HM LMRES

ISES Institute for Sustainable Energy Systems

Laboratory for Mechatronic and Renewable Energy Systems

Free-wheeling offline and online identification of machine back-EMF harmonics by analytically integrated angle-dependent permanent magnet flux linkage prototype functions

Bernd Pfeifer and Christoph M. Hackl †

Hochschule München (HM) University of Applied Sciences

Modeling

The *nonlinear* stator voltage equation in the (a,b,c)-reference frame and (d,q,γ)-reference frame is given by [2]

$$\begin{aligned} \boldsymbol{u}_{\mathrm{s}}^{abc}(t,\boldsymbol{i}_{\mathrm{s}}^{abc},\phi_{\mathrm{m}}) &= \boldsymbol{R}_{\mathrm{s}}^{abc}\boldsymbol{i}_{\mathrm{s}}^{abc}+\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_{\mathrm{s}}^{abc}(\boldsymbol{i}_{\mathrm{s}}^{abc},\phi_{\mathrm{m}}), \text{ and} \\ \boldsymbol{u}_{\mathrm{s}}^{dq\gamma}(t,\boldsymbol{i}_{\mathrm{s}}^{dq\gamma},\phi_{\mathrm{m}},\phi_{\mathrm{p}}) &= \boldsymbol{R}_{\mathrm{s}}^{dq\gamma}\boldsymbol{i}_{\mathrm{s}}^{dq\gamma}+\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_{\mathrm{s}}^{dq\gamma}(\boldsymbol{i}_{\mathrm{s}}^{dq\gamma},\phi_{\mathrm{m}},\phi_{\mathrm{p}})+\omega_{\mathrm{p}}\boldsymbol{J}_0\boldsymbol{\psi}_{\mathrm{s}}^{dq\gamma}(\boldsymbol{i}_{\mathrm{s}}^{dq\gamma},\phi_{\mathrm{m}},\phi_{\mathrm{p}}). \end{aligned} \tag{1}$$

In the identification scenario, the stator currents becomes $\boldsymbol{i}_{\mathrm{s}}^{abc}=\boldsymbol{i}_{\mathrm{s}}^{dq\gamma}=\boldsymbol{0}_3$, and (1) yields

$$\begin{aligned} \boldsymbol{u}_{\mathrm{s}}^{abc}(t,\boldsymbol{i}_{\mathrm{s}}^{abc},\phi_{\mathrm{m}}) &\overset{\boldsymbol{i}_{\mathrm{s}}^{abc}=\boldsymbol{0}_3}{=} \boldsymbol{u}_{\mathrm{emf}}^{abc}(t,\phi_{\mathrm{m}})=\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_{\mathrm{pm}}^{abc}(\phi_{\mathrm{m}}), \text{ and} \\ \boldsymbol{u}_{\mathrm{s}}^{dq\gamma}(t,\boldsymbol{i}_{\mathrm{s}}^{dq\gamma},\phi_{\mathrm{m}},\phi_{\mathrm{p}}) &\overset{\boldsymbol{i}_{\mathrm{s}}^{dq\gamma}=\boldsymbol{0}_3}{=} \boldsymbol{u}_{\mathrm{emf}}^{dq\gamma}(t,\phi_{\mathrm{m}},\phi_{\mathrm{p}})=\omega_{\mathrm{p}}\boldsymbol{J}_0\boldsymbol{\psi}_{\mathrm{pm}}^{dq\gamma}(\phi_{\mathrm{m}},\phi_{\mathrm{p}})+\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_{\mathrm{pm}}^{dq\gamma}(\phi_{\mathrm{m}},\phi_{\mathrm{p}}). \end{aligned} \tag{2}$$

