Hochschule
München (HM)
University of
Applied Sciences

Laboratory for
Mechatronic
and Renewable
Energy Systems
(LMRES)

Derivation of Clarke and mutual inductance coupling factors in (d, q)-reference frame for synchronous machines with several multi-phase winding systems

Niklas Monzen, Johannes Rossmann, and Christoph M. Hackl
{niklas.monzen, johannes.rossmann, christoph.hackl}@hm.edu

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Motivation

Example: Electrically excited synchronous machine with damper winding

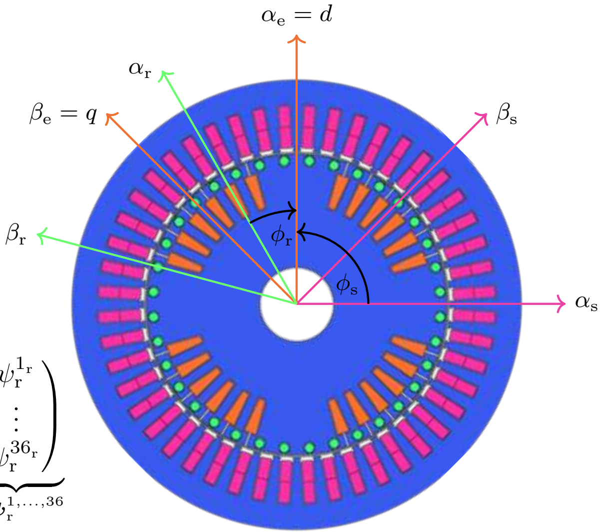
stator winding [■]:

$$\underbrace{\begin{pmatrix} u_\mathrm{s}^a \\ u_\mathrm{s}^b \\ u_\mathrm{s}^c \end{pmatrix}}_{=:\boldsymbol{u}_\mathrm{s}^{abc}} = \underbrace{\begin{bmatrix} R_\mathrm{s}^a & 0 & 0 \\ 0 & R_\mathrm{s}^b & 0 \\ 0 & 0 & R_\mathrm{s}^c \end{bmatrix}}_{=:\boldsymbol{R}_\mathrm{s}^{abc}} \underbrace{\begin{pmatrix} i_\mathrm{s}^a \\ i_\mathrm{s}^b \\ i_\mathrm{s}^c \end{pmatrix}}_{=:\boldsymbol{i}_\mathrm{s}^{abc}} + \frac{\mathrm{d}}{\mathrm{d}t} \underbrace{\begin{pmatrix} \psi_\mathrm{s}^a \\ \psi_\mathrm{s}^b \\ \psi_\mathrm{s}^c \end{pmatrix}}_{=:\boldsymbol{\psi}_\mathrm{s}^{abc}},$$

exciter winding [■]: $u_\mathrm{e} = R_\mathrm{e} i_\mathrm{e} + \frac{\mathrm{d}}{\mathrm{d}t}\psi_\mathrm{e}$

damper winding [■]:

$$\underbrace{\begin{pmatrix} u_\mathrm{r}^{1_\mathrm{r}} \\ \vdots \\ u_\mathrm{r}^{36_\mathrm{r}} \end{pmatrix}}_{=:\boldsymbol{u}_\mathrm{r}^{1,\dots,36}} = \underbrace{\begin{bmatrix} R_\mathrm{r}^{1_\mathrm{r}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & R_\mathrm{r}^{36_\mathrm{r}} \end{bmatrix}}_{=:\boldsymbol{R}_\mathrm{r}^{1,\dots,36}} \underbrace{\begin{pmatrix} i_\mathrm{r}^{1_\mathrm{r}} \\ \vdots \\ i_\mathrm{r}^{36_\mathrm{r}} \end{pmatrix}}_{=:\boldsymbol{i}_\mathrm{r}^{1,\dots,36}} + \frac{\mathrm{d}}{\mathrm{d}t} \underbrace{\begin{pmatrix} \psi_\mathrm{r}^{1_\mathrm{r}} \\ \vdots \\ \psi_\mathrm{r}^{36_\mathrm{r}} \end{pmatrix}}_{=:\boldsymbol{\psi}_\mathrm{r}^{1,\dots,36}}$$



