Grid state estimation of arbitrarily distorted grids

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German-Turkish Workshop, Technical University of Munich

Outline I

1. Introduction
2. Internal model principle
3. Observer design
4. Implementation
5. Conclusion

Outline I

1 Introduction
- Motivation
- Problem statement
- Proposed solution

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Introduction

Motivation: The future low-inertia power system—A likely guess [1]

- more distributed generation / renewable energy systems
- diminishing mechanical inertia
- hybrid grid and transmission topologies (e.g. AC & DC)
- more unbalanced faults ($> 75\%$ of faults already today)
- more loosely-coupled & unbalanced micro-grids
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$\Longrightarrow$ ... for balancing, harmonics suppression, frequency/voltage support!

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Introduction

Problem statement

For measured grid signal (voltage, current or power), given by

$$y(t) := a_0 + \sum_{\nu \in \mathbb{H}_n} \underbrace{a_\nu(t)\cos\big(\phi_\nu(t)\big)}_{=: y_\nu(t)} \quad \text{where} \quad \begin{cases} \mathbb{H}_n := \{1, \nu_2, \ldots, \nu_n\} \subset \mathbb{Q}_{>0} \\ \phi_\nu(t) := \int_0^t \nu\omega(\tau)\,\mathrm{d}\tau + \phi_{\nu,0}, \end{cases} \tag{1}$$

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the **main goals** are

(i) signal estimation after *prescribed settling time* $t_{\text{set}} > 0\,\text{s}$ for some $\varepsilon_y > 0$, i.e.

$$\forall t \geqslant t_{\text{set}}: \; |y(t) - \widehat{y}(t)| \leqslant \varepsilon_y \quad \text{with} \quad \widehat{y}(t) := \widehat{a}_0 + \sum_{\nu \in \mathbb{H}_n} \underbrace{\widehat{a}_\nu(t) \cos\left(\widehat{\phi}_\nu(t)\right)}_{=: \widehat{y}_\nu(t)}; \tag{2}$$

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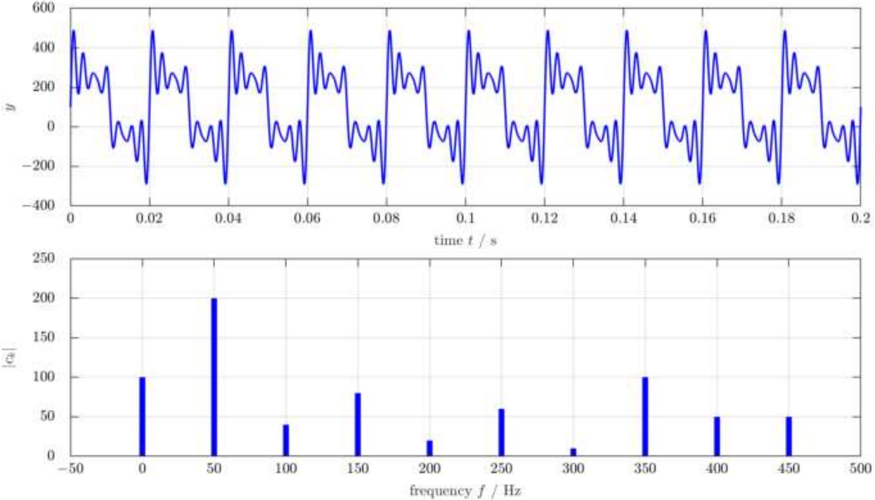
(iii) frequency and angle estimation, i.e. $\hat{\omega} \to \omega$ and $\hat{\phi}_\nu \to \phi_\nu$ for all $\nu \in \mathbb{H}_n$.

Introduction

Problem statement (illustration)



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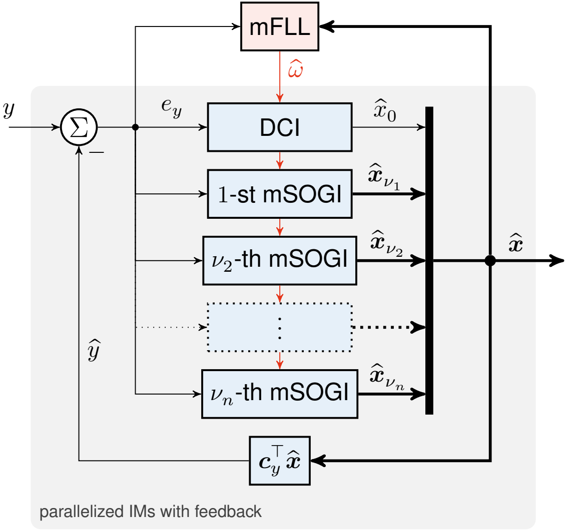
Introduction

Proposed solution: **Nonlinear observer** consisting of ...