The time derivative of the permanent magnet flux linkages can be partially derived into

$$\begin{aligned} \tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_{\mathrm{pm}}^{abc}(\phi_{\mathrm{m}}) &= \tfrac{\partial\boldsymbol{\psi}_{\mathrm{pm}}^{abc}(\phi_{\mathrm{m}})}{\partial\phi_{\mathrm{m}}}\tfrac{\mathrm{d}}{\mathrm{d}t}\phi_{\mathrm{m}}=\tfrac{\partial\boldsymbol{\psi}_{\mathrm{pm}}^{abc}(\phi_{\mathrm{m}})}{\partial\phi_{\mathrm{m}}}\omega_{\mathrm{m}}, \text{ and} \\ \tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_{\mathrm{pm}}^{dq\gamma}(\phi_{\mathrm{p}},\phi_{\mathrm{m}}) &= \tfrac{\partial\boldsymbol{\psi}_{\mathrm{pm}}^{dq\gamma}(\phi_{\mathrm{p}},\phi_{\mathrm{m}})}{\partial\phi_{\mathrm{p}}}\tfrac{\mathrm{d}}{\mathrm{d}t}\phi_{\mathrm{p}}+\tfrac{\partial\boldsymbol{\psi}_{\mathrm{pm}}^{dq\gamma}(\phi_{\mathrm{p}},\phi_{\mathrm{m}})}{\partial\phi_{\mathrm{m}}}\tfrac{\mathrm{d}}{\mathrm{d}t}\phi_{\mathrm{m}}=\tfrac{\partial\boldsymbol{\psi}_{\mathrm{pm}}^{dq\gamma}(\phi_{\mathrm{p}},\phi_{\mathrm{m}})}{\partial\phi_{\mathrm{p}}}\omega_{\mathrm{p}}+\tfrac{\partial\boldsymbol{\psi}_{\mathrm{pm}}^{dq\gamma}(\phi_{\mathrm{p}},\phi_{\mathrm{m}})}{\partial\phi_{\mathrm{m}}}\omega_{\mathrm{m}}. \end{aligned} \tag{3}$$

Thus, by inserting (3) into (2), the back-EMF voltages are given by

$$\begin{aligned} \boldsymbol{u}_{\mathrm{emf}}^{abc}(\phi_{\mathrm{m}},\omega_{\mathrm{m}}) &= \tfrac{\partial\boldsymbol{\psi}_{\mathrm{pm}}^{abc}(\phi_{\mathrm{m}})}{\partial\phi_{\mathrm{m}}}\omega_{\mathrm{m}}, \text{ and} \\ \boldsymbol{u}_{\mathrm{emf}}^{dq\gamma}(\phi_{\mathrm{m}},\phi_{\mathrm{p}},\omega_{\mathrm{m}},\omega_{\mathrm{p}}) &= \omega_{\mathrm{p}}\boldsymbol{J}_0\boldsymbol{\psi}_{\mathrm{pm}}^{dq\gamma}(\phi_{\mathrm{m}},\phi_{\mathrm{p}})+\tfrac{\partial\boldsymbol{\psi}_{\mathrm{pm}}^{dq\gamma}(\phi_{\mathrm{p}},\phi_{\mathrm{m}})}{\partial\phi_{\mathrm{p}}}\omega_{\mathrm{p}}+\tfrac{\partial\boldsymbol{\psi}_{\mathrm{pm}}^{dq\gamma}(\phi_{\mathrm{p}},\phi_{\mathrm{m}})}{\partial\phi_{\mathrm{m}}}\omega_{\mathrm{m}}. \end{aligned} \tag{4}$$

The aim is to express the partial derivative $\frac{\partial\boldsymbol{\psi}_{\mathrm{pm}}^{abc}(\phi_{\mathrm{m}})}{\partial\phi_{\mathrm{m}}}$ of the permanent magnet flux linkage with respect to mechanical angle ϕ_{m} analytically as the sum of trigonometric functions [4]

