Transient response shaping and its application to nonlinear PI-based current control systems

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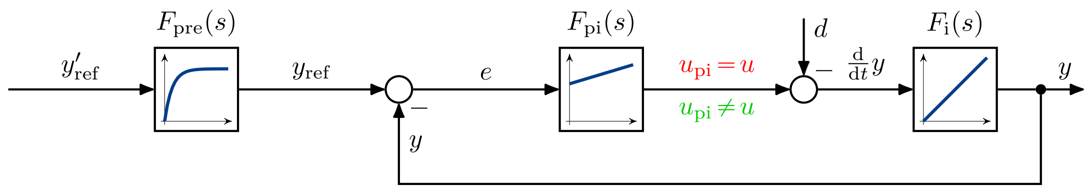
Motivation and problem statement

Example of simple closed-loop system (e.g., current/speed control or phase-locked loop)



Transfer function

$$F_{\mathrm{cl}}(s) := \frac{y(s)}{y_{\mathrm{ref}}(s)} = \frac{\frac{k_{\mathrm{p}}s+k_{\mathrm{i}}}{s}\frac{1}{s}}{1+\frac{k_{\mathrm{p}}s+k_{\mathrm{i}}}{s}\frac{1}{s}} = \frac{1+\frac{k_{\mathrm{p}}}{k_{\mathrm{i}}}s}{1+\frac{k_{\mathrm{p}}}{k_{\mathrm{i}}}s+\frac{1}{k_{\mathrm{i}}}s^2} \tag{1}$$

with prefilter [$\Rightarrow$ slow transient response! or $\Rightarrow$ alternative available?]

$$F'_{\mathrm{cl}}(s) := \frac{y(s)}{y'_{\mathrm{ref}}(s)} := \underbrace{\frac{1}{1+\frac{k_{\mathrm{p}}}{k_{\mathrm{i}}}s}}_{=:F_{\mathrm{pre}}(s)} \cdot F_{\mathrm{cl}}(s) = \frac{1}{1+\frac{k_{\mathrm{p}}}{k_{\mathrm{i}}}s+\frac{1}{k_{\mathrm{i}}}s^2} \tag{2}$$

Transient response shaping (TRS)

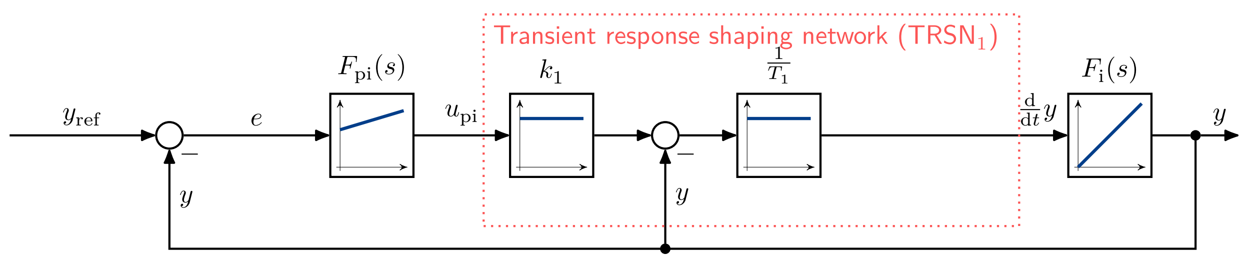
a) First-order lag system behavior by transient response shaping network (TRSN_1)

$\boxed{F_{\mathrm{fols}}(s) = \frac{u(s)}{u_{\mathrm{pi}}(s)} = \frac{k_1}{1+sT_1}}$ with $T_1 \overset{!}{=} \frac{k_{\mathrm{p}}}{k_{\mathrm{i}}}$, $k_1 \overset{!}{=} \frac{1}{k_{\mathrm{i}}T_1^\star}$ and $T_1^\star > 0$ (arbitrary)

achieves desired closed-loop behavior $\boxed{F^\star_{\mathrm{fols}}(s) := \frac{y(s)}{y_{\mathrm{ref}}(s)} := \frac{1}{1+sT_1^\star} \overset{!}{=} F_{\text{cl-fols}}(s)}$, since

