Efficient Nonlinear Torque-Slip-Curve Calculation in an Arbitrary dq-Reference Frame and Dense Inductance Matrices

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Motivation and Problem Statement

Induction machine in (d, q)-reference frame:

$$\begin{pmatrix} \boldsymbol{u}_\mathrm{s}^{dq} \\ \boldsymbol{0}_2 \end{pmatrix} = \begin{bmatrix} \boldsymbol{R}_\mathrm{s}^{dq} & \boldsymbol{0}_{2\times2} \\ \boldsymbol{0}_{2\times2} & \boldsymbol{R}_\mathrm{r}^{dq} \end{bmatrix} \begin{pmatrix} \boldsymbol{i}_\mathrm{s}^{dq} \\ \boldsymbol{i}_\mathrm{r}^{dq} \end{pmatrix} + \begin{bmatrix} \omega_\mathrm{p,s}\boldsymbol{J} & \boldsymbol{0}_{2\times2} \\ \boldsymbol{0}_{2\times2} & s\omega_\mathrm{p,s}\boldsymbol{J} \end{bmatrix} \begin{bmatrix} \boldsymbol{L}_\mathrm{s}^{dq} & \boldsymbol{M}_\mathrm{sr}^{dq} \\ \boldsymbol{M}_\mathrm{rs}^{dq} & \boldsymbol{L}_\mathrm{r}^{dq} \end{bmatrix} \begin{pmatrix} \boldsymbol{i}_\mathrm{s}^{dq} \\ \boldsymbol{i}_\mathrm{r}^{dq} \end{pmatrix}$$

$$= \begin{bmatrix} \boldsymbol{R}_\mathrm{s}^{dq} + \omega_\mathrm{p,s}\boldsymbol{J}\boldsymbol{L}_\mathrm{s}^{dq} & \omega_\mathrm{p,s}\boldsymbol{J}\boldsymbol{M}_\mathrm{sr}^{dq} \\ s\omega_\mathrm{p,s}\boldsymbol{J}\boldsymbol{M}_\mathrm{rs}^{dq} & \boldsymbol{R}_\mathrm{r}^{dq} + s\omega_\mathrm{p,s}\boldsymbol{J}\boldsymbol{L}_\mathrm{r}^{dq} \end{bmatrix} \begin{pmatrix} \boldsymbol{i}_\mathrm{s}^{dq} \\ \boldsymbol{i}_\mathrm{r}^{dq} \end{pmatrix} \quad (1)$$

$$\boldsymbol{i}_\mathrm{s}^{dq} = \left[\boldsymbol{R}_\mathrm{s}^{dq} + \omega_\mathrm{p,s}\boldsymbol{J}\left(\boldsymbol{L}_\mathrm{s}^{dq} - \boldsymbol{M}_\mathrm{sr}^{dq}\left(\boldsymbol{R}_\mathrm{r}^{dq} + s\omega_\mathrm{p,s}\boldsymbol{J}\boldsymbol{L}_\mathrm{r}^{dq}\right)^{-1}\left(s\omega_\mathrm{p,s}\boldsymbol{J}\boldsymbol{M}_\mathrm{rs}^{dq}\right)\right)\right]^{-1} \boldsymbol{u}_\mathrm{s}^{dq} \quad (2)$$

$$\boldsymbol{i}_\mathrm{r}^{dq} = -(\boldsymbol{R}_\mathrm{r}^{dq} + s\omega_\mathrm{p,s}\boldsymbol{J}\boldsymbol{L}_\mathrm{r}^{dq})^{-1}\left(s\omega_\mathrm{p,s}\boldsymbol{J}\boldsymbol{M}_\mathrm{rs}^{dq}\right)\boldsymbol{i}_\mathrm{s}^{dq} \quad (3)$$

Classic (simplified) approach: apparent (d, q)-inductances set as constant and scalar.

(i) no anisotropy,
(ii) no cross-coupling, and
(iii) no saturation.