$$\widehat{\boldsymbol{f}}_{\mathrm{emf}}^{abc}(\phi_{\mathrm{m}})=\sum_{\nu\in\mathbb{H}_{\mathrm{emf}}}\begin{pmatrix}\widehat{\Psi}_{\mathrm{emf},\nu}^{a}\cos\left(n_{\mathrm{p}}\nu\phi_{\mathrm{m}}+\widehat{\phi}_{\mathrm{emf},\nu}^{a}\right)\\ \widehat{\Psi}_{\mathrm{emf},\nu}^{b}\cos\left(n_{\mathrm{p}}\nu\phi_{\mathrm{m}}+\widehat{\phi}_{\mathrm{emf},\nu}^{b}\right)\\ \widehat{\Psi}_{\mathrm{emf},\nu}^{c}\cos\left(n_{\mathrm{p}}\nu\phi_{\mathrm{m}}+\widehat{\phi}_{\mathrm{emf},\nu}^{c}\right)\end{pmatrix} \overset{!}{:\approx} \frac{\partial\boldsymbol{\psi}_{\mathrm{pm}}^{abc}(\phi_{\mathrm{m}})}{\partial\phi_{\mathrm{m}}} \tag{5}$$

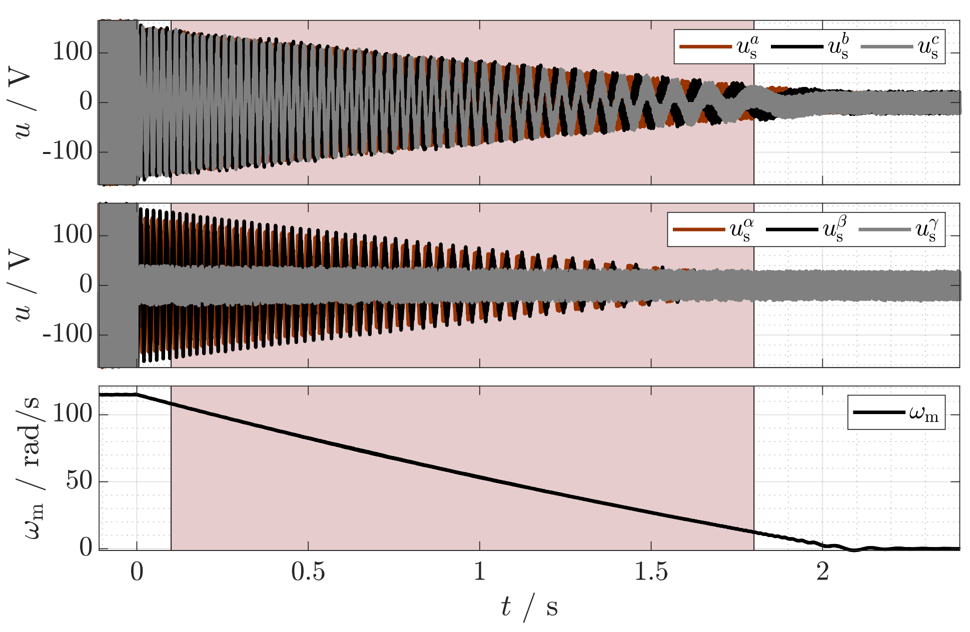
and to identify *all* harmonic orders $\nu\in\mathbb{H}_{\mathrm{emf}}:=\{\nu_1,\nu_2,...,\nu_n\}$, amplitudes $\widehat{\boldsymbol{\Psi}}_{\mathrm{emf},\nu}^{abc}$ and phase offsets $\widehat{\boldsymbol{\phi}}_{\mathrm{emf},\nu}^{abc}$. From (3) and (2), it follows

$$\frac{\partial\boldsymbol{\psi}_{\mathrm{pm}}^{abc}(\phi_{\mathrm{m}})}{\partial\phi_{\mathrm{m}}} = \frac{\boldsymbol{u}_{\mathrm{emf}}^{abc}(t,\phi_{\mathrm{m}})}{\omega_{\mathrm{m}}} \overset{!}{\approx} \widehat{\boldsymbol{f}}_{\mathrm{emf}}^{abc}(\phi_{\mathrm{m}}), \tag{6}$$

where the analytical expression $\widehat{\boldsymbol{f}}_{\mathrm{emf}}^{abc}(\phi_{\mathrm{m}})$ can be parameterized from the quotient of measured back-EMF voltages $\boldsymbol{u}_{\mathrm{emf}}^{abc}(t,\phi_{\mathrm{m}})$ and measured mechanical angular velocity ω_{m}.