Motivation

Example: Electrically excited synchronous machine with damper winding

With Clarke $\boldsymbol{T}_{\mathrm{c,z}} := \kappa_{\mathrm{z}} \begin{bmatrix} \cos(\phi_{1,\mathrm{z}}) & \cdots & \cos(\phi_{m,\mathrm{z}}) \\ \sin(\phi_{1,\mathrm{z}}) & \cdots & \sin(\phi_{m,\mathrm{z}}) \end{bmatrix}$

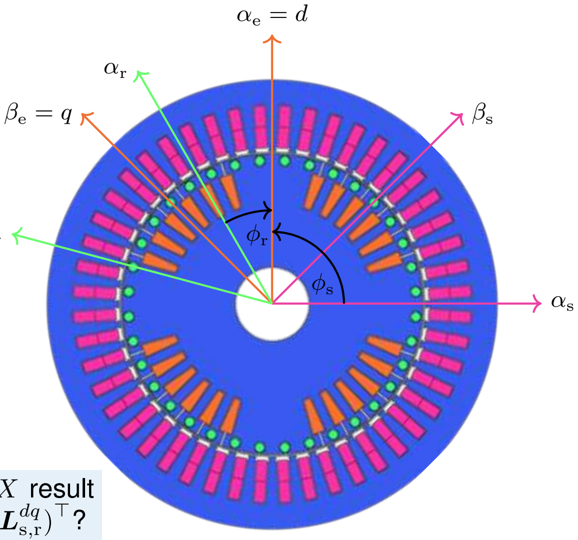
and Park $\boldsymbol{T}_{\mathrm{p}}(\phi_{\mathrm{z}}) := \begin{bmatrix} \cos(\phi_{\mathrm{z}}) & \sin(\phi_{\mathrm{z}}) \\ -\sin(\phi_{\mathrm{z}}) & \cos(\phi_{\mathrm{z}}) \end{bmatrix}$

we get in (d,q)-reference frame

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_{\mathrm{s}}^{dq} = \ldots = \boldsymbol{L}_{\mathrm{s}}^{dq}\,\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_{\mathrm{s}}^{dq} + \boldsymbol{L}_{\mathrm{s,r}}^{dq}\,\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_{\mathrm{r}}^{dq} + \boldsymbol{l}_{\mathrm{s,e}}^{dq}\,\frac{\mathrm{d}}{\mathrm{d}t}i_{\mathrm{e}}^{d},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_{\mathrm{r}}^{dq} = \ldots = \boldsymbol{L}_{\mathrm{r,s}}^{dq}\,\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_{\mathrm{s}}^{dq} + \boldsymbol{L}_{\mathrm{r}}^{dq}\,\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_{\mathrm{r}}^{dq} + \boldsymbol{l}_{\mathrm{e,r}}^{dq}\,\frac{\mathrm{d}}{\mathrm{d}t}i_{\mathrm{e}}^{d},$$

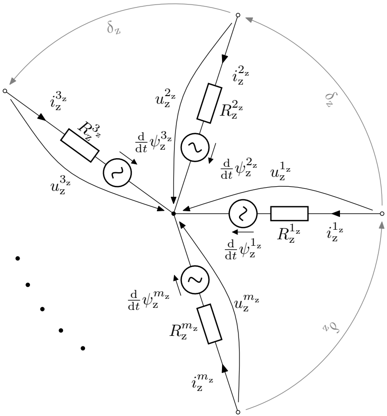
$$\frac{\mathrm{d}}{\mathrm{d}t}\psi_{\mathrm{e}}^{d} = \ldots = \boldsymbol{l}_{\mathrm{e,s}}^{dq}\,\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_{\mathrm{s}}^{dq} + \boldsymbol{l}_{\mathrm{e,r}}^{dq}\,\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_{\mathrm{r}}^{dq} + L_{\mathrm{e}}\,\frac{\mathrm{d}}{\mathrm{d}t}i_{\mathrm{e}}^{d}$$



How to select the **Clarke factors** κ_{z}?

