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Mechatronic
and Renewable
Energy Systems
(LMRES)

ISIE 2023: Nonlinear Three-Phase Reluctance Synchronous Machine Modeling With Extended Torque Equation

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Outline I

1 Introduction

2 Machine Modelling

3 Validation

4 Conclusion

Outline I

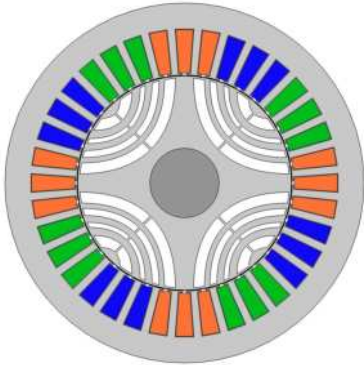
1 Introduction

Introduction

Reluctance synchronous machines

General issue and motivation

⇒ RSMs produce high torque ripple

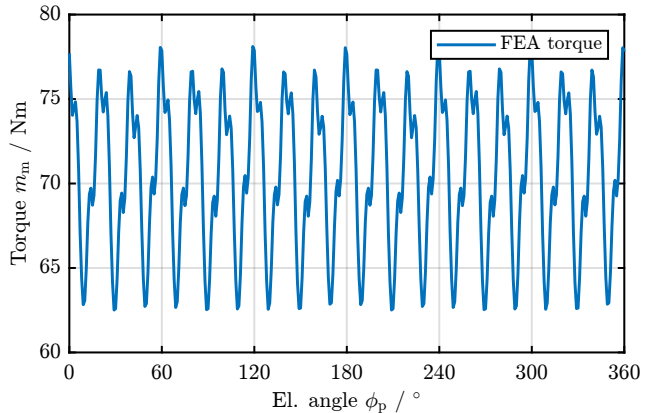
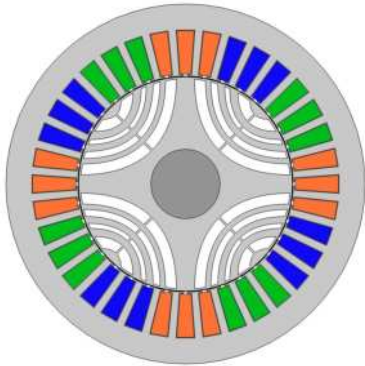


Introduction

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Introduction

Classic torque calculation

Classic torque equation in dq -reference frame:

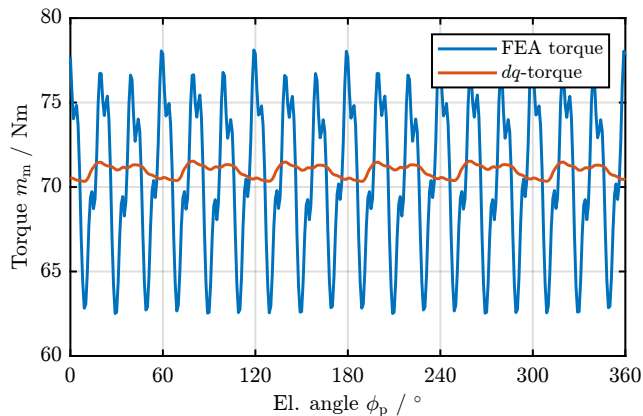
- $m_m = \frac{2n_p}{3\kappa^2} (\mathbf{i}_s^{dq})^\top \mathbf{J} \boldsymbol{\psi}_s^{dq}$
- No iron losses included
- No γ -component (or 0-component) included
- Nonlinearities are mostly neglected
- dq -torque corresponds to mean torque

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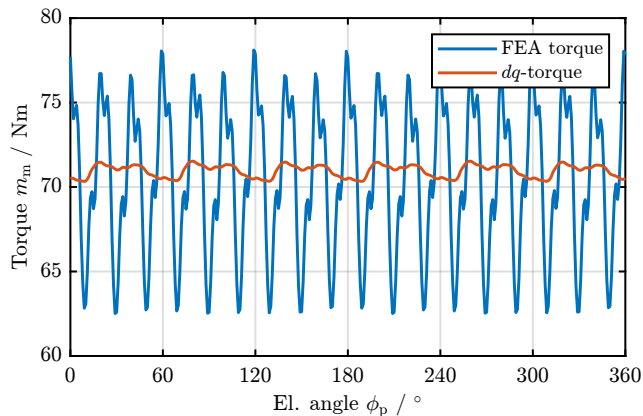