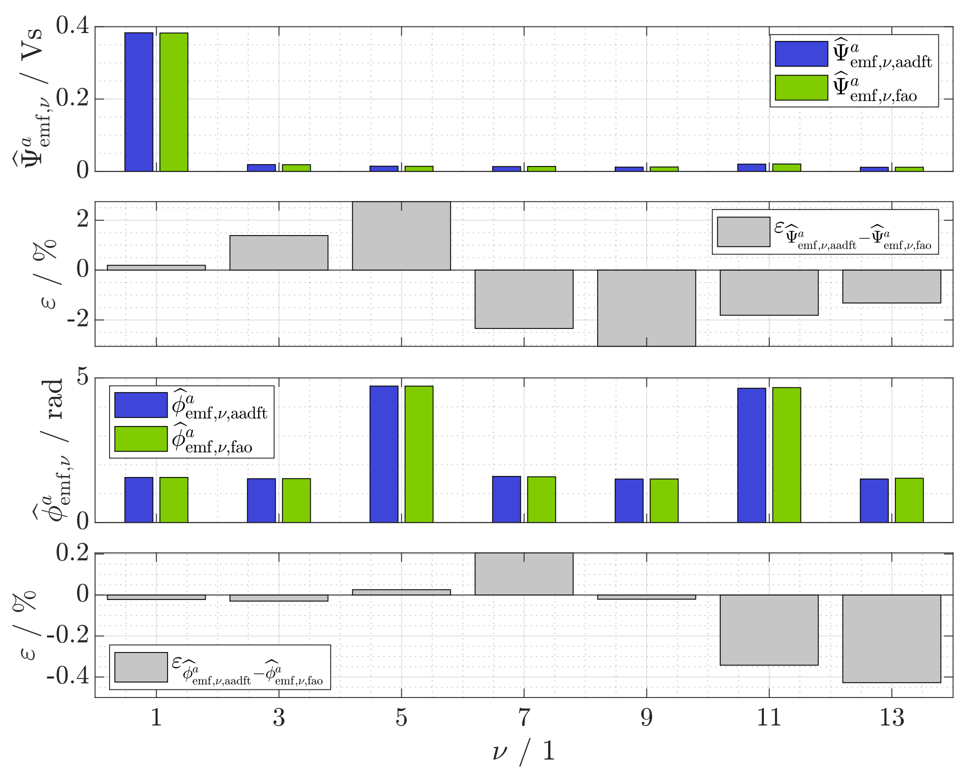
Proposed identification methods

The **Identification scenario** involves accelerating the machine to $\omega_{\mathrm{M}}=115\,\frac{\mathrm{rad}}{\mathrm{s}}$, followed by the opening of the inverter switches at $t=0\,\mathrm{s}$. Subsequently, the harmonic decomposition (6) is conducted during the time window $t=0.1\,\mathrm{s}$ and $t=1.8\,\mathrm{s}$, where $\boldsymbol{i}_{\mathrm{s}}^{abc}=\boldsymbol{0}_3$ (see red-marked region).



The **axes adjusted discrete fourier transformation (AADFT)** method uses the adjustment of x and y axes to achieve constant amplitude and frequency in the measurement time-series signal, allowing the DFT to be correctly applied afterwards.

The **frequency adaptive observer (FAO) [3]** method is capable of online parameter estimation of amplitudes and phase offsets for specified harmonics.



