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Derivation of Clarke and mutual inductance coupling factors in (d,q)-reference frame for synchronous machines with several multi-phase winding systems

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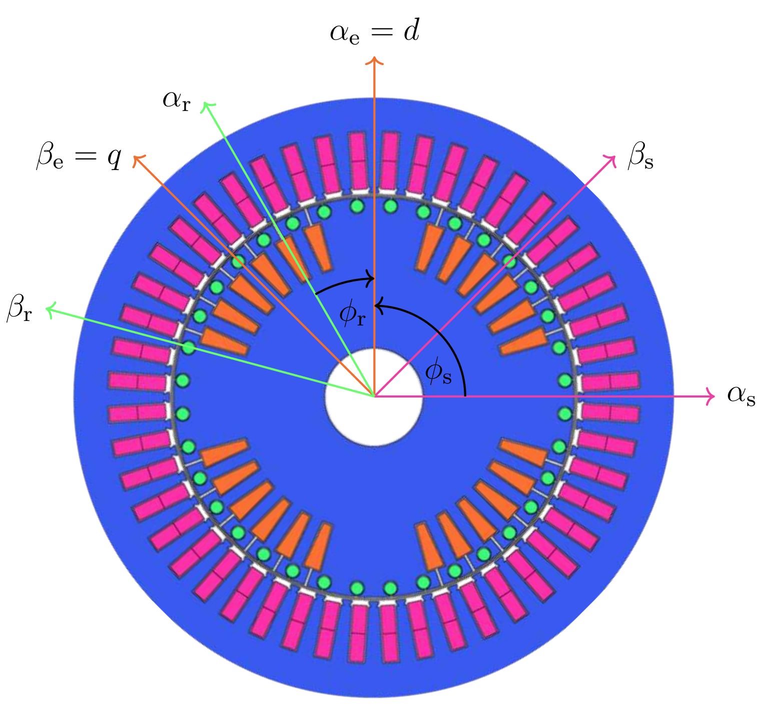
Motivation and problem statement

Example of a synchronous machine with several multi-phase winding systems showing steel sheets [■], stator winding [■], exciter winding [■], and damper winding [■].

How can we derive the Clarke factor and the mutual inductance coupling factors in the (d,q)-reference frame for such machines, considering saturation and cross-coupling effects?