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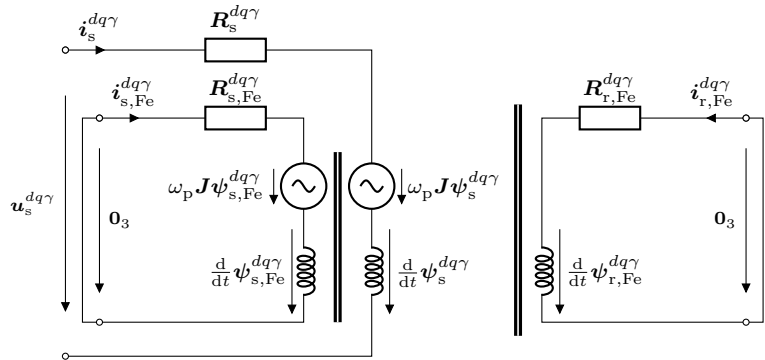
⇒ Nonlinear model required, which covers real torque and also losses and γ -component

Outline I

2 Machine Modelling

Machine Modeling

Extended transformer-like electrical model

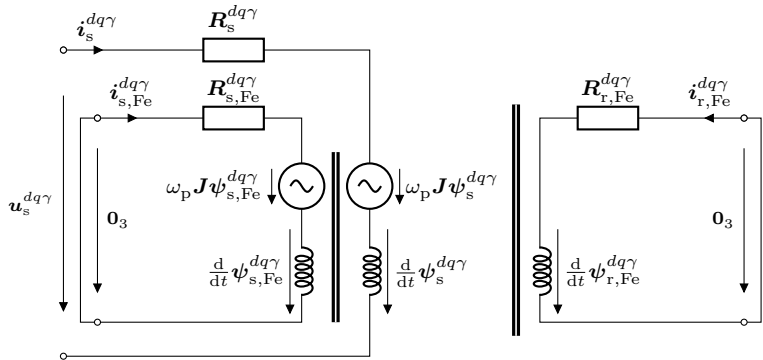


Machine Modeling

Extended transformer-like electrical model

Equivalent circuit of a RSM:

- Synchronous operation
- Iron losses in separate stator and rotor circuit
- Circuits are coupled inductively
- Resistances and fluxes depend on several inputs



Machine Modeling

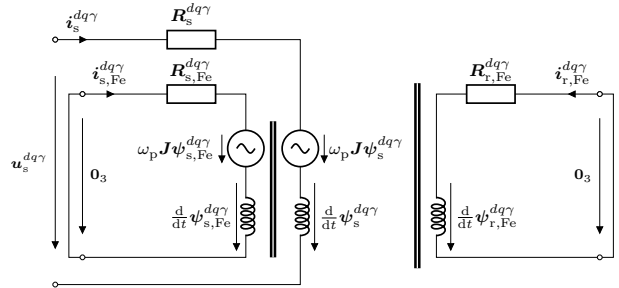
Extended transformer-like electrical model

Electric equations

$$\mathbf{u}_s^{dq\gamma} = \mathbf{R}_s^{dq\gamma} \mathbf{i}_s^{dq\gamma} + \omega_p \mathbf{J} \psi_s^{dq\gamma} + \frac{d}{dt} \psi_s^{dq\gamma}$$

$$\mathbf{0}_3 = \mathbf{R}_{s,Fe}^{dq\gamma} \mathbf{i}_{s,Fe}^{dq\gamma} + \omega_p \mathbf{J} \psi_{s,Fe}^{dq\gamma} + \frac{d}{dt} \psi_{s,Fe}^{dq\gamma}$$

$$\mathbf{0}_3 = \mathbf{R}_{r,Fe}^{dq\gamma} \mathbf{i}_{r,Fe}^{dq\gamma} + \frac{d}{dt} \psi_{r,Fe}^{dq\gamma}$$