Analytical integration and transformation

Now the analytical expression (5) is integrated to obtain an analytical permanent magnet flux linkage prototype function $\boldsymbol{\psi}_{\mathrm{pm}}^{abc}$. The analytical integration over t is

$$\int\boldsymbol{u}_{\mathrm{emf}}^{abc}(t,\phi_{\mathrm{m}})\,\mathrm{d}t=\int\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_{\mathrm{pm}}^{abc}(\phi_{\mathrm{m}})\,\mathrm{d}t=\int\tfrac{\partial\boldsymbol{\psi}_{\mathrm{pm}}^{abc}(\phi_{\mathrm{m}}(t))}{\partial\phi_{\mathrm{m}}(t)}\tfrac{\mathrm{d}}{\mathrm{d}t}\phi_{\mathrm{m}}(t)\,\mathrm{d}t \overset{\boldsymbol{\psi}_{\mathrm{pm},0}^{abc}=\boldsymbol{0}_3}{=} \boldsymbol{\psi}_{\mathrm{pm}}^{abc}(\phi_{\mathrm{m}}(t)). \tag{7}$$

and with the substitution rule [1, p. 361], (7) can be integrated over ϕ_{m} to

$$\boldsymbol{\psi}_{\mathrm{pm}}^{abc}(\phi_{\mathrm{m}})=\int\tfrac{\partial\boldsymbol{\psi}_{\mathrm{pm}}^{abc}(\phi_{\mathrm{m}})}{\partial\phi_{\mathrm{m}}}\mathrm{d}\phi_{\mathrm{m}} \overset{(5)}{=} \sum_{\nu\in\mathbb{H}_{\mathrm{emf}}}\left[\frac{1}{n_{\mathrm{p}}\nu}\begin{pmatrix}\widehat{\Psi}_{\mathrm{emf},\nu}^{a}\cos\left(n_{\mathrm{p}}\nu\phi_{\mathrm{m}}+\widehat{\phi}_{\mathrm{emf},\nu}^{a}-\frac{\pi}{2}\right)\\ \widehat{\Psi}_{\mathrm{emf},\nu}^{b}\cos\left(n_{\mathrm{p}}\nu\phi_{\mathrm{m}}+\widehat{\phi}_{\mathrm{emf},\nu}^{b}-\frac{\pi}{2}\right)\\ \widehat{\Psi}_{\mathrm{emf},\nu}^{c}\cos\left(n_{\mathrm{p}}\nu\phi_{\mathrm{m}}+\widehat{\phi}_{\mathrm{emf},\nu}^{c}-\frac{\pi}{2}\right)\end{pmatrix}\right]_{\phi_{\mathrm{m}}(t_0)}^{\phi_{\mathrm{m}}(t_1)}. \tag{8}$$