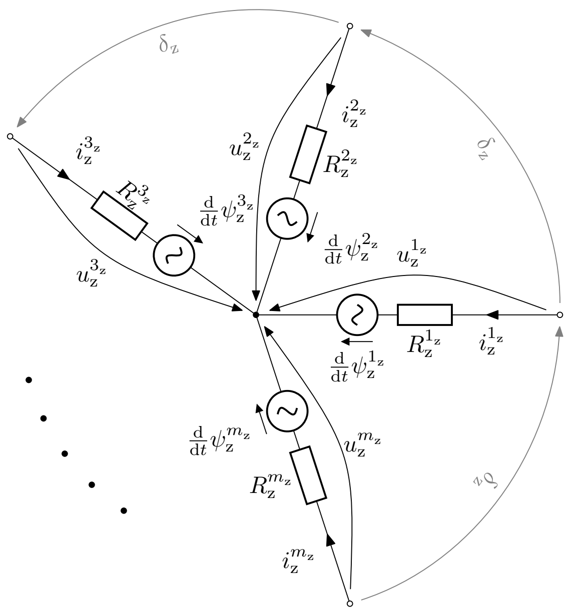


Generic modeling

Phase system with m_z phases

A multi-phase system with m_z phases is modeled by

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\begin{pmatrix}\psi_\mathrm{z}^{1_\mathrm{z}}\\ \vdots \\ \psi_\mathrm{z}^{m_\mathrm{z}}\end{pmatrix}}_{=:\boldsymbol{\psi}_\mathrm{z}^{1_\mathrm{z}\cdots m_\mathrm{z}}} = \underbrace{\begin{pmatrix}u_\mathrm{z}^{1_\mathrm{z}}\\ \vdots \\ u_\mathrm{z}^{m_\mathrm{z}}\end{pmatrix}}_{=:\boldsymbol{u}_\mathrm{z}^{1_\mathrm{z}\cdots m_\mathrm{z}}} - \underbrace{\begin{bmatrix}R_\mathrm{z}^{1_\mathrm{z}} & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & R_\mathrm{z}^{m_\mathrm{z}}\end{bmatrix}}_{=:\boldsymbol{R}_\mathrm{z}^{1_\mathrm{z}\cdots m_\mathrm{z}}}\underbrace{\begin{pmatrix}i_\mathrm{z}^{1_\mathrm{z}}\\ \vdots \\ i_\mathrm{z}^{m_\mathrm{z}}\end{pmatrix}}_{=:\boldsymbol{i}_\mathrm{z}^{1_\mathrm{z}\cdots m_\mathrm{z}}}. \quad (1)$$



Clarke transformation

By applying the Clarke transformation matrix and its pseudo-inverse

$$\boldsymbol{T}_\mathrm{c,z} := \kappa_\mathrm{z}\underbrace{\begin{bmatrix}\cos(\phi_{1,\mathrm{z}}) & \cdots & \cos(\phi_{m,\mathrm{z}})\\ \sin(\phi_{1,\mathrm{z}}) & \cdots & \sin(\phi_{m,\mathrm{z}})\end{bmatrix}}_{=:\widetilde{\boldsymbol{T}}_\mathrm{c,z}} \iff \boldsymbol{T}_\mathrm{c,z}^{-1} := \begin{bmatrix}\cos(\phi_{1,\mathrm{z}}) & \sin(\phi_{1,\mathrm{z}})\\ \vdots & \vdots\\ \cos(\phi_{m,\mathrm{z}}) & \sin(\phi_{m,\mathrm{z}})\end{bmatrix} = \widetilde{\boldsymbol{T}}_\mathrm{c,z}^\top, \quad (2)$$

the orthogonal (α,β)-reference frame is obtained, leading to

$$\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_\mathrm{z}^{\alpha_\mathrm{z}\beta_\mathrm{z}} = \boldsymbol{u}_\mathrm{z}^{\alpha_\mathrm{z}\beta_\mathrm{z}} - \boldsymbol{R}_\mathrm{z}^{\alpha_\mathrm{z}\beta_\mathrm{z}}\boldsymbol{i}_\mathrm{z}^{\alpha_\mathrm{z}\beta_\mathrm{z}}. \quad (3)$$

Derivation of the Clarke factor

Amplitude invariant transformation is if the amplitude $\hat{x}$ of the original signal is equal to the norm of the transformed signal, i.e.

$$\hat{x} \overset{!}{=} ||\boldsymbol{x}_\mathrm{z}^{\alpha_\mathrm{z}\beta_\mathrm{z}}|| = ||\boldsymbol{T}_\mathrm{c,z}\boldsymbol{x}_\mathrm{z}^{1_\mathrm{z}\cdots m_\mathrm{z}}|| = \left\Vert \kappa_\mathrm{z}\widetilde{\boldsymbol{T}}_\mathrm{c,z}\,\hat{x}\underbrace{\begin{pmatrix}\cos(\phi_{1,\mathrm{z}}+\phi_\mathrm{x}(t))\\ \vdots\\ \cos(\phi_{m,\mathrm{z}}+\phi_\mathrm{x}(t))\end{pmatrix}}_{\text{symmetric signal}}\right\Vert. \quad (4)$$

The Clarke factor can be derived as

$$\kappa_\mathrm{z} = \left\Vert \overbrace{\begin{bmatrix}\cos(\phi_{1,\mathrm{z}}) & \cdots & \cos(\phi_{m,\mathrm{z}})\\ \sin(\phi_{1,\mathrm{z}}) & \cdots & \sin(\phi_{m,\mathrm{z}})\end{bmatrix}}^{=\widetilde{\boldsymbol{T}}_\mathrm{c,z}}\begin{pmatrix}\cos(\phi_{1,\mathrm{z}}+\phi_\mathrm{x}(t))\\ \vdots\\ \cos(\phi_{m,\mathrm{z}}+\phi_\mathrm{x}(t))\end{pmatrix}\right\Vert^{-1} = \begin{cases}\frac{1}{2}, & \text{if } m_\mathrm{z}=2 \text{ and } \phi_\mathrm{x}(t)=0,\\ \frac{2}{m_\mathrm{z}}, & \text{if } m_\mathrm{z}>2.\end{cases} \quad (5)$$