Machine Modeling

Extended transformer-like electrical model

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$$\mathbf{0}_3 = \mathbf{R}_{r,Fe}^{dq\gamma} \mathbf{i}_{r,Fe}^{dq\gamma} + \frac{d}{dt} \psi_s^{dq\gamma}$$

General maps:

- Stator flux linkage

$$\psi_s^{dq\gamma} = \psi_s^{dq\gamma}(\mathbf{i}_s^{dq\gamma}, \omega_p, \phi_p)$$

- Stator winding resistances

$$\mathbf{R}_s^{dq\gamma} = \mathbf{R}_s^{dq\gamma}(\mathbf{i}_s^{dq\gamma}, \omega_p, \vartheta_s, \phi_p)$$

- Stator iron resistances

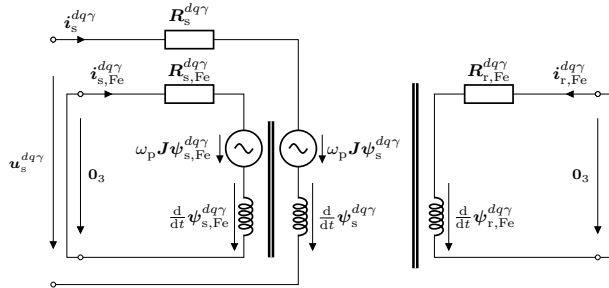
$$\mathbf{R}_{s,Fe}^{dq\gamma} = \mathbf{R}_{s,Fe}^{dq\gamma}(\mathbf{i}_s^{dq\gamma}, \omega_p, \vartheta_{s,Fe}, \phi_p)$$

- Rotor iron resistances

$$\mathbf{R}_{r,Fe}^{dq\gamma} = \mathbf{R}_{r,Fe}^{dq\gamma}(\mathbf{i}_s^{dq\gamma}, \omega_p, \vartheta_{r,Fe}, \phi_p)$$

Assumption: flux leakage is neglected

$$\Rightarrow \psi_s^{dq\gamma} = \psi_{s,Fe}^{dq\gamma} = \psi_{r,Fe}^{dq\gamma}$$



Machine Modeling

Extended mechanical model

Derivation of the extended mechanical model:

Conservation of power:

$$p_{\text{el}} = p_{\text{m}} + \underbrace{p_{\text{s,Cu}} + p_{\text{s,Fe}} + p_{\text{r,Fe}}}_{=: p_{\text{loss}}} + p_{\text{mag}}$$

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with

- $p_{\text{el}} = \frac{2}{3\kappa^2} (\mathbf{u}_{\text{s}}^{dq\gamma})^\top \mathbf{i}_{\text{s}}^{dq\gamma}$
- $p_{\text{m}} = m_{\text{m}} \omega_{\text{m}}$
- $p_{\text{s,Cu}} = \frac{2}{3\kappa^2} (\mathbf{i}_{\text{s}}^{dq\gamma})^\top \mathbf{R}_{\text{s}}^{dq\gamma} \mathbf{i}_{\text{s}}^{dq\gamma}$
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- $p_{\text{r,Fe}} = \frac{2}{3\kappa^2} (\mathbf{i}_{\text{r,Fe}}^{dq\gamma})^\top \mathbf{R}_{\text{r,Fe}}^{dq\gamma} \mathbf{i}_{\text{r,Fe}}^{dq\gamma}$
- $p_{\text{mag}} = \frac{dW_{\text{mag}}}{dt}$

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- $p_{\text{el}} = \frac{2}{3\kappa^2} (\mathbf{i}_{\text{s}}^{dq\gamma})^\top \dot{\mathbf{i}}_{\text{s}}^{dq\gamma}$
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- $p_{\text{s,Fe}} = \frac{2}{3\kappa^2} (\dot{\mathbf{i}}_{\text{s,Fe}}^{dq\gamma})^\top \mathbf{R}_{\text{s,Fe}}^{dq\gamma} \dot{\mathbf{i}}_{\text{s,Fe}}^{dq\gamma}$
- $p_{\text{r,Fe}} = \frac{2}{3\kappa^2} (\dot{\mathbf{i}}_{\text{r,Fe}}^{dq\gamma})^\top \mathbf{R}_{\text{r,Fe}}^{dq\gamma} \dot{\mathbf{i}}_{\text{r,Fe}}^{dq\gamma}$
- $p_{\text{mag}} = \frac{dW_{\text{mag}}}{dt}$



