Hochschule
München
University of
Applied Sciences

Laboratory for
Mechatronic
and Renewable
Energy Systems
(LMRES)

Optimal operation of electrical drives

A complete framework to minimize copper & iron losses

Christoph M. Hackl
(with H. Eldeeb, L. Horlbeck, J. Kullick, N. Monzen and A. Thommessen)
christoph.hackl@hm.edu

08.09.2021 – IEEE Conference “Advances in Electrical Machines, Drives & Vehicles”

ISES

Outline

Outline

Introduction

HM (www.hm.edu), ISES (ises.hm.edu) and LMRES (lmres.ee.hm.edu)

HM – Munich University of Applied Sciences

- 14 departmens
- >18.000 students
- ~480 professors
- 3 campuses

ISES – Institute for Sustainable Energy Systems

- 5 research labs
- 5 professors
- 25 PhD candidates

LMRES – Laboratory for Mechatronic and Renewable Energy Systems

Outline

2 Motivation

Motivation

Electrical machines: Widely-used, compact and efficient actuators

www.farmwood.co.uk

www.abb.com (from video)

www.bmw.de

www.siemens.com

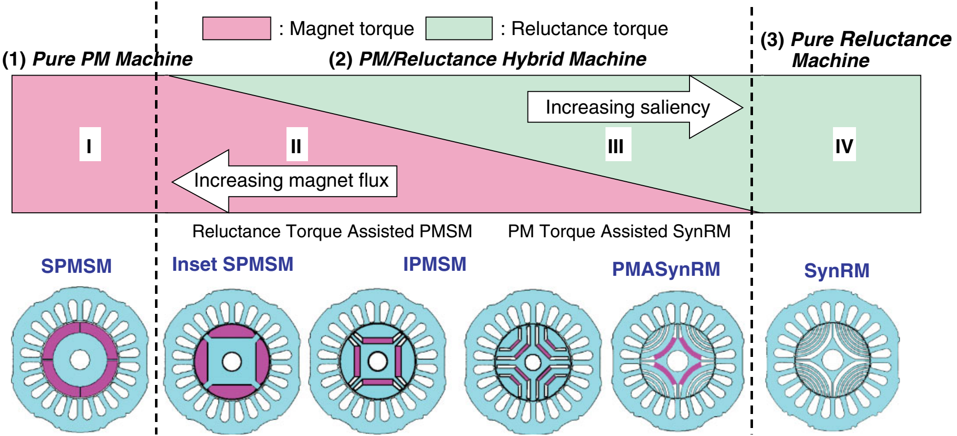
www.lilium.com

...

- buildings (e.g. air conditioning or fans)
- industrial applications (e.g. pumps)
- traction drives (e.g. electric vehicles, trains, etc.)
- power generation (e.g. conventional power plants or wind turbine systems)
- (more) electric aircrafts

Motivation

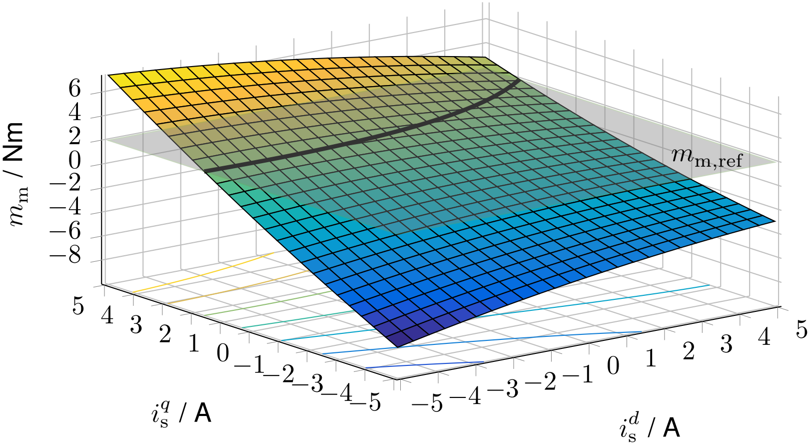
Examples of anisotropic synchronous machines [1] with “saliency ratio” $L_s^d / L_s^q \neq 1$



$$m_m(i_s^d, i_s^q) = \frac{2n_p}{3\kappa^2} (\boldsymbol{i}_s^{dq})^\top \boldsymbol{J} \boldsymbol{\psi}_s^{dq} \overset{\text{(const. Para.)}}{=} \frac{2n_p}{3\kappa^2} \Big[\underbrace{\widetilde{\psi}_{pm} i_s^q}_{\text{magnetic torque}} + \underbrace{(\widetilde{L}_s^d - \widetilde{L}_s^q) i_s^d i_s^q}_{\text{reluctance torque}} + \underbrace{\widetilde{L}_{s,m}\big((i_s^q)^2 - (i_s^d)^2\big)}_{\text{cross-coupling torque}} \Big]$$

Motivation

Optimal feedforward torque control problem: **Anisotropic, affine PMSM** (without iron losses)

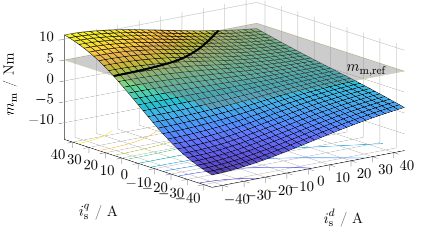


$$m_{\mathrm{m,ref}} \stackrel{!}{=} m_{\mathrm{m}}\left(\boldsymbol{i}_{\mathrm{s}}^{dq}\right) = \frac{2n_{\mathrm{p}}}{3\kappa^2}\left[\widetilde{\psi}_{\mathrm{pm}} i_{\mathrm{s}}^{q} + \left(\widetilde{L}_{\mathrm{s}}^{d} - \widetilde{L}_{\mathrm{s}}^{q}\right) i_{\mathrm{s}}^{d} i_{\mathrm{s}}^{q} + \widetilde{L}_{\mathrm{s,m}}\left((i_{\mathrm{s}}^{q})^2 - (i_{\mathrm{s}}^{d})^2\right)\right] \Longrightarrow \boxed{\boldsymbol{i}_{\mathrm{s,ref}}^{dq} := (?,?)^\top}$$

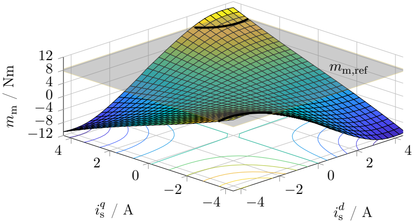
Motivation

Optimal feedforward torque control problem: **Identical for all nonlinear machines**

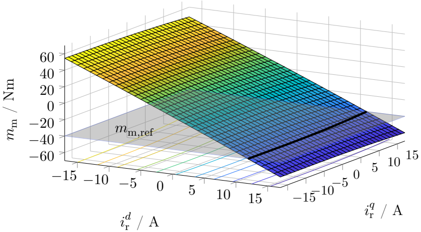
Nonlinear IPMSM (3.9 kW@5 500 rpm)



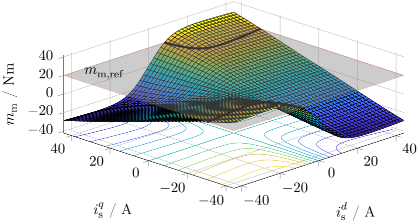
Nonlinear IM (3 kW@1 445 rpm)



Nonlinear DFIM (10 kW@1 500 rpm)



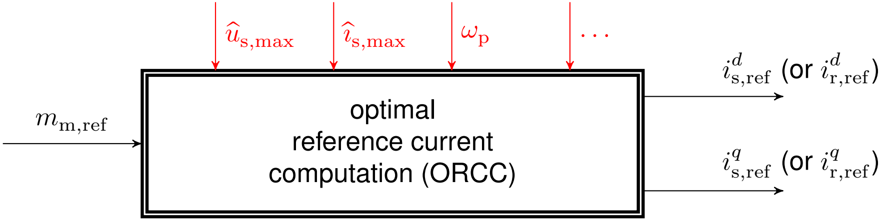
Nonlinear RSM (9.6 kW@1 500 rpm)



Outline

Problem statement and proposed solution

Optimal feedforward torque control problem: Optimal reference current computation (ORCC)



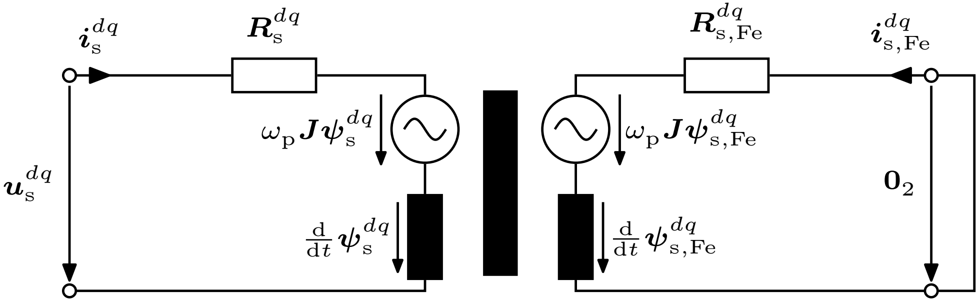
$$\boldsymbol{i}^{dq}_{\mathrm{s,ref}}(m_{\mathrm{m,ref}}, \widehat{u}_{\mathrm{s,max}}, \widehat{\imath}_{\mathrm{s,max}}, \omega_{\mathrm{p}}, \dots) = \begin{pmatrix} i^d_{\mathrm{s,ref}}(m_{\mathrm{m,ref}}, \widehat{u}_{\mathrm{s,max}}, \widehat{\imath}_{\mathrm{s,max}}, \omega_{\mathrm{p}}, \dots) \\ i^q_{\mathrm{s,ref}}(m_{\mathrm{m,ref}}, \widehat{u}_{\mathrm{s,max}}, \widehat{\imath}_{\mathrm{s,max}}, \omega_{\mathrm{p}}, \dots) \end{pmatrix}$$

- ✓ Numerical solutions and/or look-up tables
  (but: limited storage, accuracy, real-time applicability)
- **?** Analytical solutions

$\Big($some do exist but impose simplifying assumptions such as ~~$R_{\mathrm{s}} = 0$, $R_{\mathrm{s,Fe}} = 0$ and/or $L_{\mathrm{s,m}} = 0$, etc.~~$\Big)$

Problem statement and proposed solution

Considered anisotropic synchronous machines with iron losses: **Nonlinear transformer model**



with $\boldsymbol{\psi}_{\mathrm{s}}^{dq} = \boldsymbol{\psi}_{\mathrm{s,Fe}}^{dq}$ (single assumption!) and (possibly) *nonlinear*

- *current, angle, speed* and *temperature* dependent stator resistance, i.e. $\boldsymbol{R}_{\mathrm{s}}^{dq} := \boldsymbol{R}_{\mathrm{s}}^{dq}(\boldsymbol{i}_{\mathrm{s}}^{dq}, \phi_{\mathrm{p}}, \omega_{\mathrm{p}}, \vartheta_{\mathrm{s}})$
- *current, angle, speed* and *temperature* dependent iron resistance, i.e. $\boldsymbol{R}_{\mathrm{s,Fe}}^{dq} := \boldsymbol{R}_{\mathrm{s,Fe}}^{dq}(\boldsymbol{i}_{\mathrm{s}}^{dq}, \phi_{\mathrm{p}}, \omega_{\mathrm{p}}, \vartheta_{\mathrm{s}})$
- *current, angle, speed* and *temperature* dependent flux linkages, i.e. $\boldsymbol{\psi}_{\mathrm{s}}^{dq} := \boldsymbol{\psi}_{\mathrm{s}}^{dq}(\boldsymbol{i}_{\mathrm{s}}^{dq}, \phi_{\mathrm{p}}, \omega_{\mathrm{p}}, \vartheta_{\mathrm{s}})$