➡ breakdown torque will be overestimated torque-speed map is wrong

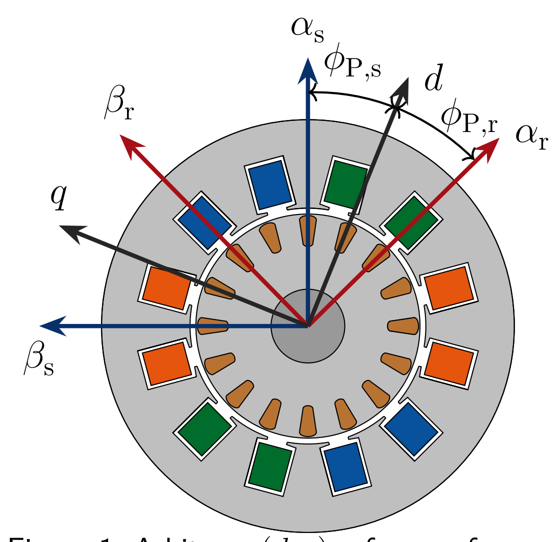

Figure 1. Arbitrary (d, q)-reference frames of an exemplary induction machine.

Mathematical Solution

NEW Approach: nonlinear, dense (d, q)-inductance matrices:

$$\boldsymbol{L}_\mathrm{s}^{dq} := \underbrace{u_\mathrm{i}\boldsymbol{M}_\mathrm{sr}^{dq}}_{\boldsymbol{L}_\mathrm{s,h}^{dq}} + \boldsymbol{L}_{\mathrm{s},\sigma}^{dq}, \quad \boldsymbol{L}_\mathrm{r}^{dq} := \underbrace{(u_\mathrm{i})^{-1}\boldsymbol{M}_\mathrm{rs}^{dq}}_{\boldsymbol{L}_\mathrm{r,h}^{dq}} + \boldsymbol{L}_{\mathrm{r},\sigma}^{dq} \quad (4)$$

with

$$\boldsymbol{M}_\mathrm{sr}^{dq} = \begin{bmatrix} M_\mathrm{sr}^{d} & M_\mathrm{sr}^{dq} \\ M_\mathrm{sr}^{qd} & M_\mathrm{sr}^{q} \end{bmatrix} \quad \boldsymbol{L}_{\mathrm{s},\sigma}^{dq} = \begin{bmatrix} L_{\mathrm{s},\sigma}^{d} & L_{\mathrm{s},\sigma}^{dq} \\ L_{\mathrm{s},\sigma}^{qd} & L_{\mathrm{s},\sigma}^{q} \end{bmatrix} \quad \boldsymbol{M}_\mathrm{rs}^{dq} = \begin{bmatrix} M_\mathrm{rs}^{d} & M_\mathrm{rs}^{dq} \\ M_\mathrm{rs}^{qd} & M_\mathrm{rs}^{q} \end{bmatrix} \quad \boldsymbol{L}_{\mathrm{r},\sigma}^{dq} = \begin{bmatrix} L_{\mathrm{r},\sigma}^{d} & L_{\mathrm{r},\sigma}^{dq} \\ L_{\mathrm{r},\sigma}^{qd} & L_{\mathrm{r},\sigma}^{q} \end{bmatrix}$$

How to obtain these dense matrices?