What **mutual inductance coupling factors** X result in orthogonal-reference frames, i.e., $\boldsymbol{L}_{\mathrm{r,s}}^{dq} = X(\boldsymbol{L}_{\mathrm{s,r}}^{dq})^{\top}$?

Derivation of Clarke and mutual inductance coupling factors



$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\begin{pmatrix}\psi_z^{1_z}\\ \vdots \\ \psi_z^{m_z}\end{pmatrix}}_{=:\boldsymbol{\psi}_z^{1_z\cdots m_z}} = \underbrace{\begin{pmatrix}u_z^{1_z}\\ \vdots \\ u_z^{m_z}\end{pmatrix}}_{=:\boldsymbol{u}_z^{1_z\cdots m_z}} - \underbrace{\begin{pmatrix}R_z^{1_z} & \cdots & 0\\ \vdots & \ddots & \vdots \\ 0 & \cdots & R_z^{m_z}\end{pmatrix}}_{=:\boldsymbol{R}_z^{1_z\cdots m_z}}\underbrace{\begin{pmatrix}i_z^{1_z}\\ \vdots \\ i_z^{m_z}\end{pmatrix}}_{=:\boldsymbol{i}_z^{1_z\cdots m_z}}$$

Clarke factor κ_z:

$$\kappa_z = \cdots = \begin{cases}\frac{1}{2}, & \text{if } m_z = 2 \text{ and } \phi_x(t) = 0,\\ \frac{2}{m_z}, & \text{if } m_z > 2.\end{cases}$$

Mutual inductance coupling factor X:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\boldsymbol{\psi}_x^{1_x\cdots m_x}\\ \boldsymbol{\psi}_y^{1_y\cdots m_y}\end{pmatrix} = \begin{bmatrix}\boldsymbol{L}_x^{1_x\cdots m_x} & \boldsymbol{M}_{xy}^{1_x\cdots m_x}\\ \boldsymbol{M}_{yx}^{1_y\cdots m_y} & \boldsymbol{L}_y^{1_y\cdots m_y}\end{bmatrix}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\boldsymbol{i}_x^{1_x\cdots m_x}\\ \boldsymbol{i}_y^{1_y\cdots m_y}\end{pmatrix}$$

with $\boldsymbol{M}_{xy}^{1_x\cdots m_x} = \left(\boldsymbol{M}_{yx}^{1_y\cdots m_y}\right)^\top$

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\boldsymbol{\psi}_x^{\alpha_x\beta_x}\\ \boldsymbol{\psi}_y^{\alpha_y\beta_y}\end{pmatrix} = \begin{bmatrix}\boldsymbol{L}_x^{\alpha_x\beta_x} & \boldsymbol{M}_{xy}^{\alpha_x\beta_x}\\ \boldsymbol{M}_{yx}^{\alpha_y\beta_y} & \boldsymbol{L}_y^{\alpha_y\beta_y}\end{bmatrix}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\boldsymbol{i}_x^{\alpha_x\beta_x}\\ \boldsymbol{i}_y^{\alpha_y\beta_y}\end{pmatrix}$$

with $\boldsymbol{M}_{xy}^{\alpha_x\beta_x} = \underbrace{\frac{\kappa_x}{\kappa_y}}_{=:X}\left(\boldsymbol{M}_{yx}^{\alpha_y\beta_y}\right)^\top$

Simulation results

I/O-linearization based nonlinear current control of EESM with damper winding

- Machine parameters (e.g., flux linkage maps) identified from finite-element-analysis (FEA)
- Nonlinear current control system with input/output (I/O) linearization
- Simulation scenario: reference current steps [- - -] applied
- Two current controllers:
  - (C_1) Controller considering the damper winding [——]
  - (C_2) Controller neglecting the damper effects [——]
- Observations:
  - Damper currents occur during stator and exciter transients
  - Considering damper winding in controller design improves control performance