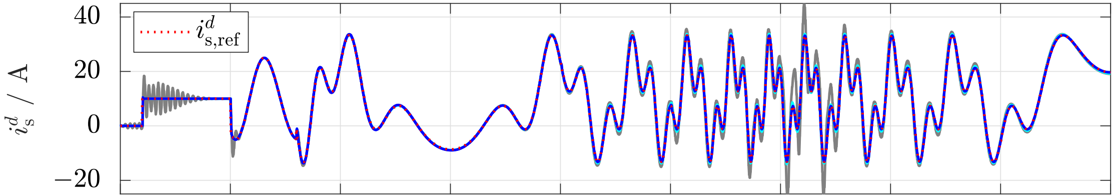
Reference signal with constant and (multiple) sinusoidal components



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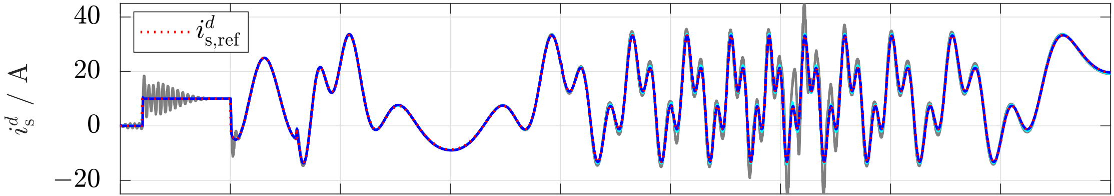

Can we achieve fast and accurate tracking ...

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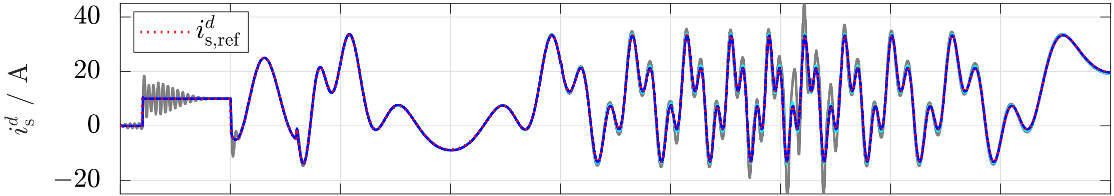

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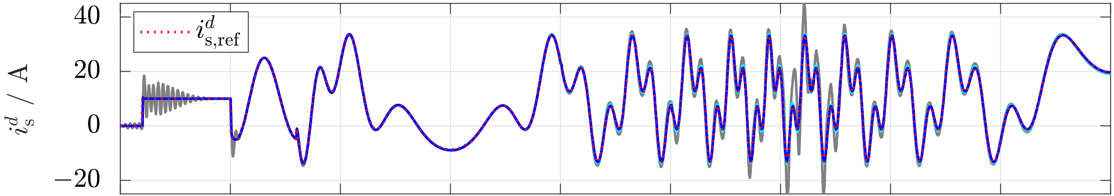

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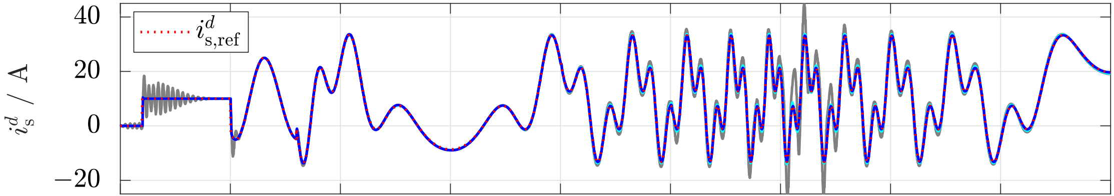

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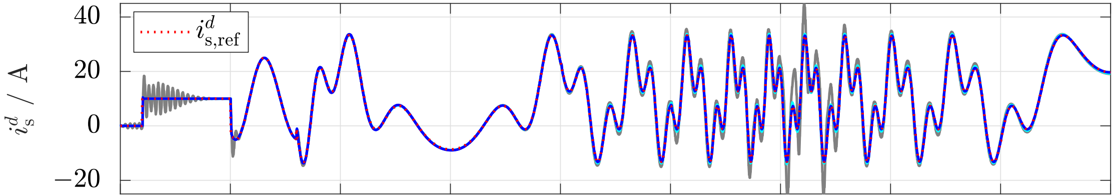

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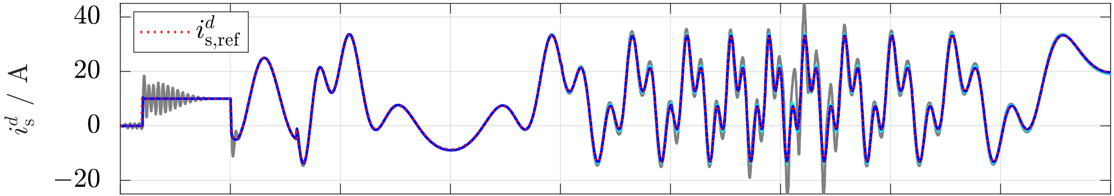

Can we achieve fast and accurate tracking ...