Problem statement and proposed solution

Considered anisotropic synchronous machines with iron losses: **Nonlinear machine dynamics** ($\Longrightarrow$ steady-state model)

Stator:

$$\overbrace{\begin{pmatrix} u_{\mathrm{s}}^{d} \\ u_{\mathrm{s}}^{q} \end{pmatrix}}^{=:\boldsymbol{u}_{\mathrm{s}}^{dq}} = \overbrace{\begin{bmatrix} R_{\mathrm{s}}^{d} & R_{\mathrm{s}}^{dq} \\ R_{\mathrm{s}}^{qd} & R_{\mathrm{s}}^{q} \end{bmatrix}}^{=:\boldsymbol{R}_{\mathrm{s}}^{dq}} \overbrace{\begin{pmatrix} i_{\mathrm{s}}^{d} \\ i_{\mathrm{s}}^{q} \end{pmatrix}}^{=:\boldsymbol{i}_{\mathrm{s}}^{dq}} + \omega_{\mathrm{p}} \overbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}^{=:\boldsymbol{J}} \overbrace{\begin{pmatrix} \psi_{\mathrm{s}}^{d} \\ \psi_{\mathrm{s}}^{q} \end{pmatrix}}^{=:\boldsymbol{\psi}_{\mathrm{s}}^{dq}} + \cancel{\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_{\mathrm{s}}^{dq}}$$

Iron:

$$\underbrace{\begin{pmatrix} 0 \\ 0 \end{pmatrix}}_{=:\boldsymbol{0}_2} = \underbrace{\begin{bmatrix} R_{\mathrm{s,Fe}}^{d} & R_{\mathrm{s,Fe}}^{dq} \\ R_{\mathrm{s,Fe}}^{qd} & R_{\mathrm{s,Fe}}^{q} \end{bmatrix}}_{=:\boldsymbol{R}_{\mathrm{s,Fe}}^{dq}} \underbrace{\begin{pmatrix} i_{\mathrm{s,Fe}}^{d} \\ i_{\mathrm{s,Fe}}^{q} \end{pmatrix}}_{=:\boldsymbol{i}_{\mathrm{s,Fe}}^{dq}} + \omega_{\mathrm{p}} \boldsymbol{J} \boldsymbol{\psi}_{\mathrm{s}}^{dq} + \cancel{\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_{\mathrm{s}}^{dq}}$$

$$\boldsymbol{i}_{\mathrm{s,Fe}}^{dq} = -\left(\boldsymbol{R}_{\mathrm{s,Fe}}^{dq}\right)^{-1} \Big[\quad + \omega_{\mathrm{p}} \boldsymbol{J} \boldsymbol{\psi}_{\mathrm{s}}^{dq} + \cancel{\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_{\mathrm{s}}^{dq}} \Big]$$

Mech.:

$$\frac{\mathrm{d}}{\mathrm{d}t}\omega_{\mathrm{m}} = \frac{1}{\Theta_{\mathrm{m}}}\left(m_{\mathrm{m}} + m_{\mathrm{l}}\right) \qquad \text{with} \quad \omega_{\mathrm{p}} = n_{\mathrm{p}}\omega_{\mathrm{m}}$$

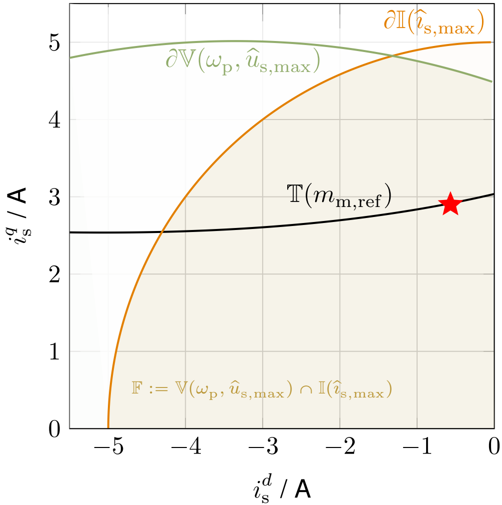
$$m_{\mathrm{m}} = \frac{2}{3\kappa^2}\left(\boldsymbol{i}_{\mathrm{s}}^{dq} + \boldsymbol{i}_{\mathrm{s,Fe}}^{dq}\right)^{\top} \boldsymbol{J} \boldsymbol{\psi}_{\mathrm{s}}^{dq}$$

Losses:

$$p_{\mathrm{s,L}} = \underbrace{\frac{2}{3\kappa^2}\left(\boldsymbol{i}_{\mathrm{s}}^{dq}\right)^{\top} \boldsymbol{R}_{\mathrm{s}}^{dq} \boldsymbol{i}_{\mathrm{s}}^{dq}}_{=:p_{\mathrm{s,Cu}}} + \underbrace{\frac{2}{3\kappa^2}\left(\boldsymbol{i}_{\mathrm{s,Fe}}^{dq}\right)^{\top} \boldsymbol{R}_{\mathrm{s,Fe}}^{dq} \boldsymbol{i}_{\mathrm{s,Fe}}^{dq}}_{=:p_{\mathrm{s,Fe}}}$$

Problem statement and proposed solution

Optimal reference current computation (ORCC): Optimization problem(s) with multiple constraints



$$\max_{\boldsymbol{i}_s^{dq}\in\mathbb{F}} \; -f(\boldsymbol{i}_s^{dq}) \quad \text{subject to}$$

$$\begin{cases} \|\boldsymbol{i}_s^{dq}\|^2 \leqslant \widehat{\imath}_{s,\max}^2, \\ \text{(current circular area)} \\ \|\boldsymbol{u}_s^{dq}(\boldsymbol{i}_s^{dq}, \omega_p, \ldots)\|^2 \leqslant \widehat{u}_{s,\max}^2, \\ \text{(voltage elliptical area)} \\ |m_m(\boldsymbol{i}_s^{dq}, \omega_p, \ldots)| \leqslant |m_{m,\mathrm{ref}}|, \\ \text{and } \operatorname{sign}(m_{m,\mathrm{ref}}) = \\ \operatorname{sign}(m_m(\boldsymbol{i}_s^{dq})). \end{cases}$$

$\Longrightarrow$ e.g. minimize copper losses, i.e., $-f(\boldsymbol{i}_s^{dq}) = -\|\boldsymbol{i}_s^{dq}\|^2$.

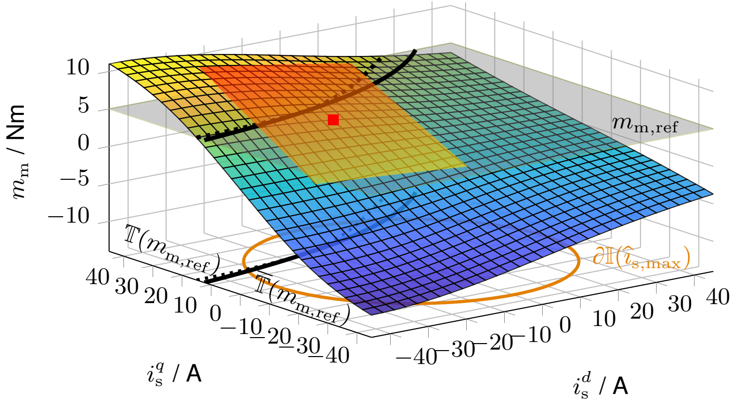
$\Longrightarrow$ $\boldsymbol{i}_{s,\mathrm{ref}}^{dq} = \boldsymbol{i}_{s,\mathrm{MTPC}}^{dq}$ at ★, i.e., MTPC with $m_{m,\mathrm{ref}} = m_m(\boldsymbol{i}_{s,\mathrm{ref}}^{dq})$ feasible.

Problem statement and proposed solution

Proposed solution: **Online linearization, implicit formulation (quadrics), optimization and intersection points**

Step 1: Online linearization of flux linkages, machine torque, iron resistance, etc.: for example:

- **flux linkages** (first-order Taylor approximation around operating point $\bar{\boldsymbol{i}}_s^{dq}$ [■])
- **machine torque** (second-order Taylor approximation around operating point $\bar{\boldsymbol{i}}_s^{dq}$ [■])



- ...

Problem statement and proposed solution

Proposed solution: **Online linearization, implicit formulation (quadrics), optimization and intersection points**

Step 2: Derivation of quadrics $Q_A(\boldsymbol{i}_\mathrm{s}^{dq}) := (\boldsymbol{i}_\mathrm{s}^{dq})^\top \boldsymbol{A}\,\boldsymbol{i}_\mathrm{s}^{dq} + 2\boldsymbol{a}^\top \boldsymbol{i}_\mathrm{s}^{dq} + \alpha$:

- Current circular area: $(i_\mathrm{s}^d)^2 + (i_\mathrm{s}^q)^2 \leqslant \widehat{\imath}_{\mathrm{s,max}}^2 \iff \|\boldsymbol{i}_\mathrm{s}^{dq}\|^2 = (\boldsymbol{i}_\mathrm{s}^{dq})^\top \boldsymbol{I}_2 \boldsymbol{i}_\mathrm{s}^{dq} \leqslant \widehat{\imath}_{\mathrm{s,max}}^2$

$$\Longrightarrow \quad \mathbb{I}(\widehat{\imath}_{\mathrm{s,max}}) := \left\{\, \boldsymbol{i}_\mathrm{s}^{dq} \in \mathbb{R}^2 \;\middle|\; (\boldsymbol{i}_\mathrm{s}^{dq})^\top \boldsymbol{I}_2 \boldsymbol{i}_\mathrm{s}^{dq} - \widehat{\imath}_{\mathrm{s,max}}^2 \leqslant 0 \,\right\}$$

- (Linearized) Voltage elliptical area: $(u_\mathrm{s}^d)^2 + (u_\mathrm{s}^q)^2 \leqslant \widehat{u}_{\mathrm{s,max}}^2 \iff \|\boldsymbol{u}_\mathrm{s}^{dq}(\boldsymbol{i}_\mathrm{s}^{dq}, \omega_\mathrm{p}, \ldots)\|^2 = \ldots \leqslant \widehat{u}_{\mathrm{s,max}}^2$

$$\Longrightarrow \quad \overline{\mathbb{V}}(\widehat{u}_{\mathrm{s,max}}) := \left\{\, \boldsymbol{i}_\mathrm{s}^{dq} \in \mathbb{R}^2 \;\middle|\; (\boldsymbol{i}_\mathrm{s}^{dq})^\top \overline{\boldsymbol{V}}(\overline{\omega}_\mathrm{p})\, \boldsymbol{i}_\mathrm{s}^{dq} + 2\,\overline{\boldsymbol{v}}(\overline{\omega}_\mathrm{p})^\top \boldsymbol{i}_\mathrm{s}^{dq} + \overline{\nu}(\overline{\omega}_\mathrm{p}, \widehat{u}_{\mathrm{s,max}}) \leqslant 0 \right\}$$

- (Linearized) Reference torque hyperbola: $m_\mathrm{m}(\boldsymbol{i}_\mathrm{s}^{dq}, \omega_\mathrm{p}, \ldots) = \frac{3}{2} n_\mathrm{p} (\boldsymbol{i}_\mathrm{s}^{dq} + \boldsymbol{i}_{\mathrm{s,Fe}}^{dq})^\top \boldsymbol{J} \boldsymbol{\psi}_\mathrm{s}^{dq} \stackrel{!}{=} m_{\mathrm{m,ref}} \iff \ldots$

$$\Longrightarrow \quad \overline{\mathbb{T}}(m_{\mathrm{m,ref}}) := \left\{\, \boldsymbol{i}_\mathrm{s}^{dq} \in \mathbb{R}^2 \;\middle|\; (\boldsymbol{i}_\mathrm{s}^{dq})^\top \overline{\boldsymbol{T}} \boldsymbol{i}_\mathrm{s}^{dq} + 2\,\overline{\boldsymbol{t}}^\top \boldsymbol{i}_\mathrm{s}^{dq} - m_{\mathrm{m,ref}} = 0 \,\right\}$$

- ...