- **DC-integrator (DCI)** for DC offset estimation
- **modified Second-Order-Generalized-Integrators (mSOGIs)** for harmonics estimation
- **modified frequency-locked loop (mFLL)** for frequency estimation

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- Motivation
- Signal reduplication
- Parallelization of internal models (IMs)

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Internal model principle

Motivation

Walter M. Wonham (1985):
"Every good regulator [or observer] must incorporate a model of the outside world being capable to reduplicate the dynamic structure of the exogenous signals which the regulator [or observer] is required to process." [2]

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Internal model principle

Constant signal reduplication

constant IM (with initial value)

$$x_{0,0} = a_0 \longrightarrow \boxed{\mathsf{IM}_{\mathrm{dc}}} \longrightarrow x_0$$

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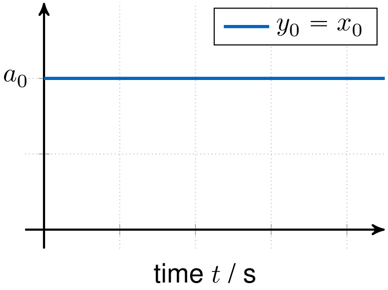

Internal model principle

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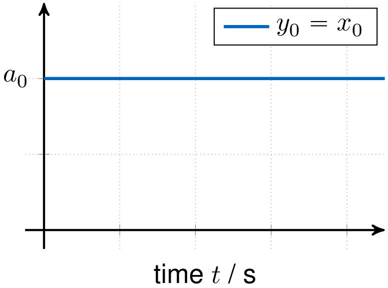

$$\frac{\mathrm{d}}{\mathrm{d}t} x_0 = 0, \quad x_0(0) = x_{0,0}$$

$$y_0 = x_0$$

Internal model principle

Example: Sinusoidal signal reduplication of the ν-th harmonic where $\nu \in \mathbb{H}_n$

sinusoidal IM (with initial values)

$$\boldsymbol{x}_{\nu,0} = a_\nu \begin{pmatrix} \cos(\phi_{\nu,0}) \\ \sin(\phi_{\nu,0}) \end{pmatrix} \longrightarrow \boxed{\nu\text{-th IM}_{\mathrm{sin}}} \xrightarrow{} \boldsymbol{x}_\nu = \begin{pmatrix} x_\nu^\alpha \\ x_\nu^\beta \end{pmatrix}$$

ω

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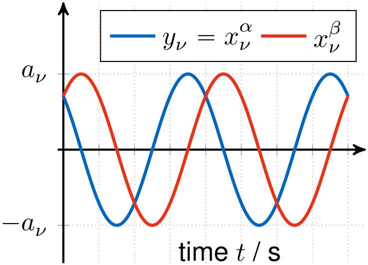

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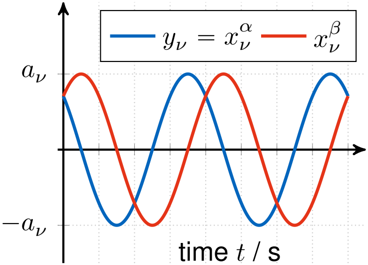

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x}_\nu = \omega(t)\begin{bmatrix} 0 & -\nu \\ \nu & 0 \end{bmatrix}\boldsymbol{x}_\nu, \quad \boldsymbol{x}_\nu(0) = \boldsymbol{x}_{\nu,0}$$

$$y_\nu = \begin{pmatrix} 1, & 0 \end{pmatrix}\boldsymbol{x}_\nu$$

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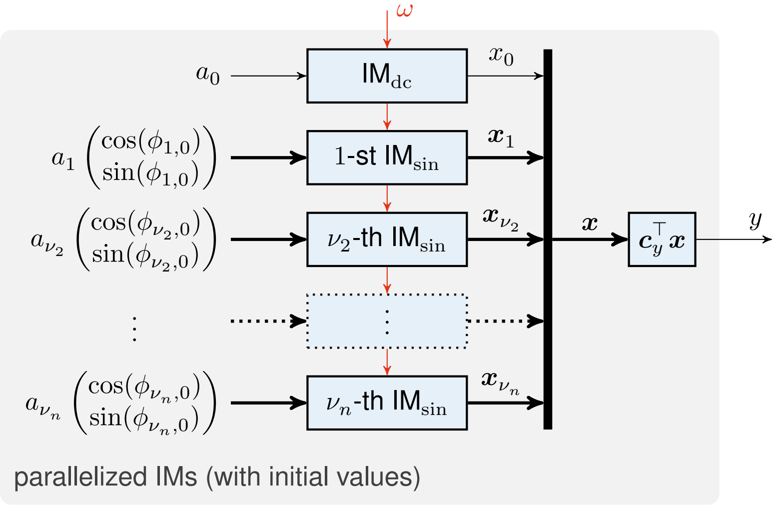
Internal model principle