- with (almost) no overshoot ?
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- while considering multiple harmonics and dc-offsets ?
- by which controller structure ?
- by which tuning even if the frequency is varying ?

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Proposed solution

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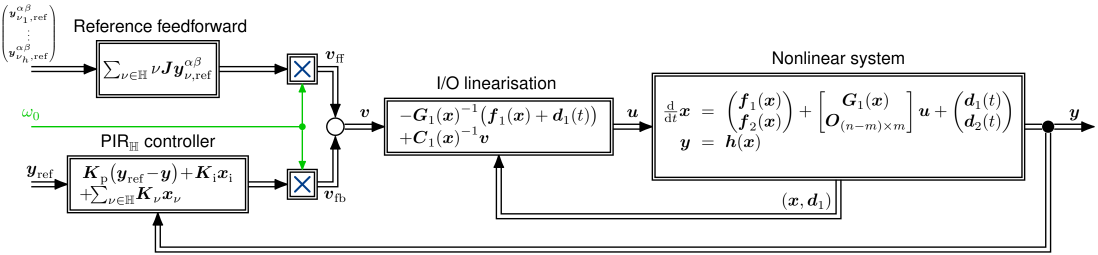
Proposed solution

Proposed controller structure



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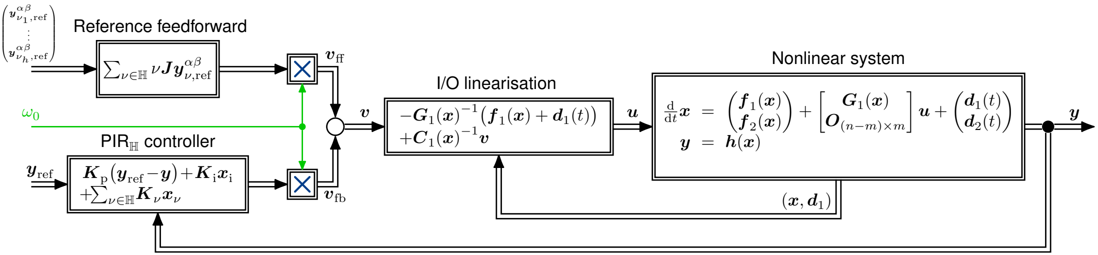
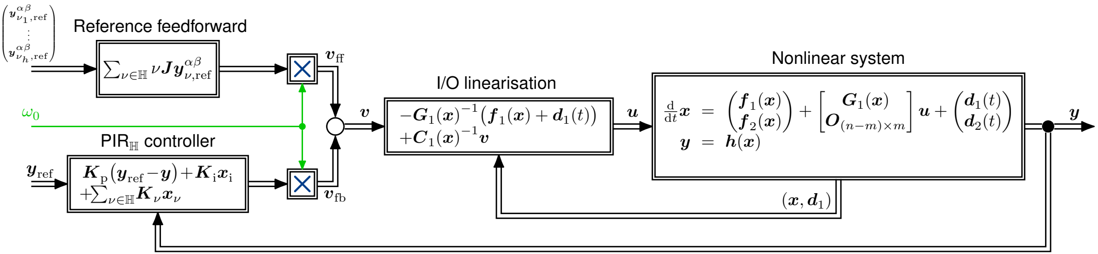


for **frequency-adaptive pole placement for parallelized proportional, integral and resonant control systems**, which is conducted in four steps:

Proposed controller structure



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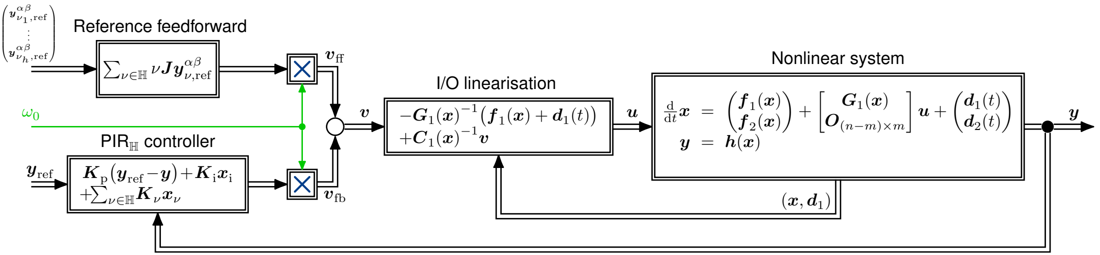