Problem statement and proposed solution

Proposed solution: **Online linearization, implicit formulation (quadrics), optimization and intersection points**

Step 3: Optimization problem with equality constraint

$$\max_{\boldsymbol{i}_\mathrm{s}^{dq}} -\Big(\underbrace{(\boldsymbol{i}_\mathrm{s}^{dq})^\top \boldsymbol{A} \boldsymbol{i}_\mathrm{s}^{dq} + 2\boldsymbol{a}^\top \boldsymbol{i}_\mathrm{s}^{dq} + \alpha}_{=:Q_A(\boldsymbol{i}_\mathrm{s}^{dq})} \Big) \text{ s.t. } \underbrace{(\boldsymbol{i}_\mathrm{s}^{dq})^\top \boldsymbol{B} \boldsymbol{i}_\mathrm{s}^{dq} + 2\boldsymbol{b}^\top \boldsymbol{i}_\mathrm{s}^{dq} + \beta}_{=:Q_B(\boldsymbol{i}_\mathrm{s}^{dq})} = 0$$

$\Longrightarrow$ Hyperbolas for e.g. MTPC, MTPL or MTPV (with $R_\mathrm{s}, R_{\mathrm{s,Fe}} \neq 0$ & $L_{\mathrm{s,m}} \neq 0$, etc.)

Step 4: Intersection of two quadrics (e.g. voltage ellipse and current circle)

$$\boldsymbol{i}_{\mathrm{s,ref}}^{dq} := \arg\min_{\|\boldsymbol{i}_\mathrm{s}^{dq}\|} \left\{ \boldsymbol{i}_\mathrm{s}^{dq} \in \mathbb{R}^2 \;\middle|\; Q_A(\boldsymbol{i}_\mathrm{s}^{dq}) = 0 \;\wedge\; Q_B(\boldsymbol{i}_\mathrm{s}^{dq}) = 0 \right\}$$

$\Longrightarrow$ Optimal operation point ★ (reference current; iteration possibly necessary)

Both lead to subproblem of solving a fourth-order (quartic) polynomial

$$\chi(\lambda) := c_4\,\lambda^4 + c_3\,\lambda^3 + c_2\,\lambda^2 + c_1\,\lambda + c_0 \overset{!}{=} 0$$

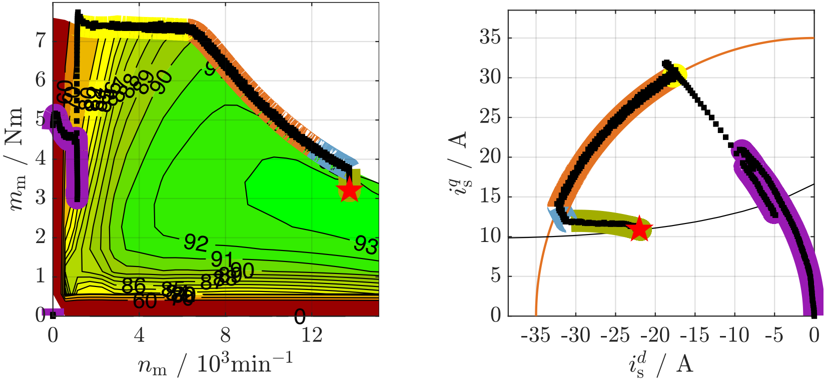
$\Longrightarrow$ Analytical solutions exist (e.g. Euler's solution, see [2])

Outline

4 Optimal operation strategies

Optimal operation strategies and operation management

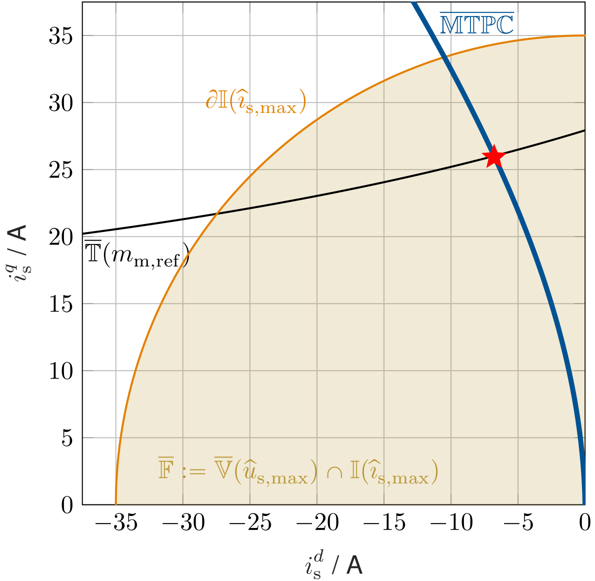
Overview of operation strategies



- Maximum Torque per Losses (MTPL) [ ] (or Maximum Torque per Current (MTPC) [ ])
- Maximum Current (MC) [ ] and Maximum Current extended ($\mathrm{MC_{ext}}$) [ ]
- Maximum Torque per Voltage (MTPV) [ ] (or Maximum Torque per Flux (MTPF)))
- Field Weakening (FW) [ ]

Optimal operation strategies and operation management

Maximum Torque per Current (**MTPC** @ $0 \cdot \omega_{\mathrm{m,R}}$): Reference torque **feasible**



Optimization problem

$$\max_{\boldsymbol{i}_{\mathrm{s}}^{dq} \in \overline{\mathbb{F}}} -p_{\mathrm{s,Cu}}(\boldsymbol{i}_{\mathrm{s}}^{dq}) \quad \text{s.t.}$$

$$\underbrace{(\boldsymbol{i}_{\mathrm{s}}^{dq})^{\top}\overline{\boldsymbol{T}}\boldsymbol{i}_{\mathrm{s}}^{dq} + 2\,\overline{\boldsymbol{t}}^{\top}\boldsymbol{i}_{\mathrm{s}}^{dq}}_{\approx m_{\mathrm{m}}(\boldsymbol{i}_{\mathrm{s}}^{dq})} - m_{\mathrm{m,ref}} = 0$$

Solution set

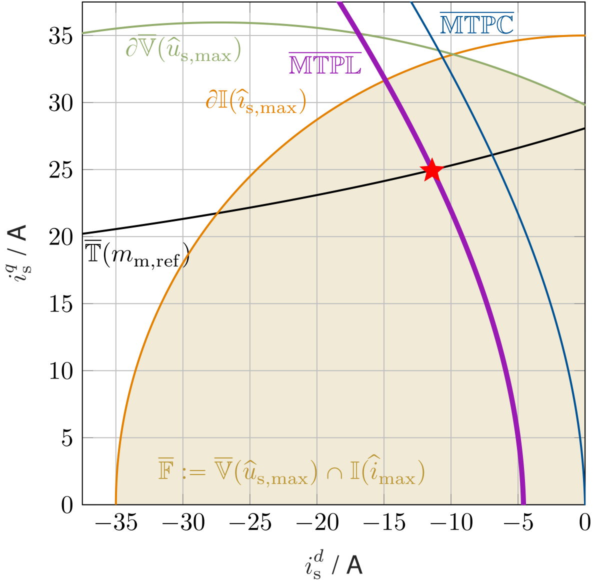
$$\overline{\mathbb{MTPC}}(\boldsymbol{i}_{\mathrm{s}}^{dq}, \overline{\omega}_{\mathrm{p}}, \dots) := \left\{ \boldsymbol{i}_{\mathrm{s}}^{dq} \in \mathbb{R}^2 \;\middle|\; (\boldsymbol{i}_{\mathrm{s}}^{dq})^{\top}\overline{\boldsymbol{M}}_{\mathrm{C}}\,\boldsymbol{i}_{\mathrm{s}}^{dq} + 2\,\overline{\boldsymbol{m}}_{\mathrm{C}}^{\top}\boldsymbol{i}_{\mathrm{s}}^{dq} = 0 \right\}$$

Optimal reference currents (★)

$$\boldsymbol{i}_{\mathrm{s,MTPC}}^{dq} = \overline{\mathbb{MTPC}} \cap \overline{\mathbb{T}}(m_{\mathrm{m,ref}})$$

Optimal operation strategies and operation management

Maximum Torque per Losses (**MTPL** @ $1 \cdot \omega_{\mathrm{m,R}}$): Reference torque **feasible**



Optimization problem

$$\max_{\boldsymbol{i}_{\mathrm{s}}^{dq} \in \overline{\mathbb{F}}} -p_{\mathrm{s,Cu}}(\boldsymbol{i}_{\mathrm{s}}^{dq}) - p_{\mathrm{s,Fe}}(\boldsymbol{i}_{\mathrm{s}}^{dq}) \qquad \text{s.t.}$$

$$\underbrace{(\boldsymbol{i}_{\mathrm{s}}^{dq})^\top \overline{\boldsymbol{T}} \boldsymbol{i}_{\mathrm{s}}^{dq} + 2\,\overline{\boldsymbol{t}}^\top \boldsymbol{i}_{\mathrm{s}}^{dq}}_{\approx m_{\mathrm{m}}(\boldsymbol{i}_{\mathrm{s}}^{dq})} - m_{\mathrm{m,ref}} = 0$$

Solution set

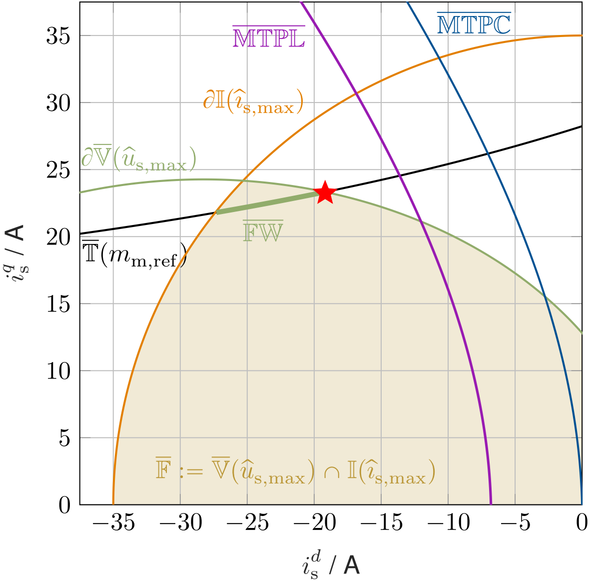
$$\overline{\mathrm{MTPL}}(\overline{\boldsymbol{i}}_{\mathrm{s}}^{dq}, \overline{\omega}_{\mathrm{p}}, \ldots) := \Big\{ \boldsymbol{i}_{\mathrm{s}}^{dq} \in \mathbb{R}^2 \;\Big|\; (\boldsymbol{i}_{\mathrm{s}}^{dq})^\top \overline{\boldsymbol{M}}_{\mathrm{L}}\, \boldsymbol{i}_{\mathrm{s}}^{dq} + 2\,\overline{\boldsymbol{m}}_{\mathrm{L}}^\top \boldsymbol{i}_{\mathrm{s}}^{dq} + \overline{\mu}_{\mathrm{L}} = 0 \Big\}$$

Optimal reference currents (★)

$$\boldsymbol{i}_{\mathrm{s,MTPL}}^{dq} = \overline{\mathrm{MTPL}} \cap \overline{\mathbb{T}}(m_{\mathrm{m,ref}})$$

Optimal operation strategies and operation management

Field Weakening (**FW** @ $1.40 \cdot \omega_{\mathrm{m,R}}$): Reference torque **feasible**



Optimization problem

$$\max_{\boldsymbol{i}_{\mathrm{s}}^{dq} \in \overline{\mathbb{F}}} -p_{\mathrm{s,Cu}}(\boldsymbol{i}_{\mathrm{s}}^{dq}) - p_{\mathrm{s,Fe}}(\boldsymbol{i}_{\mathrm{s}}^{dq}) \quad \text{s.t.}$$

$$\underbrace{(\boldsymbol{i}_{\mathrm{s}}^{dq})^\top \boldsymbol{T} \boldsymbol{i}_{\mathrm{s}}^{dq} + 2\,\boldsymbol{t}^\top \boldsymbol{i}_{\mathrm{s}}^{dq}}_{=m_{\mathrm{m}}(\boldsymbol{i}_{\mathrm{s}}^{dq})} - m_{\mathrm{m,ref}} = 0$$