Overall signal reduplication by parallelization of IMs



Internal model principle

Dynamics of parallelized IMs

$$\frac{\mathrm{d}}{\mathrm{d}t}\overbrace{\begin{pmatrix} x_0 \\ \begin{pmatrix} x_1^\alpha \\ x_1^\beta \end{pmatrix} \\ \begin{pmatrix} x_{\nu_2}^\alpha \\ x_{\nu_2}^\beta \end{pmatrix} \\ \vdots \\ \begin{pmatrix} x_{\nu_n}^\alpha \\ x_{\nu_n}^\beta \end{pmatrix} \end{pmatrix}}^{=:\boldsymbol{x}\in\mathbb{R}^{2n+1}} = \omega(t)\overbrace{\begin{bmatrix} 0 & & & & \\ & \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} & & & \\ & & \nu_2\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} & & \\ & & & \ddots & \\ & & & & \nu_n\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \end{bmatrix}}^{=:\boldsymbol{A}\in\mathbb{R}^{(2n+1)\times(2n+1)}}\boldsymbol{x}, \quad \boldsymbol{x}(0) = \boldsymbol{x}_0$$

$$y = \underbrace{\big(1, \quad (1, \quad 0), \quad (1, \quad 0), \quad \ldots \quad (1, \quad 0)\big)}_{=:\boldsymbol{c}_y^\top\in\mathbb{R}^{1\times(2n+1)}}\boldsymbol{x}$$

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$\Longrightarrow$ Observation problem of an **observable** system $\Longrightarrow$ **Observer design**

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3 Observer design

- Overall observer and pole placement
- Modified Second-order generalized integrators (mSOGIs)
- Overall observer structure

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Overall observer and pole placement: parallelized internal models with **feedback**

For $\boldsymbol{J} := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $\boldsymbol{l}_\nu := \nu \binom{k_\nu}{g_\nu}$, define the observer dynamics

$$\frac{\mathrm{d}}{\mathrm{d}t} \underbrace{\begin{pmatrix} \widehat{x}_0 \\ \widehat{\boldsymbol{x}}_1 \\ \widehat{\boldsymbol{x}}_{\nu_2} \\ \vdots \\ \widehat{\boldsymbol{x}}_{\nu_n} \end{pmatrix}}_{=:\widehat{\boldsymbol{x}} \in \mathbb{R}^{2n+1}} = \widehat{\omega}(t) \underbrace{\left[\begin{bmatrix} 0 & & & & \\ & \boldsymbol{J} & & & \\ & & \nu_2 \boldsymbol{J} & & \\ & & & \ddots & \\ & & & & \nu_n \boldsymbol{J} \end{bmatrix} - \begin{pmatrix} l_0 \\ \boldsymbol{l}_1 \\ \boldsymbol{l}_{\nu_2} \\ \vdots \\ \boldsymbol{l}_{\nu_n} \end{pmatrix} \boldsymbol{c}_y^\top \right]}_{= \boldsymbol{A} - \boldsymbol{l}\boldsymbol{c}_y^\top =: \boldsymbol{H}} \widehat{\boldsymbol{x}} - \widehat{\omega}(t) \boldsymbol{l}\, y, \quad \widehat{\boldsymbol{x}}(0) = \widehat{\boldsymbol{x}}_0$$

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- Recall: $(\boldsymbol{c}_y^\top, \boldsymbol{A})$ observable $\Longrightarrow \exists \boldsymbol{l} \in \mathbb{R}^{2n+1}$: $\boldsymbol{H} := \boldsymbol{A} - \boldsymbol{l}\boldsymbol{c}_y^\top$ Hurwitz!