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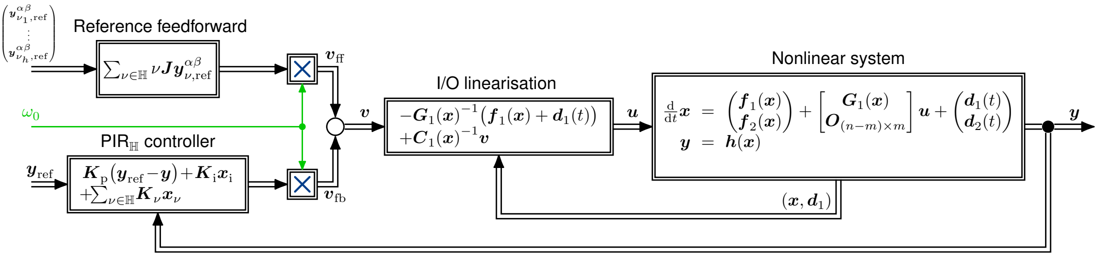

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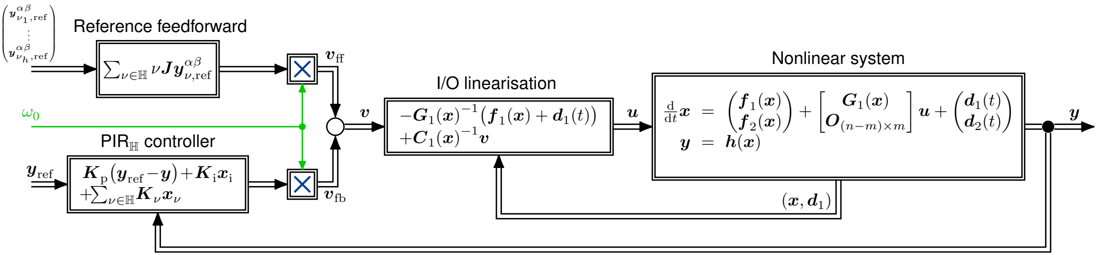

for **frequency-adaptive pole placement for parallelized proportional, integral and resonant control systems**, which is conducted in four steps:

- Step 1: Input/output (I/O) linearization;
- Step 2: Implementation of parallelized PIR$_{\mathbb{H}}$ controller;
- Step 3: Frequency-adaptive pole placement; and
- Step 4: Reference feedforward control.

Frequency-adaptive pole placement for parallelized systems

Step 1: I/O linearization

Considered system class with multi-input $\boldsymbol{u} \in \mathbb{R}^m$ multi-ouput $\boldsymbol{y} \in \mathbb{R}^m$ (MIMO) dynamics

$$\left.\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} &= \begin{pmatrix} \boldsymbol{f}_1(\boldsymbol{x}) \\ \boldsymbol{f}_2(\boldsymbol{x}) \end{pmatrix} + \begin{bmatrix} \boldsymbol{G}_1(\boldsymbol{x}) \\ \boldsymbol{O}_{(n-m)\times m} \end{bmatrix} \boldsymbol{u} + \begin{pmatrix} \boldsymbol{d}_1(t) \\ \boldsymbol{d}_2(t) \end{pmatrix} \\ \boldsymbol{y} &= \boldsymbol{h}(\boldsymbol{x}) \end{aligned}\right\} \tag{1}$$

where $\boldsymbol{f}_1 \colon \mathbb{R}^n \to \mathbb{R}^m$, $\boldsymbol{f}_2 \colon \mathbb{R}^n \to \mathbb{R}^{n-m}$, $\boldsymbol{h} \colon \mathbb{R}^n \to \mathbb{R}^m$, $\boldsymbol{G}_1 \colon \mathbb{R}^n \to \mathbb{R}^{m\times m}$ with $\det \boldsymbol{G}_1(\boldsymbol{x}) \neq 0$ and

$$\left[\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{x}}\boldsymbol{h}(\boldsymbol{x})\right]^\top = \Big[\underbrace{\frac{\mathrm{d}}{\mathrm{d}(x_1,\ldots,x_m)}\boldsymbol{h}(\boldsymbol{x}),}_{=:\boldsymbol{C}_1(\boldsymbol{x})\in\mathbb{R}^{m\times m}} \quad \boldsymbol{O}_{m\times(n-m)}\Big]$$

with $\det \boldsymbol{C}_1(\boldsymbol{x}) \neq 0$ for all $\boldsymbol{x} \in \mathbb{R}^n$ (i.e., $\boldsymbol{G}_1(\boldsymbol{x})$ and $\boldsymbol{C}_1(\boldsymbol{x})$ are invertible).