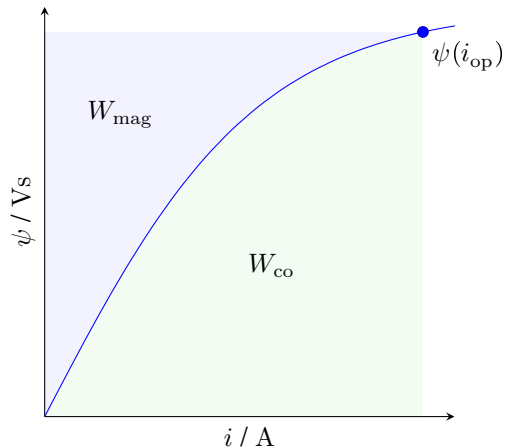
$$\begin{aligned} m_{\text{m}} &= \frac{2n_{\text{p}}}{3\kappa^2} (\mathbf{i}_{\text{s}}^{dq\gamma} + \mathbf{i}_{\text{s,Fe}}^{dq\gamma})^\top \mathbf{J} \psi_{\text{s}}^{dq\gamma} \\ &+ \frac{2}{3\kappa^2} (\dot{\mathbf{i}}_{\text{s}}^{dq\gamma} + \dot{\mathbf{i}}_{\text{s,Fe}}^{dq\gamma} + \dot{\mathbf{i}}_{\text{r,Fe}}^{dq\gamma})^\top \frac{d\psi_{\text{s}}^{dq\gamma}}{d\phi_{\text{m}}} \\ &- \frac{dW_{\text{mag}}}{d\phi_{\text{m}}} \end{aligned}$$

Machine Modeling

Extended mechanical model

Torque depends on magnetic energy

$$\blacksquare W_{\text{mag}}(\psi_s^{dq\gamma}) := \frac{2}{3\kappa^2} \int_{\mathbf{0}_3}^{\psi_s^{dq\gamma}} \mathbf{i}_s^{dq\gamma} (\tilde{\psi}_s^{dq\gamma})^\top d\tilde{\psi}_s^{dq\gamma}$$



Machine Modeling

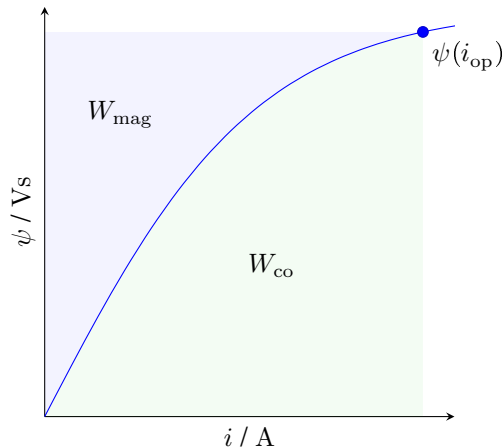
Extended mechanical model

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More practicable: magnetic co-energy

$$\blacksquare W_{\text{co}}(\mathbf{i}_s^{dq\gamma}) := \frac{2}{3\kappa^2} \int_{\mathbf{0}_3}^{\mathbf{i}_s^{dq\gamma}} \psi_s^{dq\gamma} (\tilde{\mathbf{i}}_s^{dq\gamma})^\top d\tilde{\mathbf{i}}_s^{dq\gamma}$$