$$F_{\text{cl-fols}}(s) = \frac{y(s)}{y_{\mathrm{ref}}(s)} = \frac{F_{\mathrm{pi}}(s)F_{\mathrm{fols}}(s)}{1+F_{\mathrm{pi}}(s)F_{\mathrm{fols}}(s)} = \frac{k_{\mathrm{i}}\frac{\cancel{1+k_{\mathrm{p}}/k_{\mathrm{i}}s}}{s}\frac{k_1}{\cancel{1+sT_1}}}{1+k_{\mathrm{i}}\frac{\cancel{1+k_{\mathrm{p}}/k_{\mathrm{i}}s}}{s}\frac{k_1}{\cancel{1+sT_1}}} = \frac{1}{1+s\frac{1}{k_{\mathrm{i}}k_1}}. \tag{3}$$

Implementation of TRSN_1:



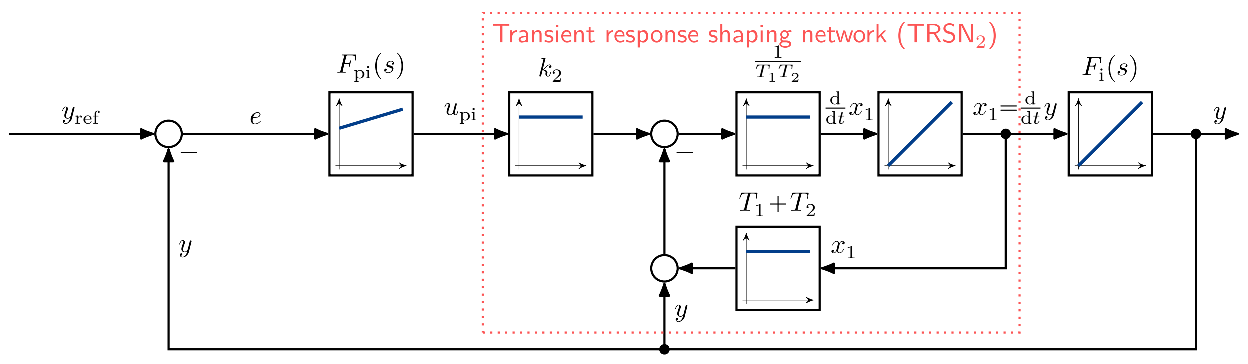
b) Second-order lag system behavior by transient response shaping network (TRSN_2)

$\boxed{F_{\mathrm{sols}}(s) = \frac{u(s)}{u_{\mathrm{pi}}(s)} = \frac{k_2}{(1+sT_1)(1+sT_2)}}$ with $T_1 \overset{!}{=} \frac{k_{\mathrm{p}}}{k_{\mathrm{i}}}$, $k_2 \overset{!}{=} \frac{\omega_0^\star}{2D^\star k_{\mathrm{i}}}$ and $T_2 = \frac{k_{\mathrm{i}}k_2}{(\omega_0^\star)^2} \overset{!}{=} \frac{1}{2D^\star\omega_0^\star}$

achieves desired closed-loop behavior $\boxed{F^\star_{\mathrm{sols}}(s) := \frac{y(s)}{y_{\mathrm{ref}}(s)} := \frac{1}{1+s\frac{2D^\star}{\omega_0^\star}+s^2\frac{1}{(\omega_0^\star)^2}} \overset{!}{=} F_{\text{cl-sols}}(s)}$, since

$$F_{\text{cl-sols}}(s) = \frac{y(s)}{y_{\mathrm{ref}}(s)} = \frac{F_{\mathrm{pi}}(s)F_{\mathrm{sols}}(s)}{1+F_{\mathrm{pi}}(s)F_{\mathrm{sols}}(s)} = \frac{k_{\mathrm{i}}\frac{\cancel{1+k_{\mathrm{p}}/k_{\mathrm{i}}s}}{s}\frac{k_2}{\cancel{(1+sT_1)}(1+sT_2)}}{1+k_{\mathrm{i}}\frac{\cancel{1+k_{\mathrm{p}}/k_{\mathrm{i}}s}}{s}\frac{k_2}{\cancel{(1+sT_1)}(1+sT_2)}} = \frac{1}{1+s\frac{1}{k_{\mathrm{i}}k_2}+s^2\frac{T_2}{k_{\mathrm{i}}k_2}}. \tag{4}$$

Implementation of TRSN_2:



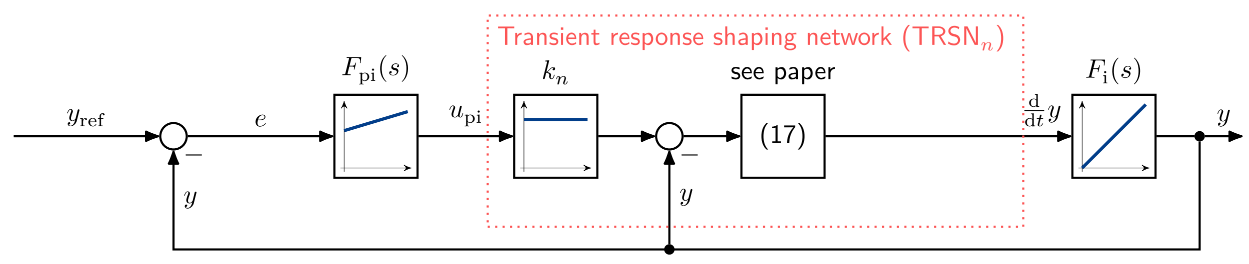
c) nth-order lag system behavior by transient response shaping network (TRSN_n)

$\boxed{F_{\mathrm{nols}}(s) = \frac{u(s)}{u_{\mathrm{pi}}(s)} = \frac{k_n}{(1+sT_1)(1+s\,a_1+\cdots+s^{n-1}\,a_{n-1})}}$ with $T_1 \overset{!}{=} \frac{k_{\mathrm{p}}}{k_{\mathrm{i}}}$, $k_n \overset{!}{=} \frac{1}{k_{\mathrm{i}}a_1^\star} \wedge \forall l \in \{1,\ldots,n{-}1\}\colon a_l = \frac{a_{l+1}^\star}{a_1^\star}$

achieves desired closed-loop behavior $\boxed{F^\star_{\mathrm{nols}}(s) := \frac{y(s)}{y_{\mathrm{ref}}(s)} := \frac{1}{1+s\,a_1^\star+\cdots+s^n\,a_n^\star} \overset{!}{=} F_{\text{cl-nols}}(s)}$, since

$$F_{\text{cl-nols}}(s) = \frac{y(s)}{y_{\mathrm{ref}}(s)} = \frac{F_{\mathrm{pi}}(s)F_{\mathrm{nols}}(s)}{1+F_{\mathrm{pi}}(s)F_{\mathrm{nols}}(s)} = \frac{k_{\mathrm{i}}\frac{\cancel{1+k_{\mathrm{p}}/k_{\mathrm{i}}s}}{s}\frac{k_n}{\cancel{(1+sT_1)}(1+s\,a_1+\cdots+s^{n-1}\,a_{n-1})}}{1+k_{\mathrm{i}}\frac{\cancel{1+k_{\mathrm{p}}/k_{\mathrm{i}}s}}{s}\frac{k_n}{\cancel{(1+sT_1)}(1+sa_1+\cdots+s^{n-1}a_{n-1})}} = \frac{1}{1+s\frac{1}{k_{\mathrm{i}}k_n}+\cdots+s^n\frac{a_{n-1}}{k_{\mathrm{i}}k_n}}.$$

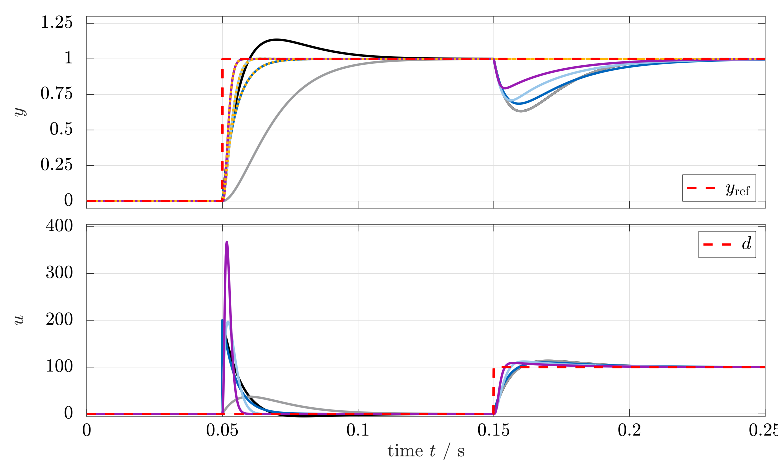
Implementation of TRSN_n:



d) Illustration of achievable closed-loop control performance

Five different controller (C_i) implementations for comparative study:

- (C_1) PI controller [—]
- (C_2) PI controller with prefilter [—]
- (C_3) PI controller with TRSN_1 [—]
- (C_4) PI controller with TRSN_2 [—]
- (C_5) PI controller with TRSN_n [—]



Application: Control of IPMSM

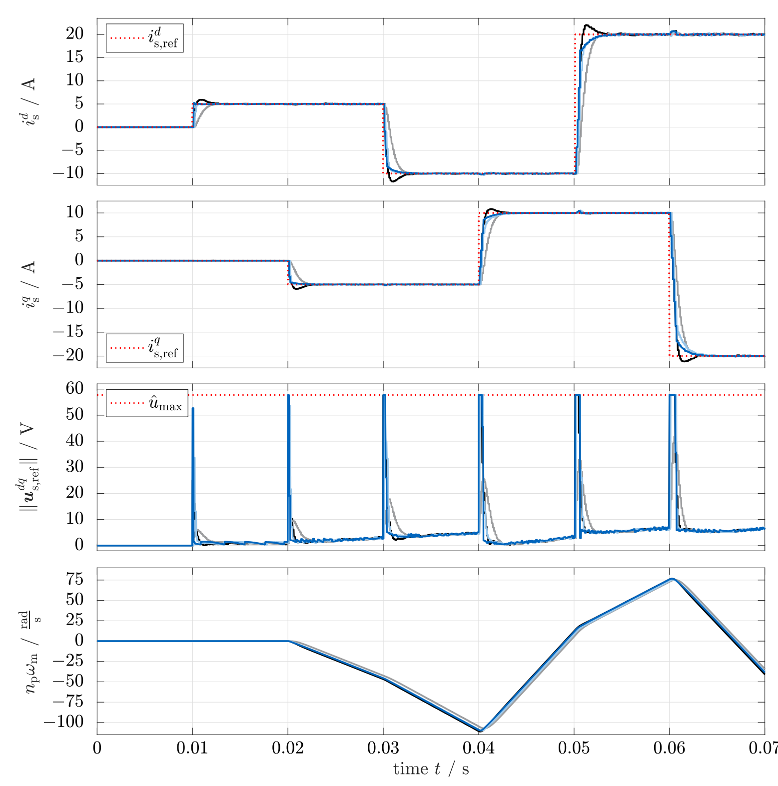
Nonlinear machine dynamics

$$\left.\begin{array}{rcl} \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_{\mathrm{s}}^{dq} & = & (\boldsymbol{L}_{\mathrm{s}}^{dq})^{-1}\left(\boldsymbol{u}_{\mathrm{s}}^{dq} - \boldsymbol{R}_{\mathrm{s}}^{dq}\boldsymbol{i}_{\mathrm{s}}^{dq} - n_{\mathrm{p}}\omega_{\mathrm{m}}\boldsymbol{J}\boldsymbol{\psi}_{\mathrm{s}}^{dq}\right) \\ \frac{\mathrm{d}}{\mathrm{d}t}\omega_{\mathrm{m}} & = & \frac{1}{\Theta_{\mathrm{m}}}\left(m_{\mathrm{m}} - m_{\mathrm{l}}\right), \\ \frac{\mathrm{d}}{\mathrm{d}t}\phi_{\mathrm{m}} & = & \omega_{\mathrm{m}}. \end{array}\right\} \tag{5}$$

Simulative validation (including VSI with SVM)

In right figure, the IPMSM time series plots (including VSI and SVM) are shown for four different controller implementations with tuning as in Tab. I (see paper):

- (C_1) PI controller [—]
- (C_2) PI controller with prefilter [—]
- (C_3) PI controller with TRSN_1 [—]
- (C_4) PI controller with TRSN_2 [—]



References

[1] Christoph M. Hackl. Transient response shaping and its application to nonlinear pi-based current control systems. In *Inproceedings of the 2025 IECON Annual conference of the IEEE Industrial Electronics Society*, 2025.

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