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Step 1: Input/output (I/O) linearization

$$\boxed{\boldsymbol{u} = -\boldsymbol{G}_1(\boldsymbol{x})^{-1}\big(\boldsymbol{f}_1(\boldsymbol{x}) + \boldsymbol{d}_1(t) + \boldsymbol{C}_1(\boldsymbol{x})^{-1}\boldsymbol{v}\big),} \tag{2}$$

gives $\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{y} \overset{(1),(2)}{=} \boldsymbol{C}_1(\boldsymbol{x})\Big[\boldsymbol{f}_1(\boldsymbol{x}) + \boldsymbol{G}_1(\boldsymbol{x}) \cdot \Big(-\boldsymbol{G}_1(\boldsymbol{x})^{-1}\big(\boldsymbol{f}_1(\boldsymbol{x}) + \boldsymbol{d}_1(t) + \boldsymbol{C}_1(\boldsymbol{x})^{-1}\boldsymbol{v}\big)\Big) + \boldsymbol{d}_1(t)\Big] = \boldsymbol{v}$.

Frequency-adaptive pole placement for parallelized systems

Step 2: Implementation of parallelized $\mathrm{PIR}_{\mathbb{H}}$ controller

Properties of controller:

- Let $m = 2 \to 2$-input, 2-output system of form (1)
- Choose set $\mathbb{H}$ of harmonic orders with cardinality $|\mathbb{H}| =: h \in \mathbb{N}$
- Choose the $\mathrm{PIR}_{\mathbb{H}}$ controller's feedback vector $\boldsymbol{v}_{\mathrm{fb}} \in \mathbb{R}^2$ with $2h + 2$ states $\boldsymbol{x}_{\mathrm{pir}} := (\boldsymbol{x}_{\mathrm{i}}^\top, \boldsymbol{x}_{\nu_1}^\top, \cdots, \boldsymbol{x}_{\nu_h}^\top)^\top \in \mathbb{R}^{2h+2}$, including integral states $\boldsymbol{x}_{\mathrm{i}} \in \mathbb{R}^2$ and h resonant double states $\boldsymbol{x}_\nu \in \mathbb{R}^2$ for all $\nu \in \mathbb{H}$.

Frequency-adaptive pole placement for parallelized systems

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Hence, the **parallelized PIR$_{\mathbb{H}}$ controller** is given by

$$
\left.
\begin{aligned}
\boldsymbol{v}_{\mathrm{fb}} &= \omega_0 \Big[\boldsymbol{K}_{\mathrm{p}} \big(\boldsymbol{y}_{\mathrm{ref}} - \boldsymbol{y} \big) + \boldsymbol{K}_{\mathrm{i}} \boldsymbol{x}_{\mathrm{i}} + \sum_{\nu \in \mathbb{H}} \boldsymbol{K}_\nu \boldsymbol{x}_\nu \Big] \\
\frac{\mathrm{d}}{\mathrm{d}t} \boldsymbol{x}_{\mathrm{pir}} &= \underbrace{\omega_0 \overbrace{\begin{bmatrix} \boldsymbol{O} & \boldsymbol{O} & \cdots & \boldsymbol{O} \\ \boldsymbol{O} & \nu_1 \boldsymbol{J} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \boldsymbol{O} \\ \boldsymbol{O} & \cdots & \boldsymbol{O} & \nu_h \boldsymbol{J} \end{bmatrix}}^{=: \boldsymbol{A}_{\mathrm{pir}}} \boldsymbol{x}_{\mathrm{pir}} + \omega_0 \overbrace{\begin{bmatrix} \boldsymbol{I}_2 \\ \boldsymbol{I}_2 \\ \vdots \\ \boldsymbol{I}_2 \end{bmatrix}}^{=: \boldsymbol{B}_{\mathrm{pir}}} \big(\boldsymbol{y}_{\mathrm{ref}} - \boldsymbol{y} \big)}_{=: \boldsymbol{f}_{\mathrm{pir}} (\boldsymbol{x}_{\mathrm{pir}}, \boldsymbol{y}_{\mathrm{ref}} - \boldsymbol{y}, \omega_0, \omega_{\mathrm{i}}, \omega_{\mathrm{f}}, \ldots, \omega_{\nu_h})}
\end{aligned}
\right\} \tag{3}
$$

Frequency-adaptive pole placement for parallelized systems

Step 2: Implementation of parallelized PIR$_{\mathbb{H}}$ controller

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Hence, the **parallelized PIR$_{\mathbb{H}}$ controller** is given by

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Remark: Note that all gain matrices $\boldsymbol{K}_{\mathrm{p}}$, $\boldsymbol{K}_{\mathrm{i}}$ and $\boldsymbol{K}_{\nu_1}, \ldots, \boldsymbol{K}_{\nu_h}$ are element of $\mathbb{R}^{2\times 2}$ and may be scaled by some angular frequency $\omega_0 \geq 1$. Anti-windup should also be implemented.