➡ approach of frozen permeabilities

$$\boldsymbol{i}^{dq} = \Bigg[\overbrace{\begin{pmatrix} \boldsymbol{i}_\mathrm{s,1}^{dq} \\ \boldsymbol{i}_\mathrm{r,1}^{dq} \end{pmatrix}}^{\mu_\mathrm{r,1} = f(B(H))} \overbrace{\begin{pmatrix} \boldsymbol{i}_\mathrm{s,2}^{dq} \\ \boldsymbol{i}_\mathrm{r,2}^{dq} \end{pmatrix} \begin{pmatrix} \boldsymbol{i}_\mathrm{s,3}^{dq} \\ \boldsymbol{i}_\mathrm{r,3}^{dq} \end{pmatrix} \begin{pmatrix} \boldsymbol{i}_\mathrm{s,4}^{dq} \\ \boldsymbol{i}_\mathrm{r,4}^{dq} \end{pmatrix}}^{\mu_\mathrm{r} = \mu_\mathrm{r,1}}\Bigg] \quad \text{and} \quad \boldsymbol{\psi}^{dq} = \Bigg[\overbrace{\begin{pmatrix} \boldsymbol{\psi}_\mathrm{s,1}^{dq} \\ \boldsymbol{\psi}_\mathrm{r,1}^{dq} \end{pmatrix}}^{\mu_\mathrm{r,1} = f(B(H))} \overbrace{\begin{pmatrix} \boldsymbol{\psi}_\mathrm{s,2}^{dq} \\ \boldsymbol{\psi}_\mathrm{r,2}^{dq} \end{pmatrix} \begin{pmatrix} \boldsymbol{\psi}_\mathrm{s,3}^{dq} \\ \boldsymbol{\psi}_\mathrm{r,3}^{dq} \end{pmatrix} \begin{pmatrix} \boldsymbol{\psi}_\mathrm{s,4}^{dq} \\ \boldsymbol{\psi}_\mathrm{r,4}^{dq} \end{pmatrix}}^{\mu_\mathrm{r} = \mu_\mathrm{r,1}}\Bigg].$$

The inductance matrices can be transformed into a vector to solve for by applying the Kronecker product

$$\boldsymbol{I}\boldsymbol{L}^{dq}\boldsymbol{i}^{dq} = \boldsymbol{\psi}^{dq} \Leftrightarrow \underbrace{((\boldsymbol{i}^{dq})^\top \otimes \boldsymbol{I})}_{\boldsymbol{K}}\mathrm{vec}(\boldsymbol{L}^{dq}) = \mathrm{vec}(\boldsymbol{\psi}^{dq}). \quad (5)$$

The inductance entries are finally calculated by solving the Least-square problem

$$\mathrm{vec}(\boldsymbol{L}^{dq}) = (\boldsymbol{K}^\top\boldsymbol{K})^{-1}\boldsymbol{K}^\top\mathrm{vec}(\boldsymbol{\psi}^{dq}). \quad (6)$$