Machine Modeling

Extended mechanical model

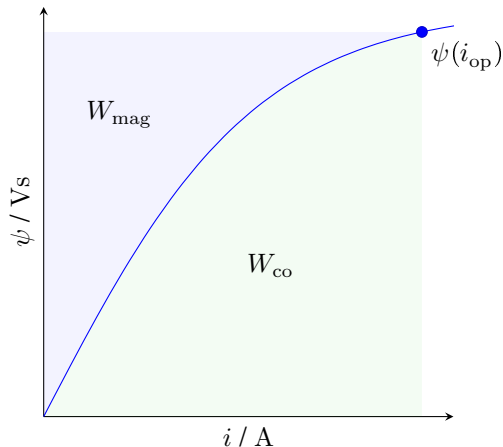
Torque depends on magnetic energy

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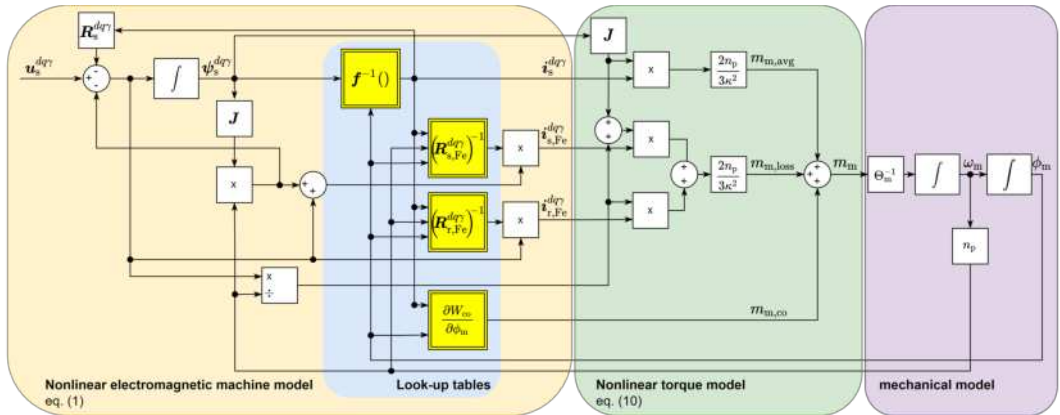
$$\blacksquare W_{\text{co}}(\mathbf{i}_s^{dq\gamma}) := \frac{2}{3\kappa^2} \int_{\mathbf{0}_3}^{\mathbf{i}_s^{dq\gamma}} \psi_s^{dq\gamma} (\tilde{\mathbf{i}}_s^{dq\gamma})^\top d\tilde{\mathbf{i}}_s^{dq\gamma}$$

$$\Rightarrow m_m = \underbrace{\frac{2n_p}{3\kappa^2} (\mathbf{i}_s^{dq\gamma})^\top \mathbf{J} \psi_s^{dq\gamma}}_{=: m_{m,\text{avg}}} + \underbrace{\frac{2}{3\kappa^2} (n_p (\mathbf{i}_{s,\text{Fe}}^{dq\gamma})^\top \mathbf{J} \psi_s^{dq\gamma} + (\mathbf{i}_{s,\text{Fe}}^{dq\gamma} + \mathbf{i}_{r,\text{Fe}}^{dq\gamma})^\top \frac{d\psi_s^{dq\gamma}}{d\phi_m})}_{=: m_{m,\text{loss}}} + \underbrace{\frac{\partial W_{\text{co}}}{\partial \phi_m}}_{=: m_{m,\text{co}}}$$



Machine Modeling

Complete model



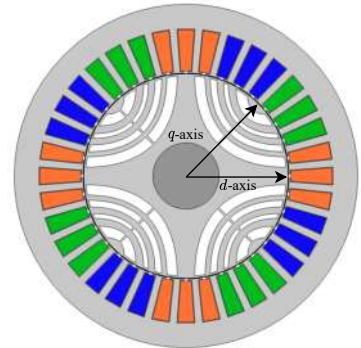
Outline I

3 Validation

Validation

Investigated motor

Parameter	Symbol	Value
Slots	Q_s	36
Pole pairs	n_p	2
Flux barriers	n_{fb}	4
Least common multiple (lcm)		36
Rated mechanical power	$p_{m,R}$	30 kW
Rated mechanical speed	$n_{m,R}$	3 600 rpm
Rated mechanical torque	$m_{m,R}$	80 Nm
Maximum current	$\hat{i}_{max}$	50 A
Stator winding resistance	$R_s^{dq\gamma}$	$0.12 I_3 \Omega$
Nonlinear flux linkages	$\psi_s^{dq\gamma}$	Maps
Nonlinear stator iron losses	$p_{s,Fe}, R_{s,Fe}$	Maps
Nonlinear rotor iron losses	$p_{r,Fe}, R_{r,Fe}$	Maps
Nonlinear co-energy	W_{co}	Maps