Feasible set

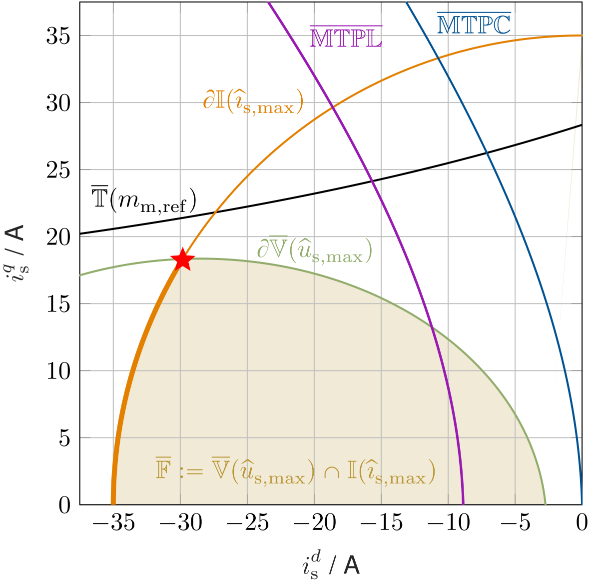
$$\overline{\mathbb{FW}}(\overline{\boldsymbol{i}}_{\mathrm{s}}^{dq}, \overline{\omega}_{\mathrm{p}}, \ldots, m_{\mathrm{m,ref}}, \widehat{u}_{\mathrm{s,max}}, \widehat{\imath}_{\mathrm{s,max}}) := \overline{\mathbb{F}}(\widehat{u}_{\mathrm{s,max}}, \widehat{\imath}_{\mathrm{s,max}}) \cap \overline{\mathbb{T}}(m_{\mathrm{m,ref}})$$

Optimal reference currents (★)

$$\boldsymbol{i}_{\mathrm{s,FW}}^{dq} = \partial\overline{\mathbb{V}}(\widehat{u}_{\mathrm{s,max}}) \cap \overline{\mathbb{T}}(m_{\mathrm{m,ref}})$$

Optimal operation strategies and operation management

Maximum Current (**MC** @ $2 \cdot \omega_{\mathrm{m,R}}$): Reference torque **not feasible**



Optimization problem

$$\max_{\boldsymbol{i}_{\mathrm{s}}^{dq} \in \overline{\mathbb{F}}} |(m_{\mathrm{m}}(\boldsymbol{i}_{\mathrm{s}}^{dq}))| \quad \text{s.t.}$$

$$\mathrm{sign}(m_{\mathrm{m}}) = \mathrm{sign}(m_{\mathrm{m,ref}})$$

Feasible set

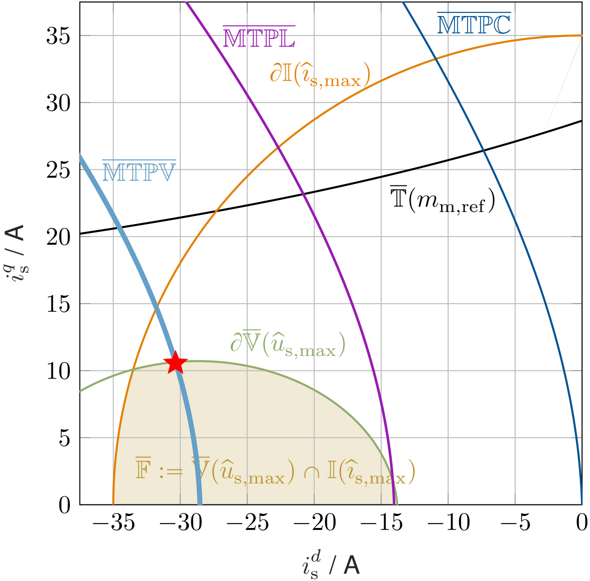
$$\overline{\mathbb{MC}}(\overline{\boldsymbol{i}}_{\mathrm{s}}^{dq}, \overline{\omega}_{\mathrm{p}}, \ldots, \widehat{u}_{\mathrm{s,max}}, \widehat{\imath}_{\mathrm{s,max}}) := \overline{\mathbb{V}}(\widehat{u}_{\mathrm{s,max}}) \cap \partial\mathbb{I}(\widehat{\imath}_{\mathrm{s,max}})$$

Optimal current reference (★)

$$\boldsymbol{i}_{\mathrm{s,MC}}^{dq} = \partial\overline{\mathbb{V}}(\widehat{u}_{\mathrm{s,max}}) \cap \partial\mathbb{I}(\widehat{\imath}_{\mathrm{s,max}})$$

Optimal operation strategies and operation management

Maximum Torque per Voltage (MTPV @ $3.50 \cdot \omega_{\mathrm{m,R}}$): Reference torque **not feasible**



Optimization problem

$$\max_{\boldsymbol{i}_{\mathrm{s}}^{dq} \in \overline{\mathbb{F}}} -\left\|\boldsymbol{u}_{\mathrm{s}}^{dq}(\boldsymbol{i}_{\mathrm{s}}^{dq})\right\|^2 \quad \text{s.t.}$$

$$\underbrace{(\boldsymbol{i}_{\mathrm{s}}^{dq})^\top \overline{\boldsymbol{T}} \boldsymbol{i}_{\mathrm{s}}^{dq} + 2\,\overline{\boldsymbol{t}}^\top \boldsymbol{i}_{\mathrm{s}}^{dq}}_{=m_{\mathrm{m}}(\boldsymbol{i}_{\mathrm{s}}^{dq})} - m_{\mathrm{m,ref}} = 0$$

Solution set

$$\overline{\mathrm{MTPV}}(\overline{\boldsymbol{i}}_{\mathrm{s}}^{dq}, \overline{\omega}_{\mathrm{p}}, \cdots) := \left\{ \boldsymbol{i}_{\mathrm{s}}^{dq} \in \mathbb{R}^2 \;\middle|\; (\boldsymbol{i}_{\mathrm{s}}^{dq})^\top \overline{\boldsymbol{M}}_{\mathrm{V}}\, \boldsymbol{i}_{\mathrm{s}}^{dq} + 2\,\overline{\boldsymbol{m}}_{\mathrm{V}}^\top \boldsymbol{i}_{\mathrm{s}}^{dq} + \overline{\mu}_{\mathrm{V}} = 0 \right\}$$

Optimal reference currents (★)

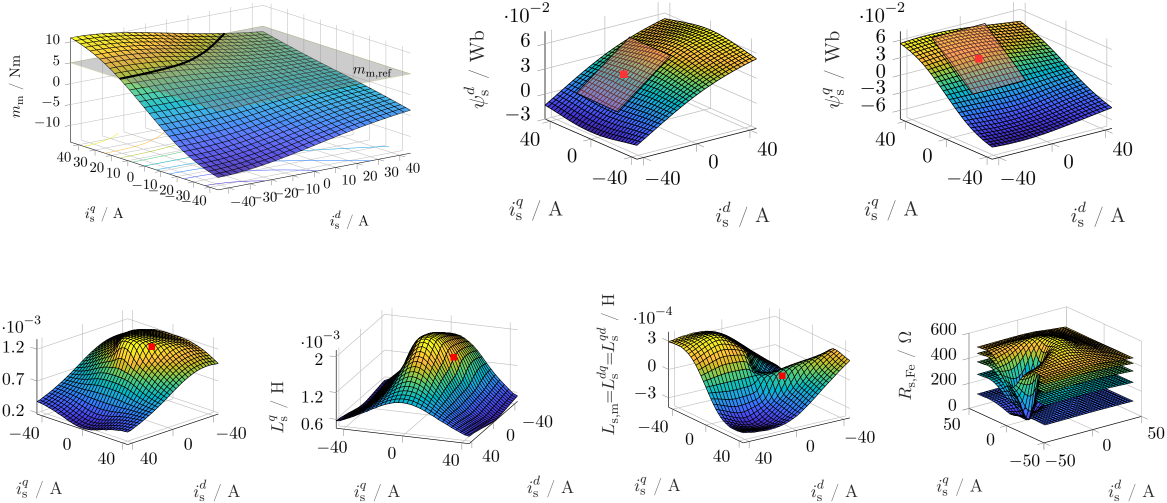
$$\boldsymbol{i}_{\mathrm{s,MTPV}}^{dq} = \overline{\mathrm{MTPV}} \cap \partial\overline{\mathbb{V}}(\widehat{u}_{\mathrm{s,max}})$$

Outline

5 Implementation results

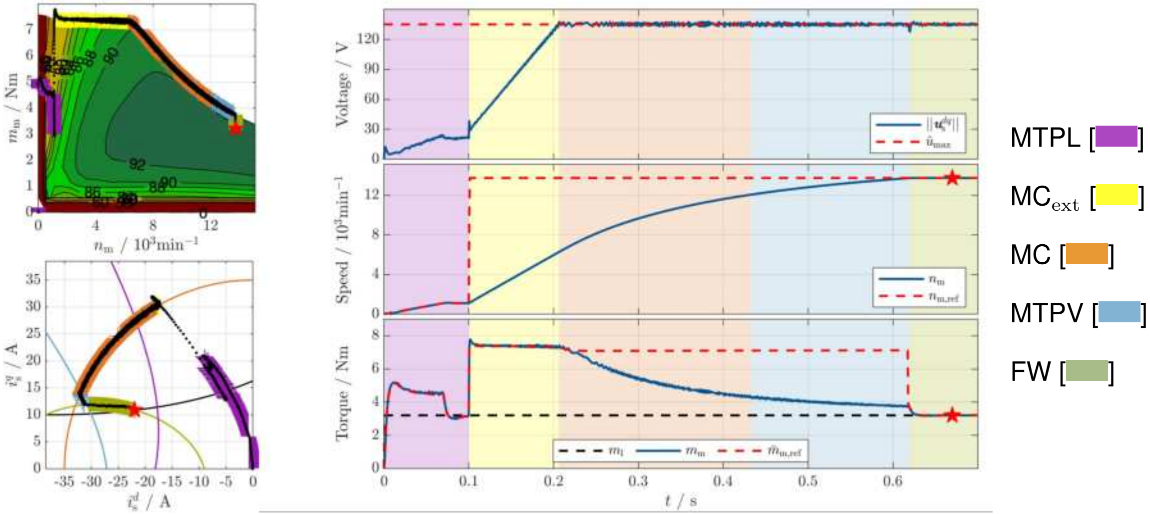
Implementation results

IPMSM (3.9 kW@5 500 rpm): Nonlinear maps of torque, flux linkages, differential inductances and iron resistance



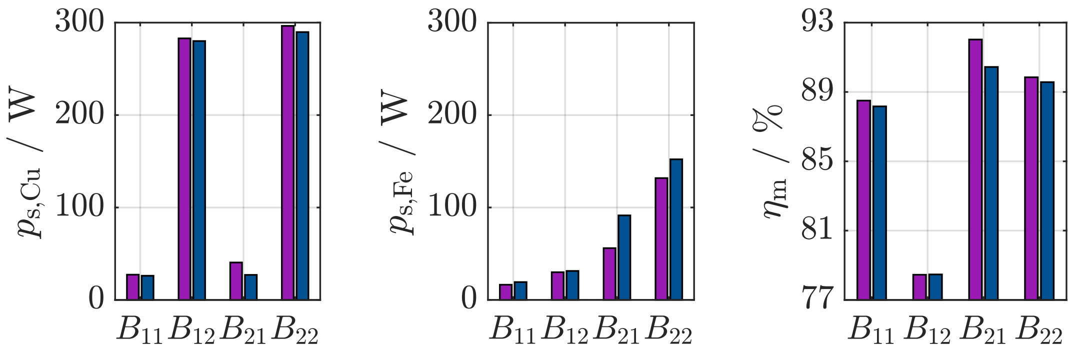
Implementation results

IPMSM (3.9 kW@5 500 rpm): Animation



Implementation results

IPMSM (3.9 kW@5 500 rpm): Comparison of MTPL [ ] & MTPC [ ] for **four** operating points (OPs)