Mutual inductance coupling factors

The partial differential equation of the flux linkages of the systems x and y is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\boldsymbol{\psi}_\mathrm{x}^{1_\mathrm{x}\cdots m_\mathrm{x}}\\ \boldsymbol{\psi}_\mathrm{y}^{1_\mathrm{y}\cdots m_\mathrm{y}}\end{pmatrix} = \begin{bmatrix}\boldsymbol{L}_\mathrm{x}^{1_\mathrm{x}\cdots m_\mathrm{x}} & \boldsymbol{M}_\mathrm{xy}^{1_\mathrm{x}\cdots m_\mathrm{x}}\\ \boldsymbol{M}_\mathrm{yx}^{1_\mathrm{y}\cdots m_\mathrm{y}} & \boldsymbol{L}_\mathrm{y}^{1_\mathrm{y}\cdots m_\mathrm{y}}\end{bmatrix}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\boldsymbol{i}_\mathrm{x}^{1_\mathrm{x}\cdots m_\mathrm{x}}\\ \boldsymbol{i}_\mathrm{y}^{1_\mathrm{y}\cdots m_\mathrm{y}}\end{pmatrix} \quad (6)$$

$$\iff \frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\boldsymbol{\psi}_\mathrm{x}^{\alpha_\mathrm{x}\beta_\mathrm{x}}\\ \boldsymbol{\psi}_\mathrm{y}^{\alpha_\mathrm{y}\beta_\mathrm{y}}\end{pmatrix} = \begin{bmatrix}\boldsymbol{L}_\mathrm{x}^{\alpha_\mathrm{x}\beta_\mathrm{x}} & \boldsymbol{M}_\mathrm{xy}^{\alpha_\mathrm{x}\beta_\mathrm{x}}\\ \boldsymbol{M}_\mathrm{yx}^{\alpha_\mathrm{y}\beta_\mathrm{y}} & \boldsymbol{L}_\mathrm{y}^{\alpha_\mathrm{y}\beta_\mathrm{y}}\end{bmatrix}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\boldsymbol{i}_\mathrm{x}^{\alpha_\mathrm{x}\beta_\mathrm{x}}\\ \boldsymbol{i}_\mathrm{y}^{\alpha_\mathrm{y}\beta_\mathrm{y}}\end{pmatrix} \quad (7)$$

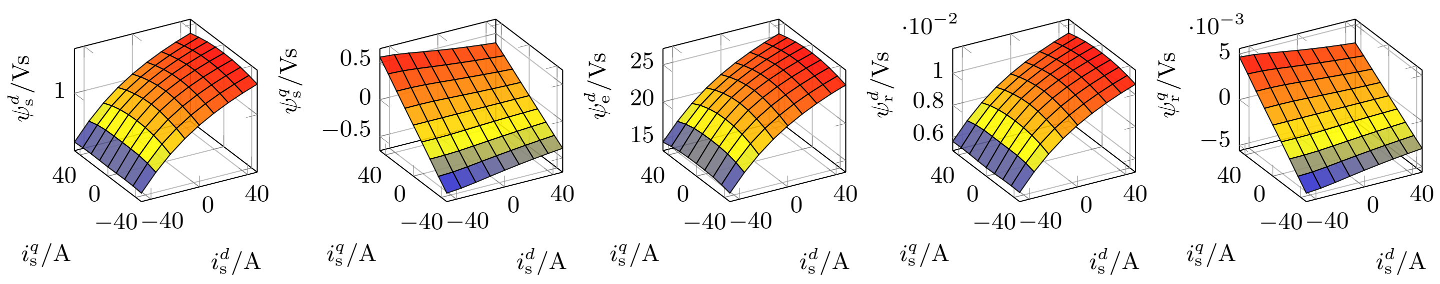
Due to the magnetic principle of reciprocity, $\boldsymbol{M}_\mathrm{xy}^{1_\mathrm{x}\cdots m_\mathrm{x}} = \left(\boldsymbol{M}_\mathrm{yx}^{1_\mathrm{y}\cdots m_\mathrm{y}}\right)^\top$ holds. However, after applying the Clarke transformation, the relationship between the mutual inductances of two arbitrary phase systems in orthogonal reference frames is characterized by the ratio $\frac{\kappa_\mathrm{x}}{\kappa_\mathrm{y}}$, i.e.