Approach Overview

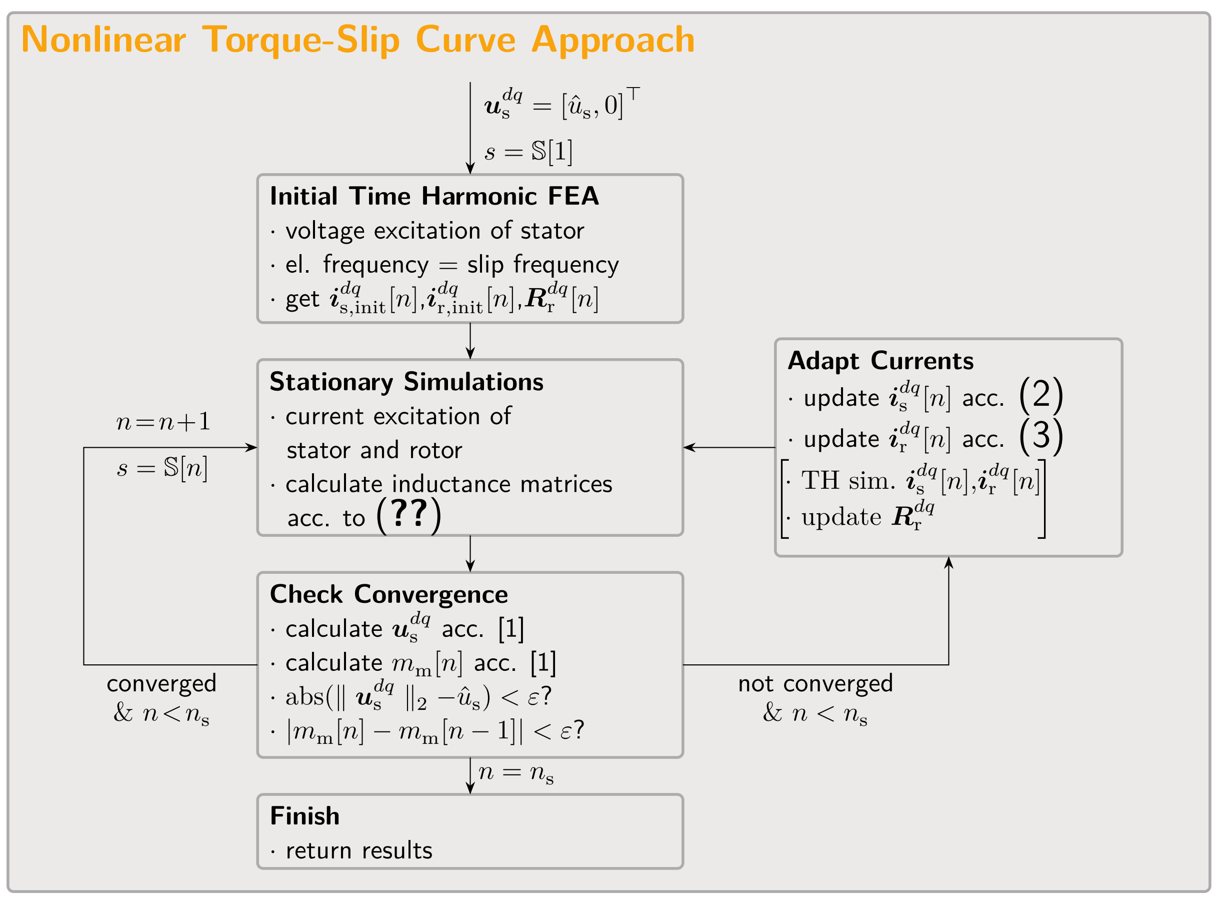

Figure 2. Illustration of Novel Approach to Find Nonlinear Torque Slip Curve.

Investigated Szenario

Table 1. Machine parameters of test model.

Parameter	Symbol	Value
nominal voltage	$U_\mathrm{r,ltl}$	400 V
nominal current	I_r	10 A
nominal stator frequency	$f_\mathrm{s,el}$	50 Hz
nominal power	P_r	5.5 kW
power factor	$\cos(\phi)$	0.8
nominal rotor speed	n_m	1410 rpm

Table 2. Overview of compared models considering different nonlinearities.

modeling	FEA_1	FEA_2	$\mathrm{FEA}_\mathrm{ref}$	analytic
nonl. inductances	yes	yes	yes	no
nonl. resistance	no	yes	yes	no
rotor skewing	no	no	yes	no

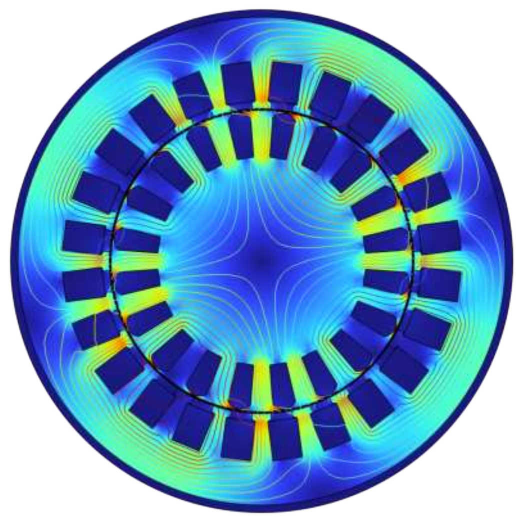

Figure 3. Illustration of the magnetic flux density of the investigated induction machine at nominal current.