Validation

Map generation

Map generation with FEA:

- Simulation with COMSOL Multiphysics
- Loss maps are not angle dependent
- Impact of eddy currents on magn. energy neglected

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Maps:

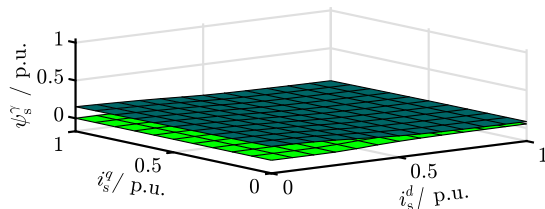
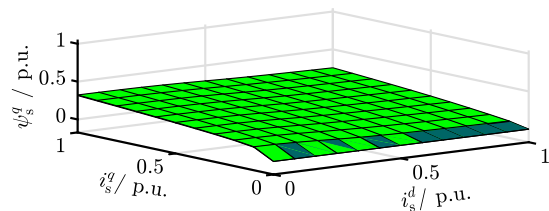
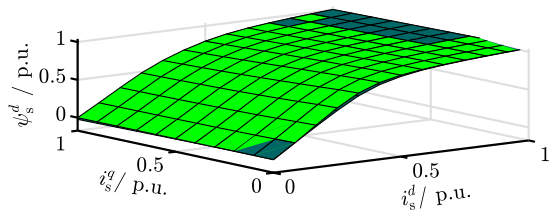
- $\psi_s^{dq\gamma} = \psi_s^{dq\gamma}(i_s^d, i_s^q, i_s^\gamma, \phi_p)$
- $p_{s,Fe} = p_{s,Fe}(i_s^d, i_s^q, i_s^\gamma, \omega_p)$
- $p_{r,Fe} = p_{r,Fe}(i_s^d, i_s^q, i_s^\gamma, \omega_p)$
- $W_{co} = W_{co}(i_s^d, i_s^q, i_s^\gamma, \phi_p)$
- $i_s^{dq\gamma}$ -mesh: $13 \times 11 \times 9$

with:

- $\phi_p \in \{x \in \mathbb{N}_0^+ \mid 0^\circ \text{ el.} \leq x \leq 360^\circ \text{ el.}\}$
- $i_s^d \in \{x \in \mathbb{R}_0^+ \mid 0 \text{ p.u.} \leq x \leq 1 \text{ p.u.}\}$
- $i_s^q \in \{x \in \mathbb{R}_0^+ \mid 0 \text{ p.u.} \leq x \leq 1 \text{ p.u.}\}$
- $i_s^\gamma \in \{x \in \mathbb{R} \mid -0.6 \text{ p.u.} \leq x \leq 0.6 \text{ p.u.}\}$

Validation

Flux linkage maps



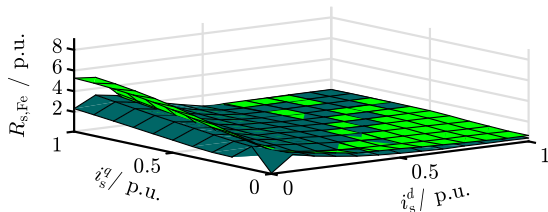
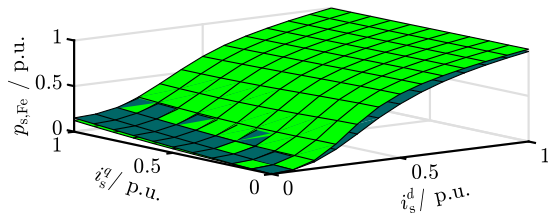
$i_s^\gamma = 0$ p.u. [light green] $i_s^\gamma = 0.6$ p.u. [dark green] at $\phi_p = 0^\circ$