Frequency-adaptive pole placement for parallelized systems

Step 3: Frequency-adaptive pole placement

The **frequency-adaptive tuning rule for the controller gain matrices**

$$\boldsymbol{K} := \left[-\boldsymbol{K}_{\mathrm{p}}, \quad \boldsymbol{K}_{\mathrm{i}}, \quad \boldsymbol{K}_{\nu_1}, \quad \cdots, \quad \boldsymbol{K}_{\nu_h}\right] \in \mathbb{R}^{2\times m} \tag{4}$$

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is obtained as

$$\boxed{\boldsymbol{K} = \left[\boldsymbol{\Lambda}^{\star}_{\mathbb{R},1} \quad \cdots \quad \boldsymbol{\Lambda}^{\star}_{\mathbb{R},l}\right] \begin{bmatrix} \boldsymbol{I}_2 & \cdots & \boldsymbol{I}_2 \\ \boldsymbol{T}_{2,1} & \cdots & \boldsymbol{T}_{2,l} \\ \vdots & \ddots & \vdots \\ \boldsymbol{T}_{l,1} & \cdots & \boldsymbol{T}_{l,l} \end{bmatrix}^{-1} \quad \text{with } \boldsymbol{\Lambda}^{\star}_{\mathbb{R},k} := \begin{bmatrix} -\sigma^{\star}_k & -\omega^{\star}_k \\ \omega^{\star}_k & -\sigma^{\star}_k \end{bmatrix} = -\sigma^{\star}_k \boldsymbol{I}_2 + \omega^{\star}_k \boldsymbol{J},} \tag{5}$$

where, for all $i \in \{2, \cdots, l\}$ and $j \in \{1, \cdots, l\}$,

$$\boxed{\boldsymbol{T}_{i,j} = \alpha_1 \left[\alpha_i \boldsymbol{J} - \boldsymbol{\Lambda}^{\star}_{\mathbb{R},j}\right]^{-1}.} \tag{6}$$

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Remark: The simple definition above is based on the introduction of $\alpha_1 := 1$, $\alpha_2 := 0$, $\alpha_3 := \nu_1$, $\alpha_4 := \nu_2, \ldots, \alpha_l := \nu_h$. All $\boldsymbol{T}_{i,j}$ depend solely on known quanteties such as the harmonic orders $\nu_1 = \alpha_3, \nu_2 = \alpha_4, \ldots, \nu_h = \alpha_l$, and real $\sigma^{\star}_j$ and imaginary $\omega^{\star}_j$ parts of the j-th desired pole pair $\lambda^{\star}_j = -\sigma^{\star}_j + \jmath\omega^{\star}_j$ and $\overline{\lambda}^{\star}_j = -\sigma^{\star}_j - \jmath\omega^{\star}_j$, respectively.

Frequency-adaptive pole placement for parallelized systems

Step 4: Reference feedforward control

Invoking the Internal Model Principle, the desired **reference feedforward control** has the following form

$$\boxed{\boldsymbol{v}_{\mathrm{ff}} = \sum\nolimits_{\nu\in\mathbb{H}} \omega_\nu \boldsymbol{J}\boldsymbol{y}^{\alpha\beta}_{\nu,\mathrm{ref}} = \omega_0 \sum\nolimits_{\nu\in\mathbb{H}} \nu \boldsymbol{J}\boldsymbol{y}^{\alpha\beta}_{\nu,\mathrm{ref}},} \tag{7}$$

which only depends on the actual reference components of the respective harmonic order $\nu \in \mathbb{H}$.

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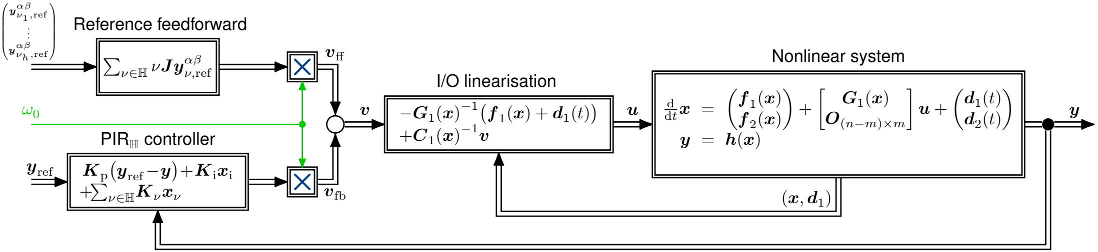
which only depends on the actual reference components of the respective harmonic order $\nu \in \mathbb{H}$. Combining all four steps gives the **overall control system** (as introduced above):



Implementation and simulative validation

Illustrative examples – Example 1: $\mathbb{H}_1 := \{3; 5\}$ with $h_1 = 2$ and $m_1 = 8$.