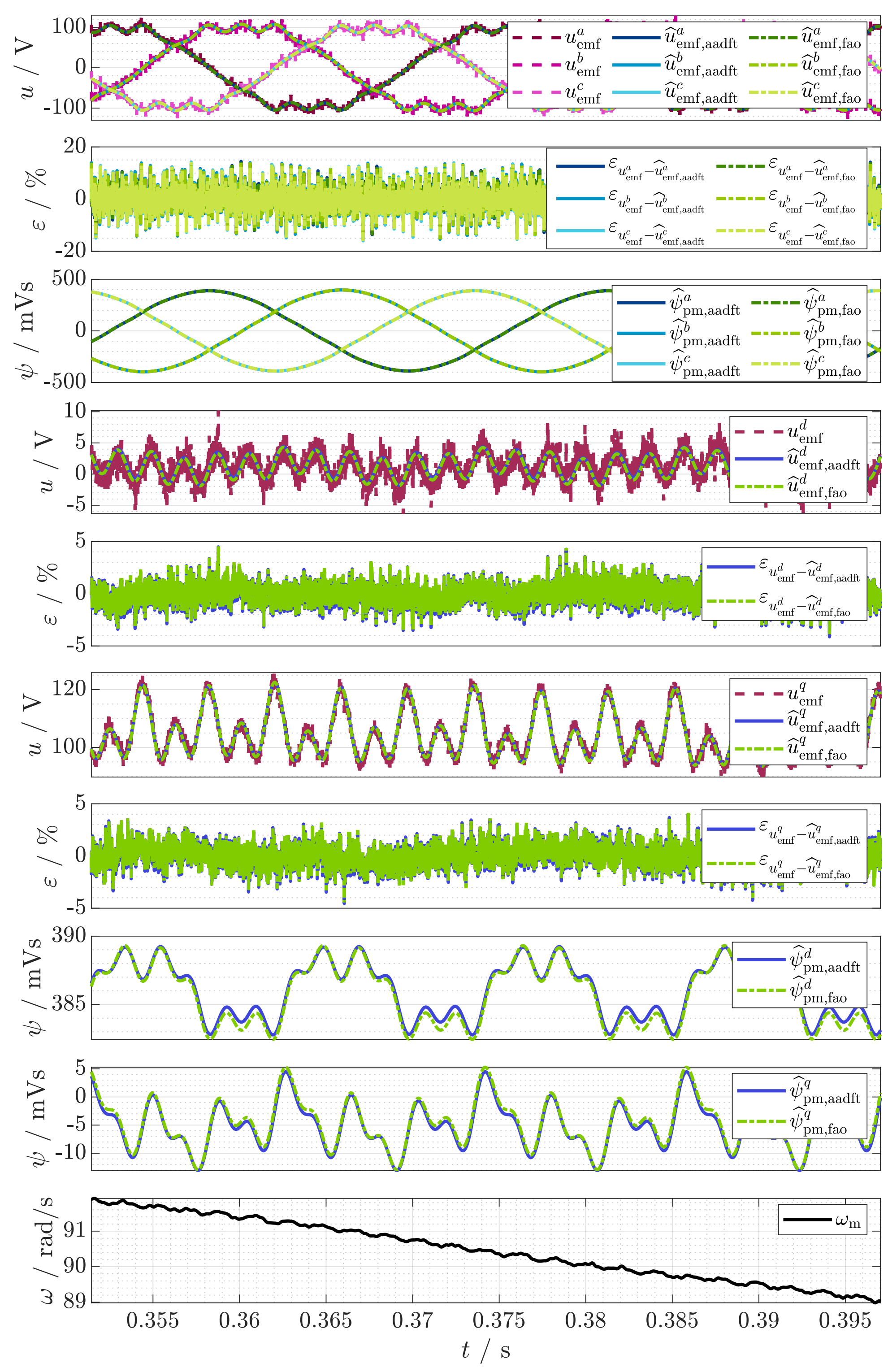
To describe the permanent magnet flux linkage in (d,q,γ)-reference frame, $\boldsymbol{\psi}_{\mathrm{pm}}^{abc}$ as in (8) is analytically transformed with Clarke and Park transformation into

$$\boldsymbol{\psi}_{\mathrm{pm}}^{dq\gamma}(\phi_{\mathrm{p}},\phi_{\mathrm{m}})\overset{!}{=}\sum_{\nu\in\mathbb{H}_{\mathrm{emf}}}\begin{pmatrix}\widehat{\Psi}_{\mathrm{pm},\nu,+}^{d}\cos\left(\phi_{\mathrm{p}}+n_{\mathrm{p}}\nu\phi_{\mathrm{m}}+\widehat{\phi}_{\mathrm{pm},\nu,+}^{d}\right)\\ \widehat{\Psi}_{\mathrm{pm},\nu,+}^{q}\cos\left(\phi_{\mathrm{p}}+n_{\mathrm{p}}\nu\phi_{\mathrm{m}}+\widehat{\phi}_{\mathrm{pm},\nu,+}^{q}\right)\\ \widehat{\Psi}_{\mathrm{pm},\nu,+}^{\gamma}\cos\left(\phi_{\mathrm{p}}+n_{\mathrm{p}}\nu\phi_{\mathrm{m}}+\widehat{\phi}_{\mathrm{pm},\nu,+}^{\gamma}\right)\end{pmatrix}+\sum_{\nu\in\mathbb{H}_{\mathrm{emf}}}\begin{pmatrix}\widehat{\Psi}_{\mathrm{pm},\nu,-}^{d}\cos\left(\phi_{\mathrm{p}}+n_{\mathrm{p}}\nu\phi_{\mathrm{m}}+\widehat{\phi}_{\mathrm{pm},\nu,-}^{d}\right)\\ \widehat{\Psi}_{\mathrm{pm},\nu,-}^{q}\cos\left(\phi_{\mathrm{p}}+n_{\mathrm{p}}\nu\phi_{\mathrm{m}}+\widehat{\phi}_{\mathrm{pm},\nu,-}^{q}\right)\\ \widehat{\Psi}_{\mathrm{pm},\nu,-}^{\gamma}\cos\left(\phi_{\mathrm{p}}+n_{\mathrm{p}}\nu\phi_{\mathrm{m}}+\widehat{\phi}_{\mathrm{pm},\nu,-}^{\gamma}\right)\end{pmatrix}. \tag{9}$$