$$\boldsymbol{M}_\mathrm{xy}^{\alpha_\mathrm{x}\beta_\mathrm{x}}\left(\boldsymbol{M}_\mathrm{yx}^{\alpha_\mathrm{y}\beta_\mathrm{y}}\right)^{-\top} = \frac{\kappa_\mathrm{x}}{\kappa_\mathrm{y}}\underbrace{\left(\widetilde{\boldsymbol{T}}_\mathrm{c,x}\boldsymbol{M}_\mathrm{xy}^{1_\mathrm{x}\cdots m_\mathrm{x}}\widetilde{\boldsymbol{T}}_\mathrm{c,y}^\top\right)\left(\widetilde{\boldsymbol{T}}_\mathrm{c,x}\boldsymbol{M}_\mathrm{xy}^{1_\mathrm{x}\cdots m_\mathrm{x}}\widetilde{\boldsymbol{T}}_\mathrm{c,y}^\top\right)^{-1}}_{=\boldsymbol{I}_2}. \quad (8)$$

Tailoring to EESM with damper winding

Nonlinear modeling and parameter identification by FEA

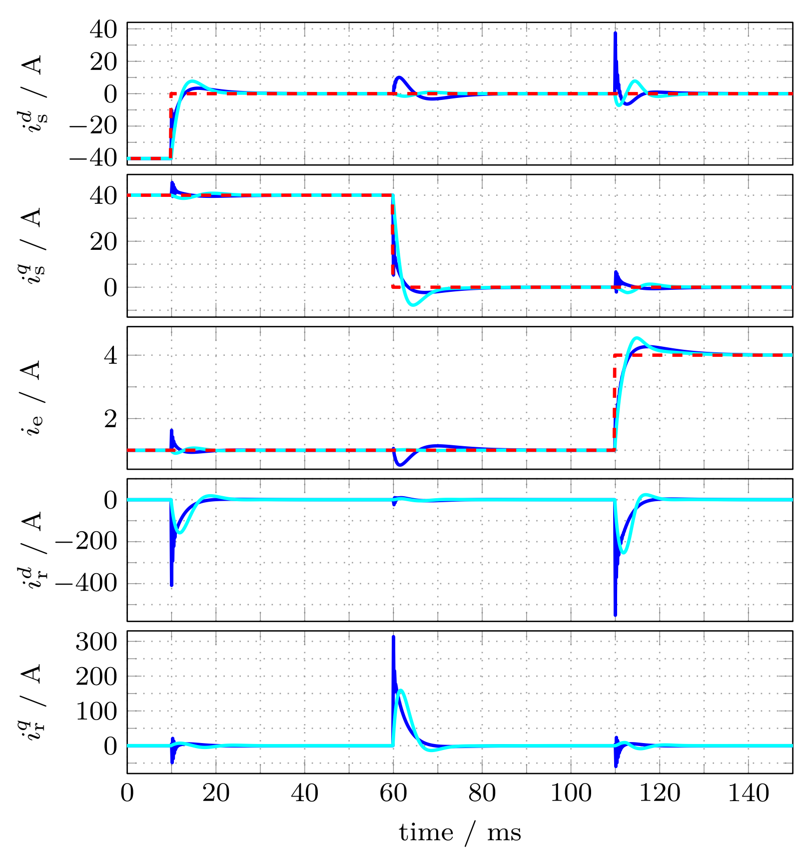
$$\left.\begin{aligned}
\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_\mathrm{s}^{dq} &= \boldsymbol{u}_\mathrm{s}^{dq} - \boldsymbol{R}_\mathrm{s}^{dq}\boldsymbol{i}_\mathrm{s}^{dq} - \omega_\mathrm{p}\boldsymbol{J}\boldsymbol{\psi}_\mathrm{s}^{dq} &&= \boldsymbol{L}_\mathrm{s}^{dq}\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_\mathrm{s}^{dq} + \boldsymbol{L}_\mathrm{s,r}^{dq}\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_\mathrm{r}^{dq} + \boldsymbol{l}_\mathrm{s,e}^{dq}\tfrac{\mathrm{d}}{\mathrm{d}t}i_\mathrm{e}^{d},\\
\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_\mathrm{r}^{dq} &= -\boldsymbol{R}_\mathrm{r}^{dq}\boldsymbol{i}_\mathrm{r}^{dq} + \boldsymbol{l}_\mathrm{r,e}^{dq}\tfrac{\mathrm{d}}{\mathrm{d}t}i_\mathrm{e}^{d} &&= \boldsymbol{L}_\mathrm{r,s}^{dq}\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_\mathrm{s}^{dq} + \boldsymbol{L}_\mathrm{r}^{dq}\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_\mathrm{r}^{dq} + \boldsymbol{l}_\mathrm{e,r}^{dq}\tfrac{\mathrm{d}}{\mathrm{d}t}i_\mathrm{e}^{d},\\