Considered simple dynamical system (pure integrator; e.g., after I/O linearization):

$$\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{y} = \tfrac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} y^\alpha \\ y^\beta \end{pmatrix} = \boldsymbol{u} := \begin{pmatrix} u^\alpha \\ u^\beta \end{pmatrix}$$

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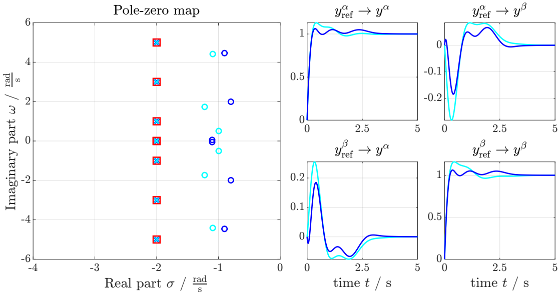
Eigenvalue Selection Guideline 1:

$$\sigma_j^\star = \alpha^\star \wedge \omega_j^\star = \nu_j^\star \Rightarrow \lambda_j^\star = -\alpha^\star \pm \jmath\nu_j^\star$$



Legend: `place` [—] and FAPP [—]: poles [×], zeros [○]

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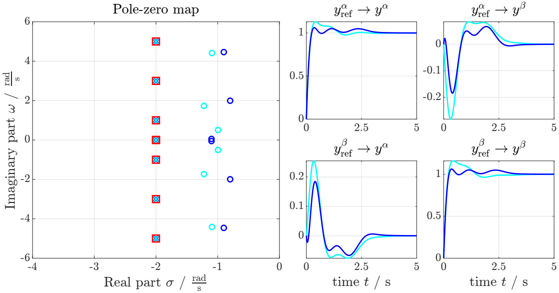
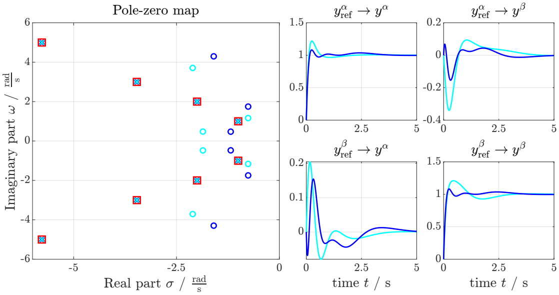


Eigenvalue Selection Guideline 2:

$$\sigma_j^\star = \frac{\nu_j^\star}{\sqrt{1-(D^\star)^2}} \wedge \omega_j^\star = \nu_j \Rightarrow \lambda_j^\star = -\nu_j^\star\left(\frac{1}{\sqrt{1-(D^\star)^2}} \pm \jmath\right)$$



Legend: `place` [—] and FAPP [—]: poles [×], zeros [∘]

Implementation and simulative validation

Illustrative examples – Example 2: $\mathbb{H}_2 := \{2; 3; 4; 5; 6; 7\}$ with $h_2 = 6$ and $m_2 = 16$.

Considered simple dynamical system (pure integrator; e.g., after I/O linearization):

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{y} = \frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} y^\alpha \\ y^\beta \end{pmatrix} = \boldsymbol{u} := \begin{pmatrix} u^\alpha \\ u^\beta \end{pmatrix}$$

Implementation and simulative validation

Illustrative examples – Example 2: $\mathbb{H}_2 := \{2;3;4;5;6;7\}$ with $h_2 = 6$ and $m_2 = 16$.

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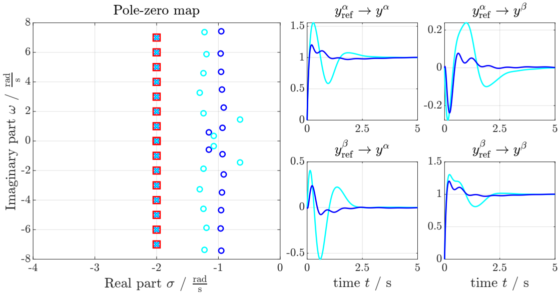

———————- Legend: `place` [—] and FAPP [—]: poles [×], zeros [○] ———————

Implementation and simulative validation

Illustrative examples – Example 2: $\mathbb{H}_2 := \{2; 3; 4; 5; 6; 7\}$ with $h_2 = 6$ and $m_2 = 16$.

Considered simple dynamical system (pure integrator; e.g., after I/O linearization):

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{y} = \frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} y^\alpha \\ y^\beta \end{pmatrix} = \boldsymbol{u} := \begin{pmatrix} u^\alpha \\ u^\beta \end{pmatrix}$$

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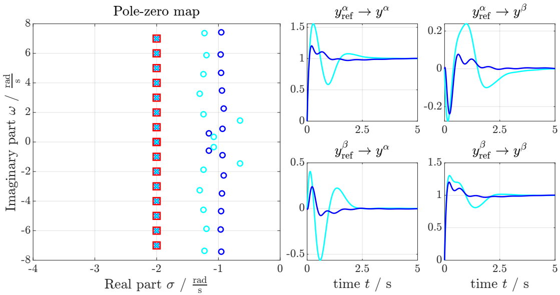
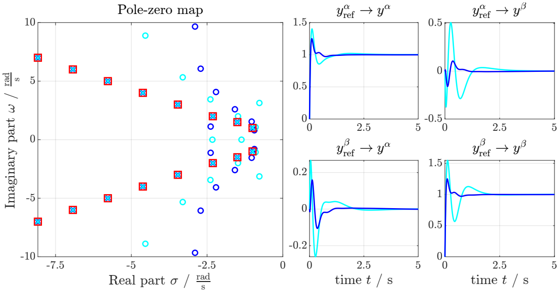