Please note that for a generic modeling where ϕ_{p} is an independent variable, the Park transformation angle ϕ_{p} and mechanical angle ϕ_{m} remain separate. It can be seen that $\phi_{\mathrm{p}}+n_{\mathrm{p}}\nu\phi_{\mathrm{m}}$ and $\phi_{\mathrm{p}}-n_{\mathrm{p}}\nu\phi_{\mathrm{m}}$ correspond to the positive and negative sequence, respectively.

Experimental Validation

The experimental validation was performed for an interior permanent magnet synchronous machine controlled by a dSpace real-time system. The measured signals $\boldsymbol{u}_{\mathrm{s}}^{abc}$, ϕ_{m}, and ω_{m} were recorded with a Tektronix MSO58 oscilloscope. In order to obtain the identified permanent magnet flux linkages the measured signals were reconstructed with (4). The relative errors of $\boldsymbol{u}_{\mathrm{emf,aadft}}^{abc}$ and $\boldsymbol{u}_{\mathrm{emf,fao}}^{abc}$ against $\boldsymbol{u}_{\mathrm{emf}}^{abc}$ with respect to the peak voltages are displayed in the second subplot, with a maximum error of $15.98\,\%$ for AADFT and maximum error of $16\,\%$ for FAO. The relative errors of $\boldsymbol{u}_{\mathrm{emf,aadft}}^{dq\gamma}$ and $\boldsymbol{u}_{\mathrm{emf,fao}}^{dq\gamma}$ against $\boldsymbol{u}_{\mathrm{emf}}^{dq\gamma}$ with respect to the peak voltages are displayed in the fifth and seventh subplots, with a maximum error of $4.55\,\%$ for AADFT and maximum error of $4.62\,\%$ for FAO.



References

[1] William L. Briggs and Lyle Cochran. *Calculus*. Addison-Wesley, Boston, Mass. [u.a.], 2011.

[2] Christoph Hackl, Julian Kullick, and Niklas Monzen. Generic loss minimization for nonlinear synchronous machines by analytical computation of optimal reference currents considering copper and iron losses. In *2021 22nd IEEE International Conference on Industrial Technology (ICIT)*, pages 1348–1355. IEEE, March 2021.

[3] Christoph M. Hackl and Markus Landerer. Modified second-order generalized integrators with modified frequency locked loop for fast harmonics estimation of distorted single-phase signals. *IEEE Transactions on Power Electronics*, 35(3):3298–3309, March 2020.

[4] Jian Zhang, Xuhui Wen, and Meng Peng. Nonlinear modeling of interior permanent magnet synchronous motors considering magnetic field harmonics and non-sinusoidal winding distribution. In *2018 21st International Conference on Electrical Machines and Systems (ICEMS)*, pages 344–349. IEEE, October 2018.

† Corresponding author: Prof. Dr.-Ing. habil. Christoph M. Hackl +49 (0)89 1265-3417 christoph.hackl@hm.edu https://lmres.ee.hm.edu