OP	Speed	Torque
B_{11}	$30\%\, n_{m,R}$	$30\%\, m_{m,R}$
B_{12}	$30\%\, n_{m,R}$	$100\%\, m_{m,R}$
B_{21}	$100\%\, n_{m,R}$	$30\%\, m_{m,R}$
B_{22}	$100\%\, n_{m,R}$	$100\%\, m_{m,R}$

- Copper losses increase with torque
- Iron losses increase with speed *and* torque
- MTPL outperforms MTPC for three OPs (equal for B_{12})
- highest efficiency enhancement for high speeds and low(er) torque (i.e., B_{21} with $\Delta\eta_m > 2\%$)

Outline

Conclusion

Summary and future work

To take home

- **unified** framework for MTPL (MTPC), FW, MC$_{\mathrm{(ext)}}$ & MTPV (MTPF) based on [3–7] and [8, Chapt. 6.9]
  - **online linearization**,
  - **quadrics** (implicit formulation),
  - **optimization problems** with **equality constraint**, and
  - **intersection** of two quadrics
- **analytical** solution for reference currents (& transition points)
- Consideration of $R_{\mathrm{s}} \neq 0$, $R_{\mathrm{s,Fe}} \neq 0$ and $L_{\mathrm{s,m}} \neq 0$ and current, speed, angle & temperature dependency (all **simultaneously**)
- **fast(er)** and **more accurate** computation
- **applicable** in real-world (e.g. nonlinear RSM, PME-RSM or IPMSM; also IMs or DFIMs)

Future work

- convergence (mathematical proof for nonlinear flux linkages)
- more extensive experimental validation
- impact of (parameter/modelling/alignment) uncertainties
- consideration of *rotor* iron losses (e.g. for induction machines), *current transients* during operation and *multi-phase* machines
- combination with minimization of *conduction and switching losses* [9]

References

Journal article [4] (published 2017; doi: 10.1080/00207179.2017.1338359)

INTERNATIONAL JOURNAL OF CONTROL, 2017
https://doi.org/10.1080/00207179.2017.1338359

Taylor & Francis
Taylor & Francis Group

OPEN ACCESS Check for updates

A unified theory for optimal feedforward torque control of anisotropic synchronous machines

Hisham Eldeeb[a], Christoph M. Hackl[a,*], Lorenz Horlbeck[b] and Julian Kullick[a]

[a]Research Group 'Control of Renewable Energy Systems (CRES)', Munich School of Engineering (MSE), Technical University of Munich (TUM), Munich, Germany; [b]Institute of Automotive Technology (FTM), Technical University of Munich (TUM), Munich, Germany

ABSTRACT

A unified theory for optimal feedforward torque control of anisotropic synchronous machines with *non-negligible stator resistance* and *mutual inductance* is presented which allows to *analytically* compute (1) the optimal direct and quadrature reference currents for all operating strategies, such as maximum torque per current (MTPC), maximum current, field weakening, maximum torque per voltage (MTPV) or maximum torque per flux (MTPF), and (2) the transition points indicating when to switch between the operating strategies due to speed, voltage or current constraints. The analytical solutions allow for an (almost) instantaneous selection and computation of actual operation strategy and corresponding reference currents. Numerical methods (approximating these solutions only) are *no longer* required. The unified theory is based on one simple idea: all optimisation problems, their respective constraints and the computation of the intersection point(s) of voltage ellipse, current circle or torque, MTPC, MTPV, MTPF hyperbolas are reformulated implicitly as *quadrics* which allows to invoke the Lagrangian formalism and to find the roots of fourth-order polynomials analytically. The proposed theory is suitable for any anisotropic synchronous machine. Implementation and measurement results illustrate effectiveness and applicability of the theoretical findings in real world.

ARTICLE HISTORY
Received 18 March 2017
Accepted 31 May 2017

KEYWORDS
Maximum torque per ampere (MTPA); maximum torque per current (MTPC); maximum torque per voltage (MTPV); maximum torque per flux (MTPF); maximum current (MC); field weakening (FW); analytical solution; optimal feedforward torque control; efficiency; copper losses; anisotropy; synchronous machine; interior

References

Conference publication (published in ICIT 2021 proceedings; not yet on IEEEexplore, see techrxiv.org)

Generic loss minimization for nonlinear synchronous machines by analytical computation of optimal reference currents considering copper and iron losses

Christoph Hackl, Julian Kullick and Niklas Monzen
Laboratory for Mechatronic and Renewable Energy Systems (LMRES)
Munich University of Applied Sciences (MUAS)
Munich, Germany
{christoph.hackl, julian.kullick, niklas.monzen}@hm.edu

***Abstract*—The unified theory introduced in [1] allows to solve *analytically* the optimal feedforward torque control (OFTC) problem of anisotropic synchronous machines (SMs). In this paper, the theory is extended by considering relevant machine nonlinearities and incorporating copper *and* iron losses, thus minimizing the overall (steady-state) losses in the machine. Instead of the well known maximum torque per current (MTPC) operation strategy, *maximum torque per losses (MTPL)* is realized. The unified theory for the derivation of the analytical solution is briefly recapitulated. Moreover, current *and* speed dependent iron losses, as well as magnetic saturation and cross-coupling effects are considered. The resulting nonlinear optimization problem**

for synchronous machines (SMs) with anisotropic rotor designs, e.g. interior permanent magnet synchronous machines (IPMSMs), reluctance synchronous machines (RSMs) or PM-assisted RMSs (PMA-RSMs), efficiency can be increased by optimal feedforward torque control (OFTC) [3, 4].

The main idea of OFTC is to exploit the ambiguity in the selection of the stator current's *direct* and *quadrature* components (producing the same amount of torque), such that losses are minimized while physical constraints are satisfied (e.g. current or voltage limits). Depending on the actual operating

References

Conference publication (submitted to ISIE 2021; see techrxiv.org)

Optimal feedforward torque control for nonlinear induction machines considering stator & rotor copper losses and current & voltage limits

Andre Thommessen, Christoph Hackl
Laboratory for Mechatronic and Renewable Energy Systems (LMRES)
Munich University of Applied Sciences (MUAS)
Munich, Germany
{andre.thommessen, christoph.hackl}@hm.edu

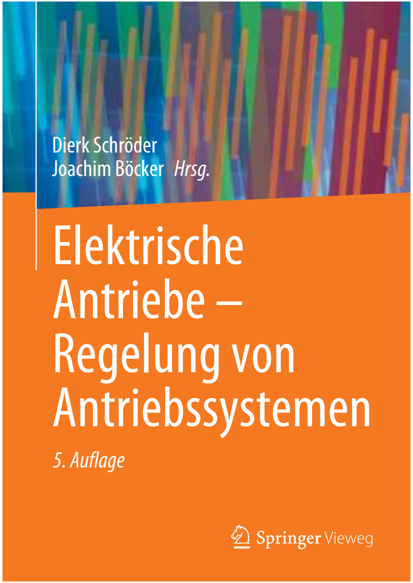
***Abstract*—In order to *analytically* solve the optimal feedforward torque control (OFTC) problem of induction machines (IMs), the unified theory for synchronous machine introduced in [1] is extended by considering relevant IM *nonlinearities* and incorporating stator *and* rotor copper losses. Instead of the well known *Maximum Torque per (stator) Current* (MTPC) operation strategy, *Maximum Torque per (copper) Losses* ($\mathrm{MTPL_{Cu}}$) is realized and extended by the *Maximum (rotor) Current* ($\mathrm{MC_{r,\,ext}}$) strategy due to stator *and* rotor current limitations. Modeling magnetic saturation and cross-coupling effects leads to a constrained *nonlinear* optimization problem which is solved based on the idea of *sequential quadratic programming* (SQP). The second order Taylor approximations are formulated in implicit**

generic approach can also be applied to other types of inverter-fed IMs, such as squirrel-cage induction machines (SCIMs).

The highest priority of the OFTC is to provide the reference torque while minimizing losses. In a wide operating range, there usually exist (infinitely many) different combinations of reference currents resulting in the same torque. Thus, an optimal reference current computation (ORCC) is desirable which minimizes the current-dependent IM losses while reaching the reference torque and taking into account operating (e.g. current & voltage) limits. To do so, this paper proposes a physics-based nonlinear IM model for analytical ORCC.

References

Chapter 6.9 in [8] (published in 2020 in Schröder “Elektrische Antriebssysteme”; doi: 10.1007/978-3-662-62700-6)



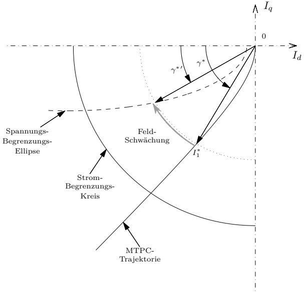


Abb. 6.101 Wirkungsweise der Feldschwächregelung mit Spannungsrückkopplung

6.9 Optimale Betriebsführung von nichtlinearen Synchronmaschinen

C. M. Hackl, J. Kullick, N. Monzen

Der folgende Abschnitt basiert auf den Publikationen [HH16; Eld+16; EHK16; Eld+17b; Hac+17; Eld+17a]. Darin wurde die erste allgemeine Theorie zur analytischen Berechnung der optimalen (verlustminimierenden) Sollströme für anisotrope Synchronmaschinen mit konstanter Erregung vorgestellt. Die weithin verbreiteten und vereinfachenden Annahmen wie z. B. die Vernachlässigung des Statorwiderstandes oder der magnetischen Kreuzkopplung konnten aufgehoben werden. Zusätzlich erlaubt die Theorie die Berücksichtigung von Eisenverlusten und Nicht-

References

Monograph [10] (published May 2017; doi: 10.1007/978-3-319-55036-7)

References I

[1] Shigeo Morimoto. Trend of permanent magnet synchronous machines. *IEEJ Transactions on Electrical and Electronic Engineering*, 2(2):101–108, mar 2007.

[2] R.W.D Nickalls. The quartic equation: Invariants and Euler's solution revealed. *Mathematical Gazette*, 93(526):66–75, mar 2009.

[3] Lorenz Horlbeck and Christoph M. Hackl. Analytical solution for the MTPV hyperbola including the stator resistance. In *Proceedings of the IEEE International Conference on Industrial Technology (ICIT 2016)*, pages 1060–1067, Taipei, Taiwan, mar 2016. Institute of Electrical and Electronics Engineers (IEEE).

[4] Hisham Eldeeb, Christoph M. Hackl, Lorenz Horlbeck, and Julian Kullick. A unified theory for optimal feedforward torque control of anisotropic synchronous machines. *International Journal of Control*, 2017.

[5] C. M. Hackl, J. Kullick, H. Eldeeb, and L. Horlbeck. Analytical computation of the optimal reference currents for MTPC/MTPA, MTPV and MTPF operation of anisotropic synchronous machines considering stator resistance and mutual inductance. In *2017 19th European Conference on Power Electronics and Applications (EPE'17 ECCE Europe)*, pages P.1–P.10, Sept 2017.

[6] Hisham Eldeeb, Christoph M. Hackl, Lorenz Horlbeck, and Julian Kullick. Analytical solutions for the optimal reference currents for MTPC/MTPA, MTPV and MTPF control of anisotropic synchronous machines. In *Proceedings of the IEEE International Electric Machines & Drives Conference (IEMDC 2017)*, pages 1–6, Miami, FL, USA, 2017.