Eigenvalue Selection Guideline 2:

$$\sigma_j^\star = \frac{\nu_j^\star}{\sqrt{1-(D^\star)^2}} \wedge \omega_j^\star = \nu_j \Rightarrow \lambda_j^\star = -\nu_j^\star\Big(\frac{1}{\sqrt{1-(D^\star)^2}} \pm \jmath\Big)$$



Legend: `place` [—] and FAPP [—]: poles [×], zeros [○]

Implementation and simulative validation

Illustrative examples – Example 3: $\mathbb{H}_3 := \{\pm 2; \pm 3; \pm 4; 5; -6; \pm 7; \pm 8; \pm 9; \pm 10; 11; -12; \pm 13\}$ with $h_3 = 20$ and $m_3 = 44$

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Implementation and simulative validation

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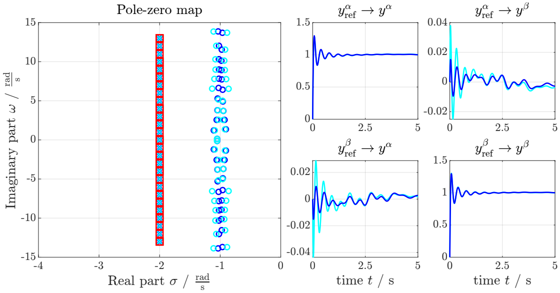

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Implementation and simulative validation

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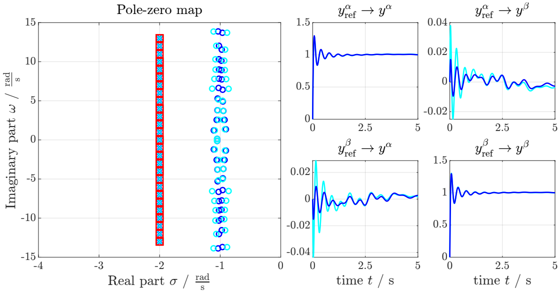

Eigenvalue Selection Guideline 2: ("`place`" fails!)

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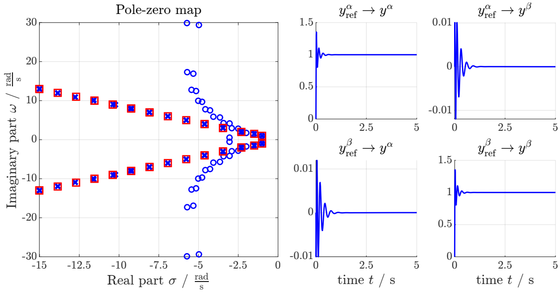

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Implementation and simulative validation

Multi-harmonic current control of nonlinear IPMSM – machine model

Implementation and simulative validation

Multi-harmonic current control of nonlinear IPMSM – machine model

Nonlinear machine dynamics

$$\left.\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_{\mathrm{s}}^{dq} &= \left(\boldsymbol{L}_{\mathrm{s}}^{dq}\right)^{-1}\left(\boldsymbol{u}_{\mathrm{s}}^{dq} - \boldsymbol{R}_{\mathrm{s}}^{dq}\boldsymbol{i}_{\mathrm{s}}^{dq} - n_{\mathrm{p}}\omega_{\mathrm{m}}\boldsymbol{J}\boldsymbol{\psi}_{\mathrm{s}}^{dq}\right) \\
\frac{\mathrm{d}}{\mathrm{d}t}\omega_{\mathrm{m}} &= \frac{1}{\Theta_{\mathrm{m}}}\left(m_{\mathrm{m}} - m_{\mathrm{l}}\right), \\
\frac{\mathrm{d}}{\mathrm{d}t}\phi_{\mathrm{m}} &= \omega_{\mathrm{m}}.
\end{aligned}\right\} \tag{8}$$

Implementation and simulative validation

Nonlinear machine dynamics

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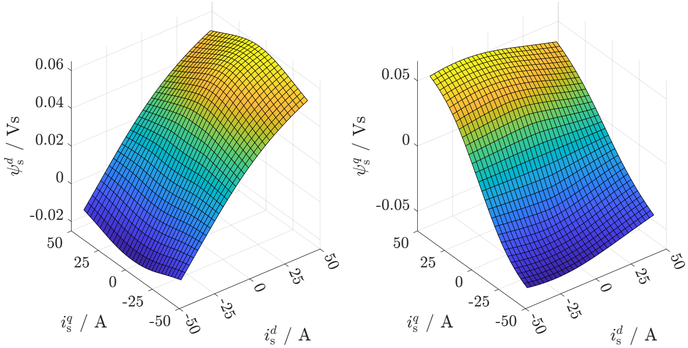
with nonlinear flux linkage maps