\tfrac{\mathrm{d}}{\mathrm{d}t}\psi_\mathrm{e}^{d} &= u_\mathrm{e}^{d} - R_\mathrm{e}^{d}i_\mathrm{e}^{d} &&= \boldsymbol{l}_\mathrm{e,s}^{dq}\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_\mathrm{s}^{dq} + \boldsymbol{l}_\mathrm{e,r}^{dq}\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_\mathrm{r}^{dq} + L_\mathrm{e}\tfrac{\mathrm{d}}{\mathrm{d}t}i_\mathrm{e}^{d},\\
\tfrac{\mathrm{d}}{\mathrm{d}t}\omega_\mathrm{m} &= \tfrac{1}{\Theta_\mathrm{m}}(m_\mathrm{m}-m_\mathrm{l}), \text{ and } \tfrac{\mathrm{d}}{\mathrm{d}t}\phi_\mathrm{m} = \omega_\mathrm{m}.
\end{aligned}\right\} \quad (9)$$



FEA results confirm expected mutual inductance coupling factors:
$\boldsymbol{L}_\mathrm{s,r}^{dq} = \frac{\kappa_\mathrm{s}}{\kappa_\mathrm{r}}(\boldsymbol{L}_\mathrm{r,s}^{dq})^\top = 12(\boldsymbol{L}_\mathrm{r,s}^{dq})^\top$, $\boldsymbol{l}_\mathrm{s,e}^{dq} = \frac{\kappa_\mathrm{s}}{\kappa_\mathrm{e}}(\boldsymbol{l}_\mathrm{e,s}^{dq})^\top = \frac{4}{3}(\boldsymbol{l}_\mathrm{e,s}^{dq})^\top$, $\boldsymbol{l}_\mathrm{e,r}^{dq} = \frac{\kappa_\mathrm{e}}{\kappa_\mathrm{r}}(\boldsymbol{l}_\mathrm{r,e}^{dq})^\top = 9(\boldsymbol{l}_\mathrm{r,e}^{dq})^\top$.

Simulation results: I/O-linearization based nonlinear current control

The right figure illustrates the time series results for reference current steps [- - -] within the current control system, involving the proposed model and identified parameters.
Two current controllers are compared:
(C_1) Controller considering the damper winding [—]
(C_2) Controller neglecting the damper effects [—]



References

[1] Niklas Monzen, Johannes Rossmann, and Christoph M. Hackl. Derivation of clarke and mutual inductance coupling factors in (d, q)-reference frame for synchronous machines with several multi-phase winding systems. In *Inproceedings of the 2025 IECON Annual conference of the IEEE Industrial Electronics Society*, 2025.

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