[7] C. M. Hackl, J. Kullick, and N. Monzen. Generic loss minimization for nonlinear synchronous machines by analytical computation of optimal reference currents considering copper and iron losses. In *2021 IEEE International Conference on Industrial Technology (ICIT)*. IEEE, mar 2021.

[8] Dierk Schröder and Joachim Böcker, editors. *Elektrische Antriebe – Regelung von Antriebssystemen*. Springer Berlin Heidelberg, 2020.

[9] M. Fakharmanesh, C. M. Hackl, and R. Perini. Instantaneous conduction and switching losses in two-level voltage source inverters. In *2017 IEEE International Conference on Environment and Electrical Engineering and 2017 IEEE Industrial and Commercial Power Systems Europe (EEEIC / I CPS Europe)*, pages 1–6, June 2017.

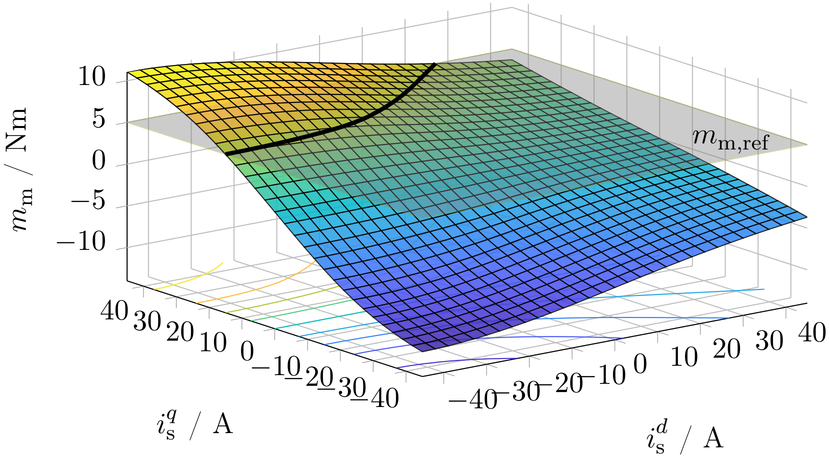
[10] Christoph M. Hackl. *Non-identifier based adaptive control in mechatronics: Theory and Application*. Number 466 in Lecture Notes in Control and Information Sciences. Springer International Publishing, Berlin, 2017.

[11] Simon Krüner and Christoph M. Hackl. Experimental identification of the optimal current vectors for a permanent-magnet synchronous machine in wave energy converters. *Energies*, 12(862), 2019.

Outline

Motivation

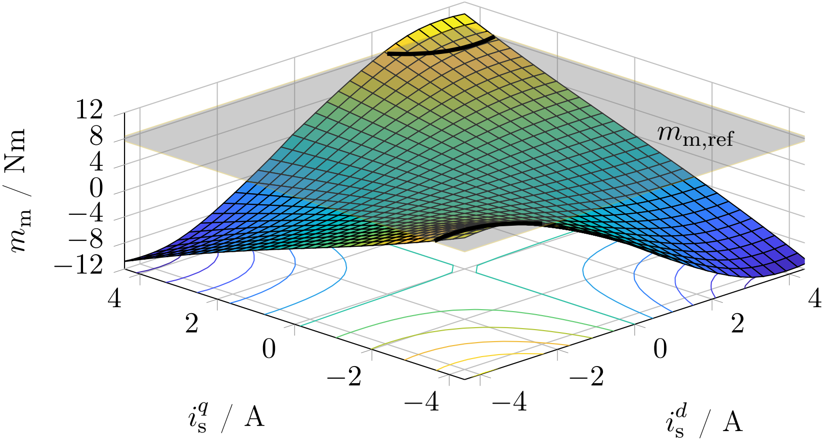
Optimal feedforward torque control problem: **Nonlinear anisotropic IPMSM** (3.9 kW@5 500 rpm)



$$m_{\mathrm{m,ref}} \stackrel{!}{=} m_{\mathrm{m}}\left(\boldsymbol{i}_{\mathrm{s}}^{dq}, \omega_{\mathrm{p}}\right) = \frac{2 n_{\mathrm{p}}}{3\kappa^2}\left(\boldsymbol{i}_{\mathrm{s}}^{dq} + \boldsymbol{i}_{\mathrm{s,Fe}}^{dq}(\boldsymbol{i}_{\mathrm{s}}^{dq}, \omega_{\mathrm{p}})\right)^{\top} \boldsymbol{J}\boldsymbol{\psi}_{\mathrm{s}}^{dq} \quad \Longrightarrow \quad \boxed{\boldsymbol{i}_{\mathrm{s,ref}}^{dq} := \left(i_{\mathrm{s,ref}}^{d}, i_{\mathrm{s,ref}}^{q}\right)^{\top} = (?, ?)^{\top}}$$

Motivation

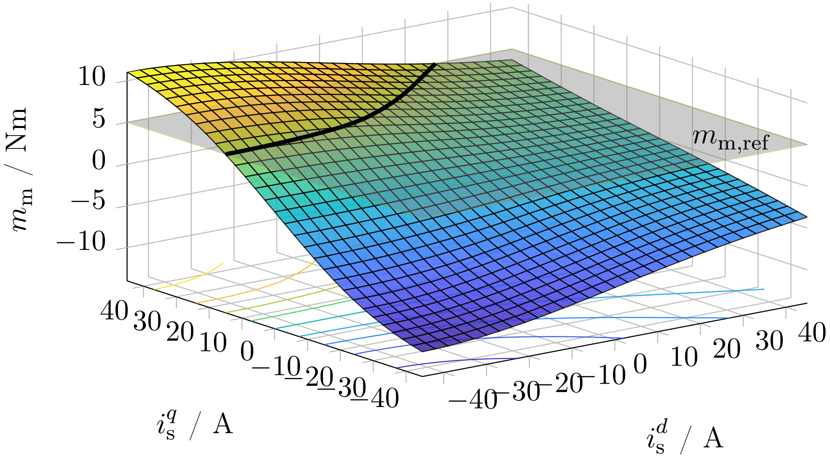
Optimal feedforward torque control problem: **Nonlinear induction machine** (IM; 3 kW@1 445 rpm)



$$m_{\mathrm{m,ref}} \overset{!}{=} m_{\mathrm{m}}\left(\boldsymbol{i}_{\mathrm{s}}^{dq}, \omega_{\mathrm{p}}\right) = \frac{2n_{\mathrm{p}}}{3\kappa^2}\left(\boldsymbol{i}_{\mathrm{s}}^{dq} + \boldsymbol{i}_{\mathrm{s,Fe}}^{dq}(\boldsymbol{i}_{\mathrm{s}}^{dq}, \omega_{\mathrm{p}})\right)^{\top} \boldsymbol{J}\boldsymbol{\psi}_{\mathrm{s}}^{dq} \quad \Longrightarrow \quad \boxed{\boldsymbol{i}_{\mathrm{s,ref}}^{dq} := \left(i_{\mathrm{s,ref}}^{d}, i_{\mathrm{s,ref}}^{q}\right)^{\top} = (?, ?)^{\top}}$$

Motivation

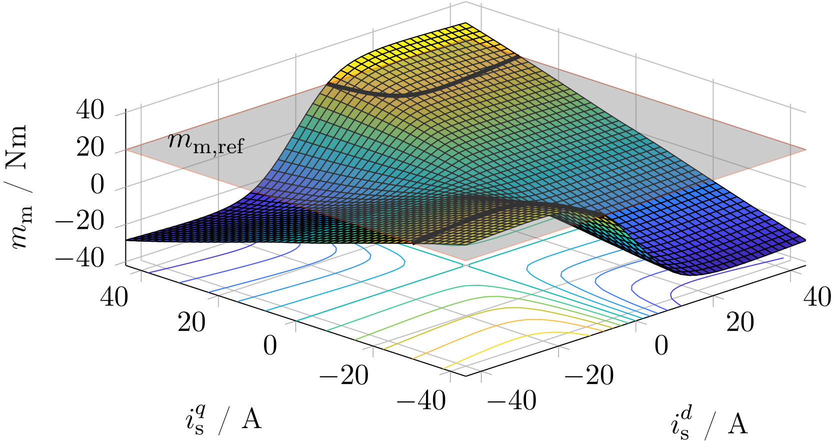
Optimal feedforward torque control problem: **Nonlinear doubly-fed induction machine** (DFIM; 10 kW@1 500 rpm)



$$m_{\mathrm{m,ref}} \overset{!}{=} m_{\mathrm{m}}\left(\boldsymbol{i}_{\mathrm{r}}^{dq}, \omega_{\mathrm{p}}\right) = -\frac{2n_{\mathrm{p}}}{3\kappa^2}\left(\boldsymbol{i}_{\mathrm{r}}^{dq} + \boldsymbol{i}_{\mathrm{r,Fe}}^{dq}\left(\boldsymbol{i}_{\mathrm{r}}^{dq}, \omega_{\mathrm{p}}, \ldots\right)\right)^{\top} \boldsymbol{J}\boldsymbol{\psi}_{\mathrm{r}}^{dq} \implies \boxed{\boldsymbol{i}_{\mathrm{r,ref}}^{dq} := \left(i_{\mathrm{r,ref}}^{d}, i_{\mathrm{r,ref}}^{q}\right)^{\top} = (?,?)^{\top}}$$

Motivation

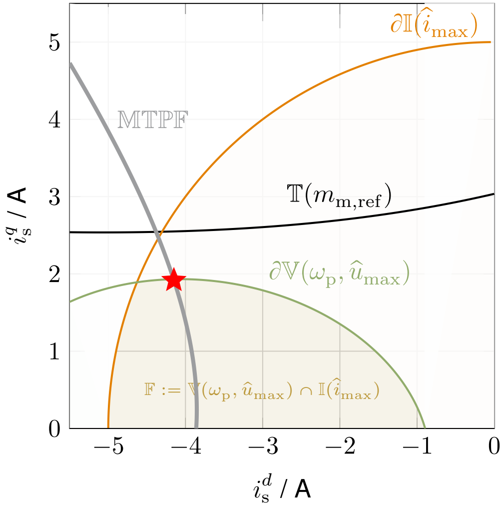
Optimal feedforward torque control problem: **Nonlinear reluctance synchronous machine** (RSM; 9.6 kW@1 500 rpm)



$$m_{\mathrm{m,ref}} \stackrel{!}{=} m_{\mathrm{m}}(\boldsymbol{i}_{\mathrm{s}}^{dq}, \omega_{\mathrm{p}}) = \frac{2n_{\mathrm{p}}}{3\kappa^2}\Big(\boldsymbol{i}_{\mathrm{s}}^{dq} + \boldsymbol{i}_{\mathrm{s,Fe}}^{dq}(\boldsymbol{i}_{\mathrm{s}}^{dq}, \omega_{\mathrm{p}})\Big)^{\top} \boldsymbol{J}\boldsymbol{\psi}_{\mathrm{s}}^{dq} \quad \Longrightarrow \quad \boxed{\boldsymbol{i}_{\mathrm{s,ref}}^{dq} := (i_{\mathrm{s,ref}}^{d}, i_{\mathrm{s,ref}}^{q})^{\top} = (?, ?)^{\top}}$$

Optimal operation strategies and operation management

Maximum Torque per Flux (**MTPF**): Reference torque **not** feasible



Optimization problem

$$\max_{\boldsymbol{i}_{\mathrm{s}}^{dq} \in \mathbb{F}} -\left\|\boldsymbol{\psi}_{\mathrm{s}}^{dq}(\boldsymbol{i}_{\mathrm{s}}^{dq})\right\|^2 \quad \text{s.t.}$$

$$\underbrace{(\boldsymbol{i}_{\mathrm{s}}^{dq})^\top \boldsymbol{T} \boldsymbol{i}_{\mathrm{s}}^{dq} + 2\,\boldsymbol{t}^\top \boldsymbol{i}_{\mathrm{s}}^{dq}}_{=m_{\mathrm{m}}(\boldsymbol{i}_{\mathrm{s}}^{dq})} - m_{\mathrm{m,ref}} = 0$$