Results

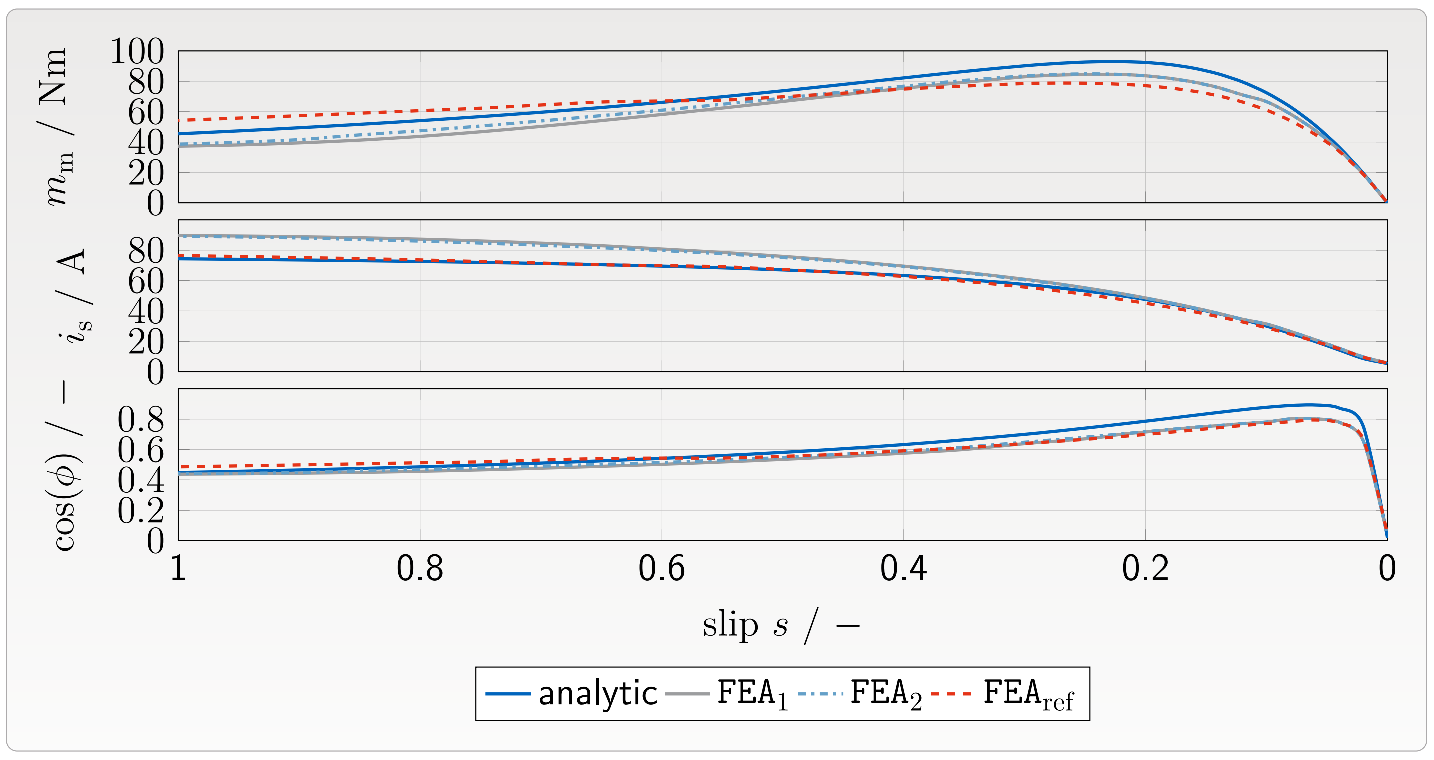

Figure 4. Resulting torque, stator current amplitude and power factor obtained from the analytic model and the novel FEA approach with different settings.