at rated speed

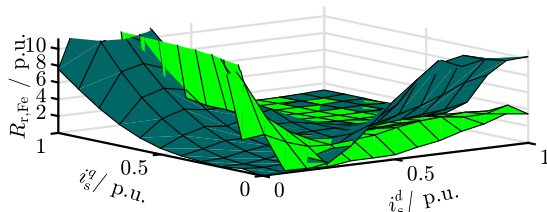
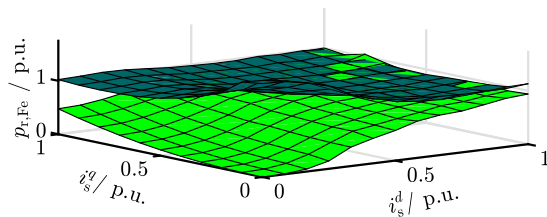
Validation

Iron loss maps

Stator loss maps:



Rotor loss maps:



$i_s^d = 0 \text{ p.u.}$ [light green] $i_s^d = 0.6 \text{ p.u.}$ [dark green] at $\phi_p = 0^\circ$ at rated speed

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Co-energy map

Co-energy map:

- Co-energy can be obtained directly by FEA

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- Co-energy can be obtained directly by FEA
- Analytic calculation is also possible

$$W_{\text{co},\text{f}}(i_s^{dq\gamma}) = \frac{2}{3\kappa^2} \left(\int_{i_s^d=0}^{\hat{i}_s^d} \psi_s^d(\hat{i}_s^d, i_s^{q\gamma} = 0_2) d\hat{i}_s^d + \int_{i_s^q=0}^{\hat{i}_s^q} \psi_s^q(\hat{i}_s^d, \hat{i}_s^q, i_s^\gamma = 0) d\hat{i}_s^q + \int_{i_s^\gamma=0}^{\hat{i}_s^\gamma} \psi_s^\gamma(\hat{i}_s^{dq}, \hat{i}_s^\gamma) d\hat{i}_s^\gamma \right)$$

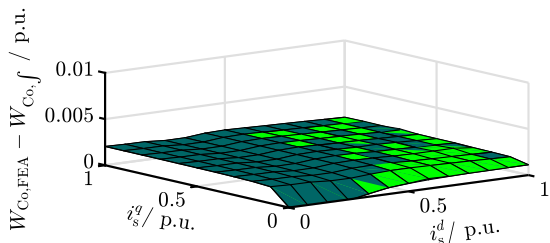
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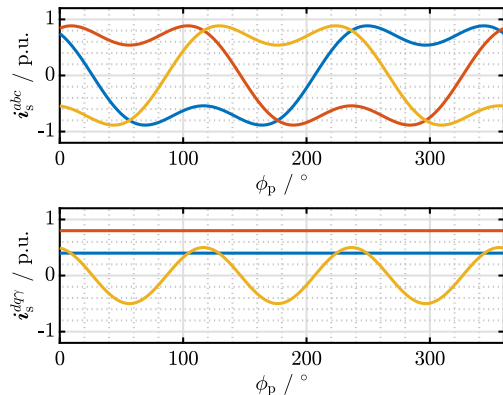
$$W_{\text{Co},f}(i_s^{dq\gamma}) = \frac{2}{3\kappa^2} \left(\int_{i_s^d=0}^{\hat{i}_s^d} \psi_s^d(\hat{i}_s^d, i_s^{q\gamma} = 0_2) d\hat{i}_s^d + \int_{i_s^q=0}^{\hat{i}_s^q} \psi_s^q(\hat{i}_s^d, \hat{i}_s^q, i_s^\gamma = 0) d\hat{i}_s^q + \int_{i_s^\gamma=0}^{\hat{i}_s^\gamma} \psi_s^\gamma(\hat{i}_s^{dq}, \hat{i}_s^\gamma) d\hat{i}_s^\gamma \right)$$