Solution set

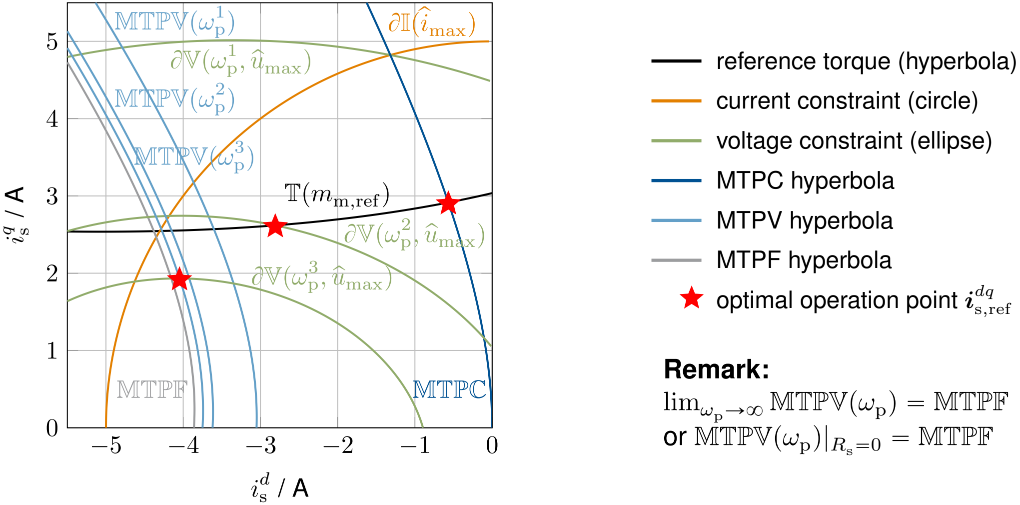
$$\mathbb{MTPF} := \left\{ \boldsymbol{i}_{\mathrm{s}}^{dq} \in \mathbb{R}^2 \;\middle|\; (\boldsymbol{i}_{\mathrm{s}}^{dq})^\top \boldsymbol{M}_{\mathrm{F}}\, \boldsymbol{i}_{\mathrm{s}}^{dq} + 2\,\boldsymbol{m}_{\mathrm{F}}^\top \boldsymbol{i}_{\mathrm{s}}^{dq} + \mu_{\mathrm{F}} = 0 \right\}$$

Optimal current reference (★)

$$\boldsymbol{i}_{\mathrm{s,ref}}^{dq} = \mathbb{MTPF} \cap \partial\mathbb{V}(\omega_{\mathrm{p}}, \widehat{u}_{\max})$$

Optimal operation strategies and operation management

Illustration of MTPC ($\omega_\mathrm{p}^1 = \omega_\mathrm{p,R}$), FW ($\omega_\mathrm{p}^2 = 2\omega_\mathrm{p,R}$) and MTPV ($\omega_\mathrm{p}^3 = 3\omega_\mathrm{p,R}$)



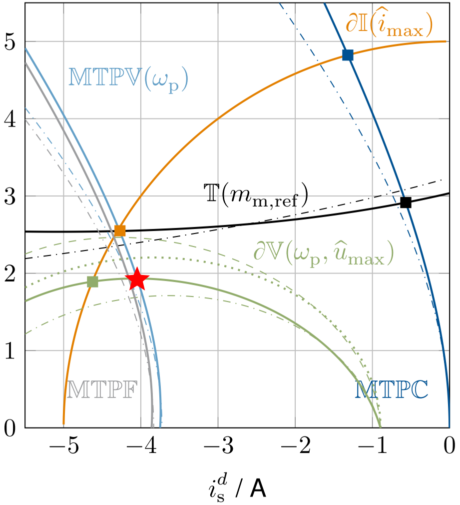
Remark:

$\lim_{\omega_\mathrm{p}\to\infty} \mathbb{MTPV}(\omega_\mathrm{p}) = \mathbb{MTPF}$

or $\mathbb{MTPV}(\omega_\mathrm{p})|_{R_\mathrm{s}=0} = \mathbb{MTPF}$

Optimal operation strategies and operation management

Illustration of the effect of neglecting R_s and/or $L_{s,m}$



- - - - $R_s = 0$ (and $L_{s,m} \neq 0$)

-·-·- $L_{s,m} = 0$ (and $R_s \neq 0$)

······ $R_s = 0$ and $L_{s,m} = 0$

$\Longrightarrow$ **Reduced efficiency**!
(and wrong torque and/or possibly wrong operation strategies)

Analytical solution

Example: Analytical solution for intersection points of two quadrics

Problem

$$\mathbb{X} := \left\{ \boldsymbol{x} \in \mathbb{R}^2 \;\middle|\; \underbrace{\boldsymbol{x}^\top \boldsymbol{A}\boldsymbol{x} + 2\boldsymbol{a}^\top \boldsymbol{x} + \alpha}_{=:Q_A(\boldsymbol{x})} = 0 \;\wedge\; \underbrace{\boldsymbol{x}^\top \boldsymbol{B}\boldsymbol{x} + 2\boldsymbol{b}^\top \boldsymbol{x} + \beta}_{=:Q_B(\boldsymbol{x})} = 0 \right\}$$

Solution: Relate both (e.g., if $\alpha \neq 0$ and $\beta \neq 0$)

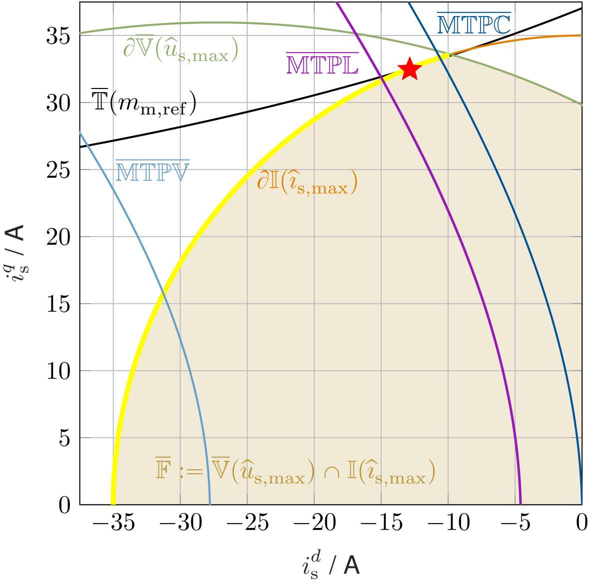
$$Q_D(\boldsymbol{x}) := \frac{Q_A(\boldsymbol{x})}{\alpha} - \frac{Q_B(\boldsymbol{x})}{\beta} = \boldsymbol{x}^\top \underbrace{\left(\frac{\boldsymbol{A}}{\alpha} - \frac{\boldsymbol{B}}{\beta}\right)}_{=:\boldsymbol{D}} \boldsymbol{x} + 2 \underbrace{\left(\frac{\boldsymbol{a}}{\alpha} - \frac{\boldsymbol{b}}{\beta}\right)^\top}_{=:\boldsymbol{d}^\top} \boldsymbol{x} + \underbrace{\left(\frac{\alpha}{\alpha} - \frac{\beta}{\beta}\right)}_{=0} = 0$$

and note that

$$Q_D(\boldsymbol{x}) := \boldsymbol{x}^\top \big(\underbrace{\boldsymbol{D}\boldsymbol{x} + 2\boldsymbol{d}}_{\stackrel{!}{=}\lambda \boldsymbol{J}\boldsymbol{x}} \big) \stackrel{!}{=} 0 \quad \Longleftrightarrow \quad \boldsymbol{x}^\star(\lambda) = -2\big(\boldsymbol{D} - \lambda \boldsymbol{J}\big)^{-1} \boldsymbol{d}$$

Optimal operation strategies and operation management

Maximum Current extended (**MC**$_{\mathrm{ext}}$ @ $1 \cdot \omega_{\mathrm{m,R}}$): Reference torque **feasible**



Optimization problem

$$\max_{\boldsymbol{i}_s^{dq} \in \overline{\mathbb{F}}} |(m_{\mathrm{m}}(\boldsymbol{i}_s^{dq}))| \quad \text{s.t.}$$

$$\mathrm{sign}(m_{\mathrm{m}}) = \mathrm{sign}(m_{\mathrm{m,ref}})$$

Solution set

$$\overline{\mathrm{MC}}_{\mathrm{ext}}(\overline{\boldsymbol{i}}_s^{dq}, \overline{\omega}_{\mathrm{p}}, \ldots, \widehat{u}_{\mathrm{s,max}}, \widehat{\imath}_{\mathrm{s,max}}) := \overline{\mathbb{V}}(\widehat{u}_{\mathrm{s,max}}) \cap \partial\mathbb{I}(\widehat{\imath}_{\mathrm{s,max}})$$

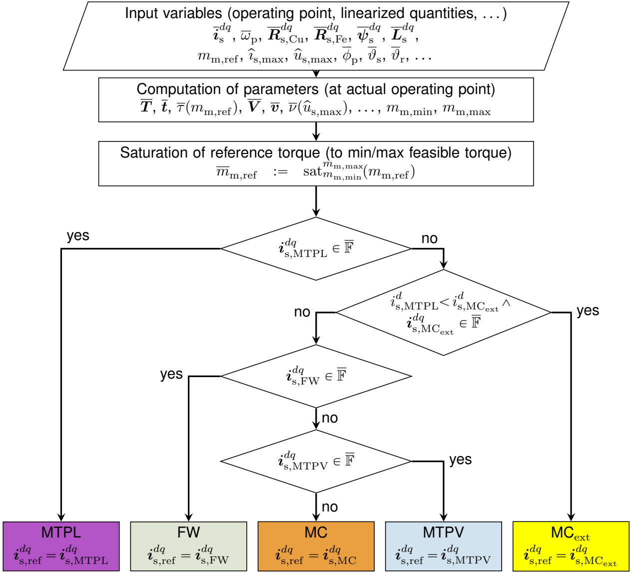
Optimal current reference (★)

$$\boldsymbol{i}_{\mathrm{s,MC_{ext}}}^{dq} = \overline{\mathbb{T}}(m_{\mathrm{m,ref}}) \cap \partial\mathbb{I}(\widehat{\imath}_{\mathrm{s,max}})$$

with $i_{\mathrm{s,MTPL}}^d < i_{\mathrm{s,MC_{ext}}}^d$

Optimal operation strategies and operation management

Operation management and decision tree



Analytical solution

Example: Analytical solution for intersection points of two quadrics (cont'd)

then insert $\boldsymbol{x}^\star(\lambda)$ in $Q_A(\boldsymbol{x})$ (or $Q_B(\boldsymbol{x})$), i.e.

$$Q_A(\boldsymbol{x}^\star(\lambda)) =$$

$$4\boldsymbol{d}^\top\big(\boldsymbol{D}-\lambda\boldsymbol{J}\big)^{-\top}\boldsymbol{A}\big(\boldsymbol{D}-\lambda\boldsymbol{J}\big)^{-1}\boldsymbol{d} - 4\boldsymbol{a}^\top\big(\boldsymbol{D}-\lambda\boldsymbol{J}\big)^{-1}\boldsymbol{d} + \alpha = 0 \,\Big|\, \cdot \Big[\underbrace{\det\big(\boldsymbol{D}-\lambda\boldsymbol{J}\big)}_{\propto\lambda^2}\Big]^2$$

$$\Longrightarrow \boxed{\chi(\lambda) := c_4\,\lambda^4 + c_3\,\lambda^3 + c_2\,\lambda^2 + c_1\,\lambda + c_0 \overset{!}{=} 0}$$

$\Longrightarrow$ Analytical (real) solutions $\lambda^\star \in \{\lambda_1, \ldots, \lambda_4\}$ can be computed [2]:

$$\Longrightarrow \boxed{\boldsymbol{x}^\star(\lambda^\star) = -2\big(\boldsymbol{D} - \lambda^\star\boldsymbol{J}\big)^{-1}\,\boldsymbol{d}}$$