Implementation and simulative validation

Controller specifications / tuning:

- Desired harmonics $\mathbb{H}_{\mathrm{ipmsm}} := \{1; 3\}$ $\Rightarrow$ number of harmonics $h_{\mathrm{ipmsm}} = 2$ and states $m_{\mathrm{ipmsm}} = 8$;
- Pole placement with (unscaled) desired poles $\sigma_j^\star$, $\omega_j^\star$ as in figure below (see [□])
- Varying speeds $\Rightarrow$ frequency adaptation $\alpha_{\mathrm{m}} n_{\mathrm{p}} \omega_{\mathrm{m,R}} \big(\beta_{\mathrm{m}} + \frac{|\omega_{\mathrm{m}}(t)|}{\omega_{\mathrm{m,R}}}\big)$ with $\alpha_{\mathrm{m}} = 1$ and $\beta_{\mathrm{m}} = 0.05$

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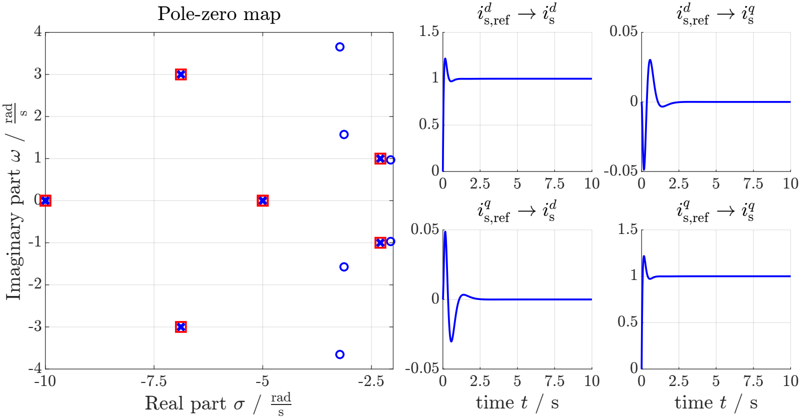

Implementation and simulative validation

Multi-harmonic current control of nonlinear IPMSM

High-fidelity simulation results:
Three different controller implementations to compare multi-harmonic current control performace for nonlinear IPMSM:

Implementation and simulative validation

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(C1) PIR$_{\mathbb{H}}$ controller *without* reference feedforward control and *without* frequency adaption (w/o RFF & w/o FA) [—];

Implementation and simulative validation

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Implementation and simulative validation

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Implementation and simulative

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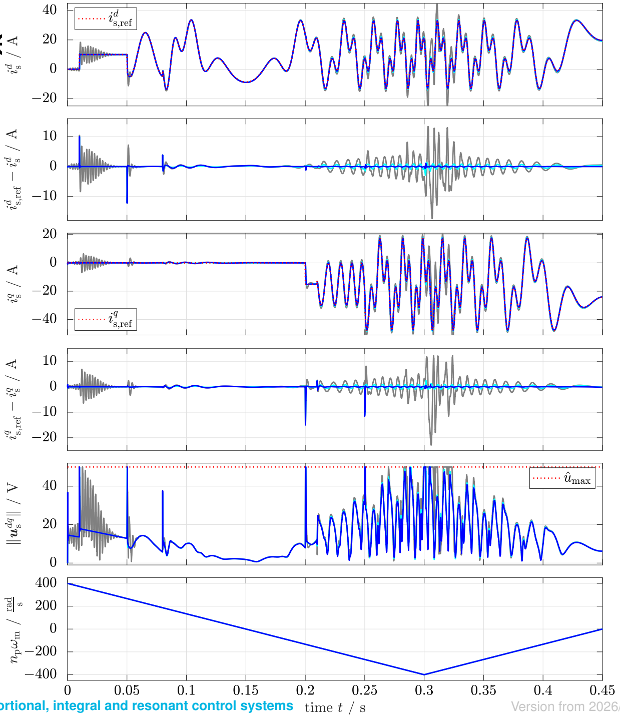

Conclusion

To take home

- Novel and generic frequency-adaptive pole placement (FAPP) algorithm (beats `place`!)
- Applicable for for parallelized proportional, integral and resonant (PIR) control systems has been presented. Additionally, a simple conditional-integration based anti-windup strategy
- Effective reference feedforward control

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- Effective reference feedforward control

Future work

- descrete-time and real-time implementation;
- experimental validation in the laboratory;
- sensitivity and robustness analysis / consideration of parameter/model uncertainties; and
- avoidance of the need of matrix inversion (to avoid risk of numerical instability).