$i_s^\gamma = 0$ p.u. [red] $i_s^\gamma = 0.6$ p.u. [blue] at $\phi_p = 0^\circ$

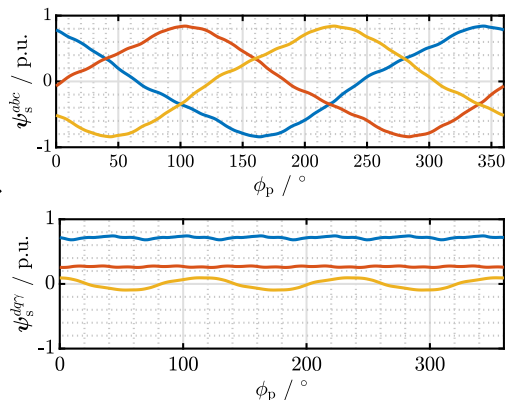
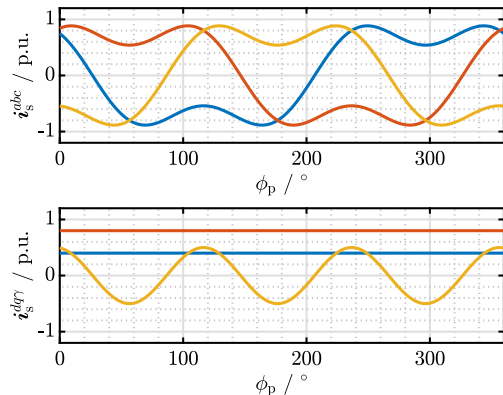
Validation

Simulation scenario



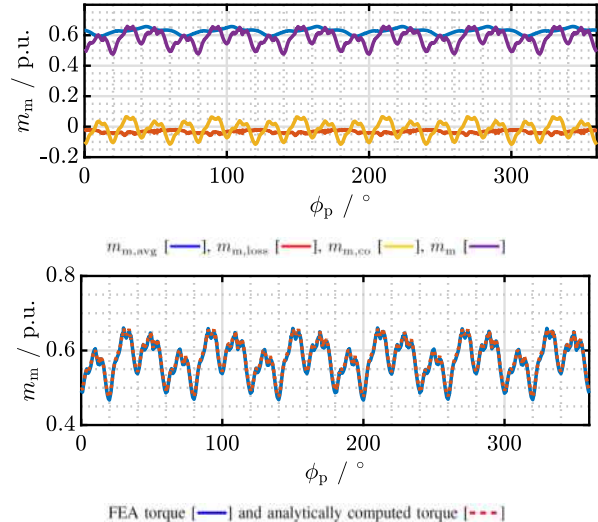
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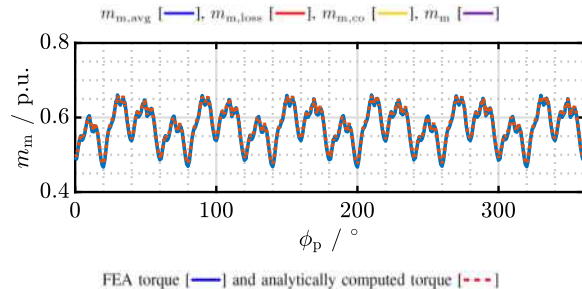
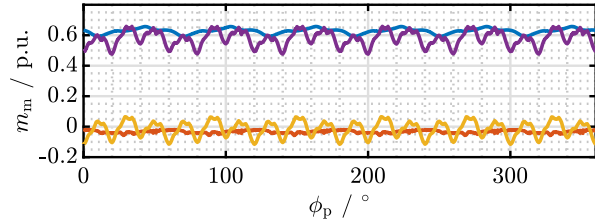
Torque comparison and conclusion



Validation

Torque comparison and conclusion

- Analytic torque comprises three parts:
 - Mean torque
 - Loss torque
 - Co-energy torque
- Analytic torque equals FEA torque
- Maximum error < 0.01 p.u.



Outline I

4 Conclusion

Conclusion

Summary and Outlook

To take home:

- ✓ Unified electromagnetic and torque model
- ✓ Saturation and cross-coupling included
- ✓ New model covers iron losses and γ -component
- ✓ Torque ripple represented correctly

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Next steps:

- 🔧 Consideration of eddy currents on magnetic energy
- 🔧 Consideration of leakage flux linkages
- 🔧 Measurement of angle dependent flux maps on testbench