Roots of a fourth-order polynomial

Euler's solution [2]

Problem: Find roots of

$$\chi(\lambda) := c_4\,\lambda^4 + c_3\,\lambda^3 + c_2\,\lambda^2 + c_1\,\lambda + c_0 \overset{!}{=} 0$$

Solution:
Step 1: Transform $\chi(\lambda)$ to *depressed* polynomial $\chi_{\mathrm{dep}}(y)$

$$\chi_{\mathrm{dep}}(y) := y^4 + p\,y^2 + q\,y + r \overset{!}{=} 0 \quad \text{(where } \lambda := y - \tfrac{c_3}{4c_4}\text{)}$$

with coefficients

$$p := \tfrac{1}{c_4^2}\left(c_2\,c_4 - \tfrac{3\,c_3^2}{8}\right), \quad q := \tfrac{1}{c_4^3}\left(\tfrac{c_3^3}{8} - \tfrac{c_2\,c_3\,c_4}{2} + c_1\,c_4^2\right) \quad \text{and}$$
$$r := \tfrac{1}{c_4^4}\left(-\tfrac{3c_3^4}{256} + c_4^3 c_0 - \tfrac{c_4^2 c_3 c_2}{4} + \tfrac{c_4 c_3^2 c_2}{16}\right).$$

Roots of a fourth-order polynomial

Euler's solution [2] (cont'd)

Step 2: Find roots $z_1^\star, z_2^\star, z_3^\star \in \mathbb{C}$ of Euler's resolvent cubic polynomial

$$\chi_{\mathrm{res}}(z) := z^3 + \underbrace{2p}_{=:d_2} z^2 + \underbrace{(p^2 - 4r)}_{=:d_1} z \underbrace{-q^2}_{=:d_0} \stackrel{!}{=} 0$$

by defining

$$\tilde{q} := \frac{d_1}{3} - \frac{d_2^2}{9} \quad \text{and} \quad \tilde{r} := \frac{d_1 d_2 - 3d_0}{6} - \frac{d_2^3}{27} \quad \text{and}$$

$$s_1 := \sqrt[3]{\tilde{r} + \sqrt{\tilde{q}^3 + \tilde{r}^2}} \quad \text{and} \quad s_2 := \sqrt[3]{\tilde{r} - \sqrt{\tilde{q}^3 + \tilde{r}^2}}.$$

Roots of resolvent are then given by

$$z_1^\star = (s_1 + s_2) - \frac{d_2}{3} \quad \text{and} \quad z_{2,3}^\star = -\frac{1}{2}(s_1 + s_2) - \frac{d_2}{3} \pm \jmath \frac{\sqrt{3}}{2}(s_1 - s_2).$$

Roots of a fourth-order polynomial

Euler's solution [2] (cont'd)

Step 3: With the roots $z_1^\star, z_2^\star, z_3^\star \in \mathbb{C}$ of $\chi_{\mathrm{res}}(z)$, the roots $\lambda_1^\star, \ldots, \lambda_4^\star \in \mathbb{C}$ of the fourth-order polynomial $\chi(\lambda)$ are given by

$$
\begin{aligned}
\lambda_1^\star &= \frac{(-1)^l}{2}\left(\sqrt{z_1^\star} + \sqrt{z_2^\star} + \sqrt{z_3^\star} \right) - \frac{c_3}{4c_4}, \\
\lambda_2^\star &= \frac{(-1)^l}{2}\left(\sqrt{z_1^\star} - \sqrt{z_2^\star} - \sqrt{z_3^\star} \right) - \frac{c_3}{4c_4}, \\
\lambda_3^\star &= \frac{(-1)^l}{2}\left(-\sqrt{z_1^\star} + \sqrt{z_2^\star} - \sqrt{z_3^\star} \right) - \frac{c_3}{4c_4}, \qquad \text{and} \\
\lambda_4^\star &= \frac{(-1)^l}{2}\left(-\sqrt{z_1^\star} - \sqrt{z_2^\star} + \sqrt{z_2^\star} \right) - \frac{c_3}{4c_4},
\end{aligned}
$$

where $l \in \{0, 1\}$ must be chosen such that [2]

$$
(-1)^l \left(\lambda_1^\star \lambda_2^\star \lambda_3^\star + \lambda_1^\star \lambda_2^\star \lambda_4^\star + \lambda_1^\star \lambda_3^\star \lambda_4^\star + \lambda_2^\star \lambda_3^\star \lambda_4^\star \right) = -q.
$$

Implementation results

Exemplary laboratory setup

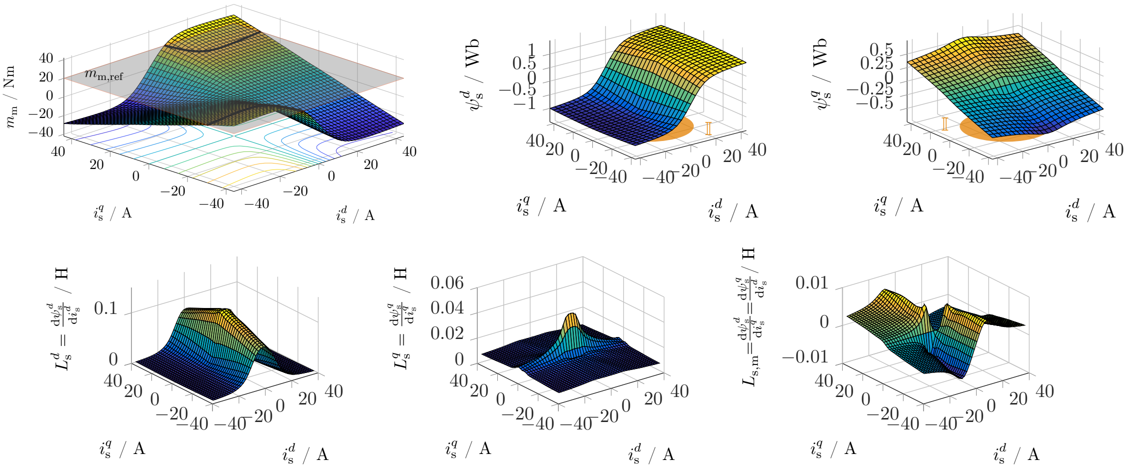
Reluctance SM (RSM) and Permanent-magnet SM

Host PC (rapid prototyping)

Real-time system and VSIs

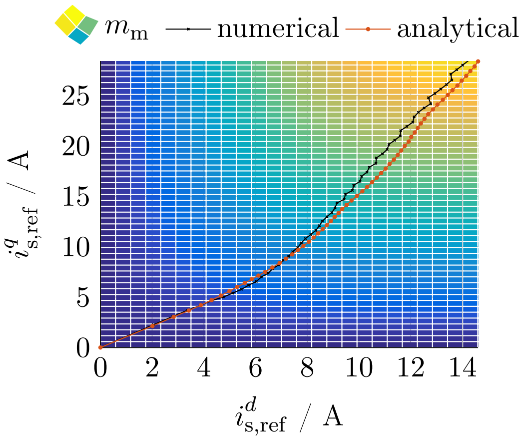
Implementation results

RSM (9.6 kW@1 500 rpm): Nonlinear maps of torque, flux linkages and differential inductances

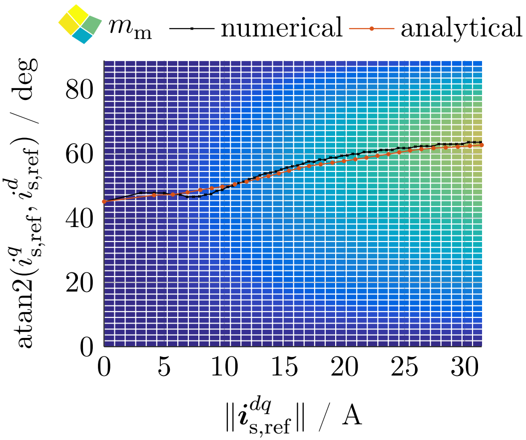


Implementation results

RSM (9.6 kW@1 500 rpm): Comparison of numerical and analytical solutions for MTPC



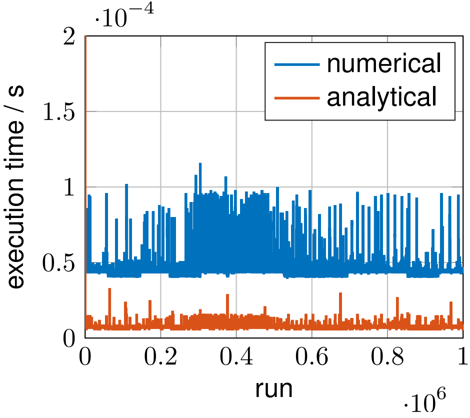
Cartesian coordinates

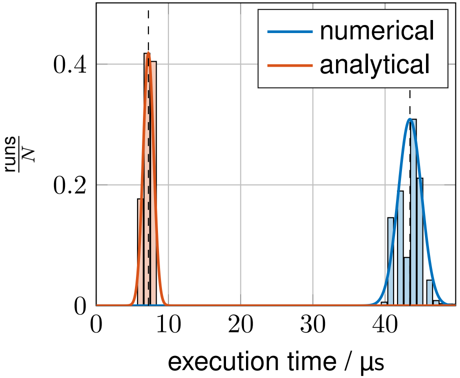


polar coordinates

Implementation results

RSM (9.6 kW@1 500 rpm): Comparison of computational load

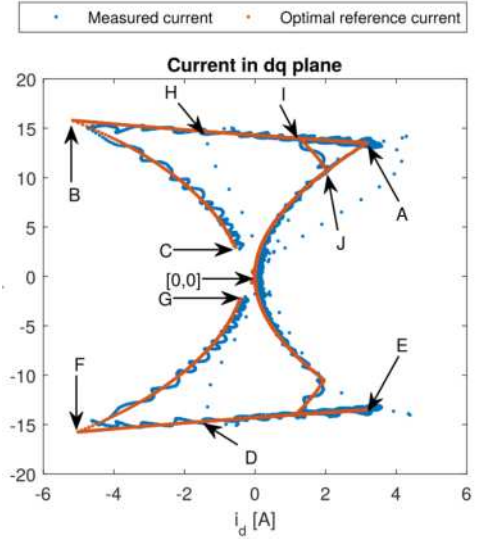


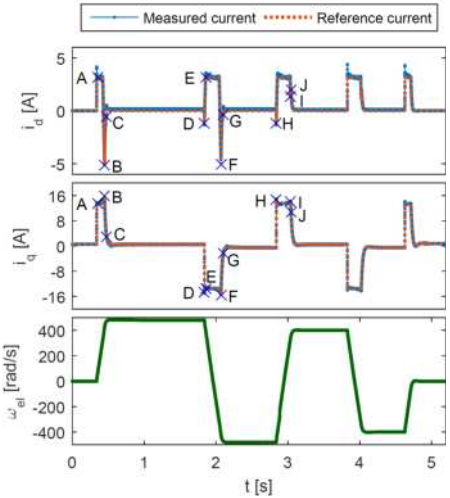


- Execution times for $N = 10^6$ runs (downsampled by factor 100)
- Standard deviations $\sigma_n = 1.61 \cdot 10^{-6}$ s and $\sigma_a = 0.73 \cdot 10^{-6}$ s
- Average execution times $\mu_n = 43.4 \cdot 10^{-6}$ s and $\mu_a = 7.23 \cdot 10^{-6}$ s ($>$**6x faster**)

Implementation results

PME-RSM (4.5 kW@1 500 rpm): Implementation by colleagues $[\max\left(L_s^d(\boldsymbol{i}_s^{dq})/L_s^q(\boldsymbol{i}_s^{dq})\right) = 1.43]$ [?]





Implementation results

IPMSM (generator mode): Efficiency enhancements considering iron losses [11]