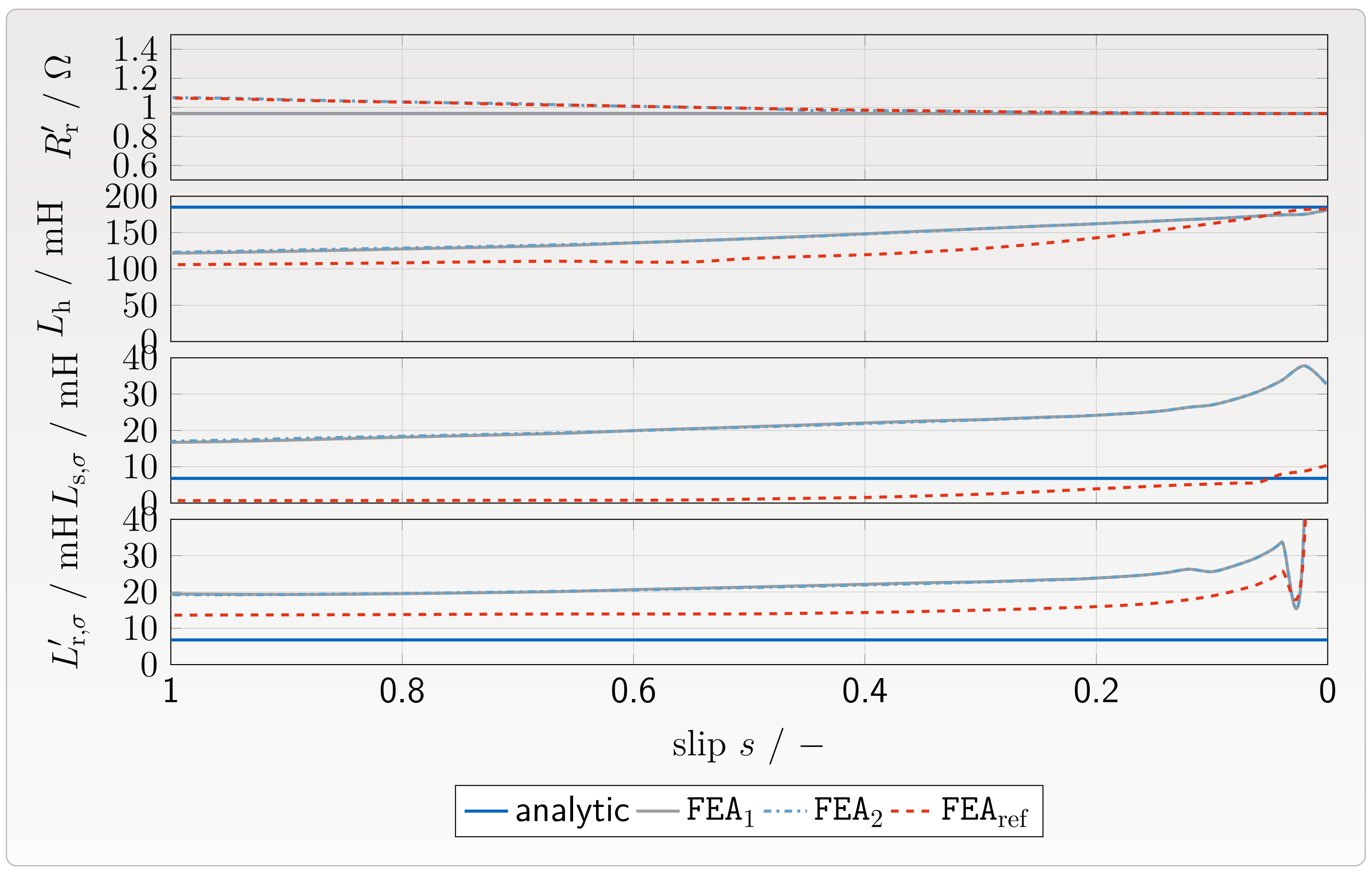

Figure 5. Comparison of the nonlinear parameters obtained by the novel FEA approach.

References

[1] Johannes Rossmann and Christoph M. Hackl. Efficient steady-state rotor bar current estimation based on dense apparent (d, q)-inductance matrices [extended version], 2025. DOI: 10.36227/techrxiv.174586888.85472929.

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