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- Pole placement: $\boldsymbol{l} = \boldsymbol{S}\boldsymbol{p}_{\boldsymbol{H}}^* \Longrightarrow s_0^* = -p_0^* \wedge \forall \nu \in \mathbb{H}_n: \; s_{\nu,1/2}^* = -p_\nu^* \pm \jmath\nu$

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Observer design

Standard SOGI (sSOGI) for ν-th harmonic estimation: ν-th IM$_{\sin}$ with **feedback**

$$\frac{\mathrm{d}}{\mathrm{d}t}\widehat{\boldsymbol{x}}_\nu(t) = \widehat{\omega}(t)\overbrace{\begin{bmatrix} -\nu k_\nu & -\nu \\ \nu & 0 \end{bmatrix}}^{=:\boldsymbol{A}_\nu - \boldsymbol{l}_\nu(k_\nu)\boldsymbol{c}_y^\top}\widehat{\boldsymbol{x}}_\nu(t) + \widehat{\omega}(t)\overbrace{\begin{pmatrix} \nu k_\nu \\ 0 \end{pmatrix}}^{=:\boldsymbol{l}_\nu(k_\nu)} y_\nu(t), \quad \widehat{\boldsymbol{x}}_\nu(0) = \widehat{\boldsymbol{x}}_{\nu,0}$$

$$\widehat{y}_\nu(t) = \boldsymbol{c}_\nu^\top \widehat{\boldsymbol{x}}_\nu(t)$$

Observer design

Modified SOGI (mSOGI) for ν-th harmonic estimation: ν-th $\mathrm{IM}_{\mathrm{sin}}$ **with** extended **feedback**

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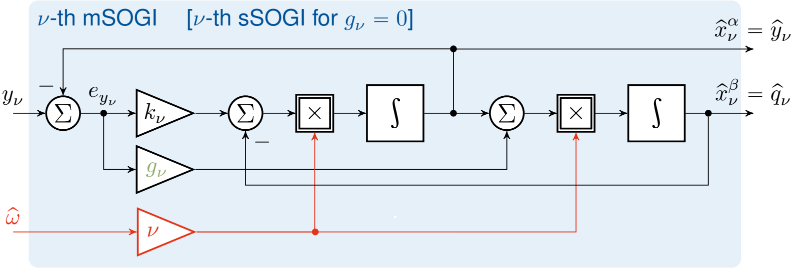

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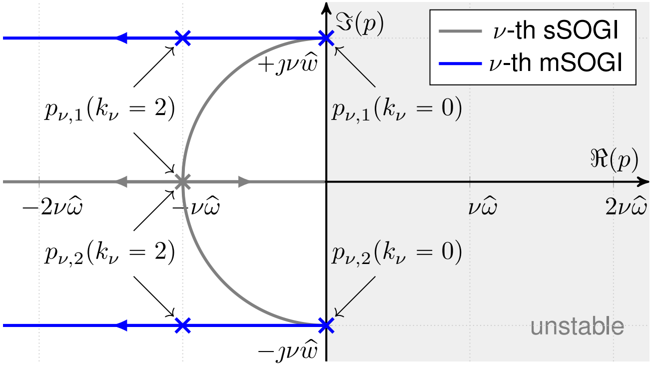
Observer design

Comparison of root loci of ν-th sSOGI and mSOGI



sSOGI

$$p_{\nu,1/2} = -\frac{\nu\widehat{\omega}k_\nu}{2}\left(1 \pm \sqrt{1 - \frac{4}{k_\nu^2}}\right)$$

mSOGI (for $g_\nu = -\frac{k_\nu^2}{4}$)

$$p_{\nu,1/2} = -\frac{\nu\widehat{\omega}k_\nu}{2} \pm \jmath\nu\widehat{\omega}.$$

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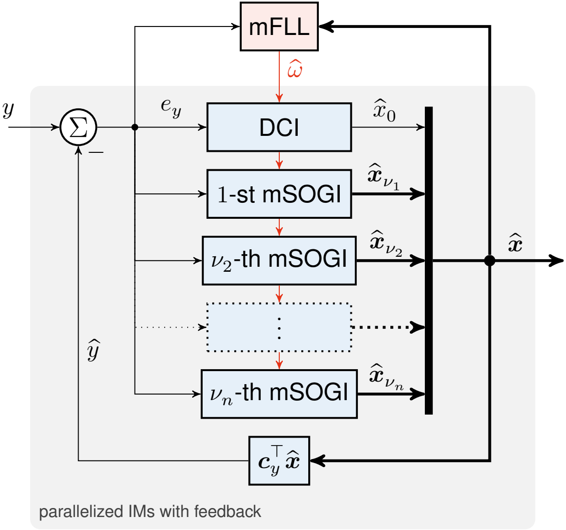
Overall observer structure



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4 Implementation
- Measurement results

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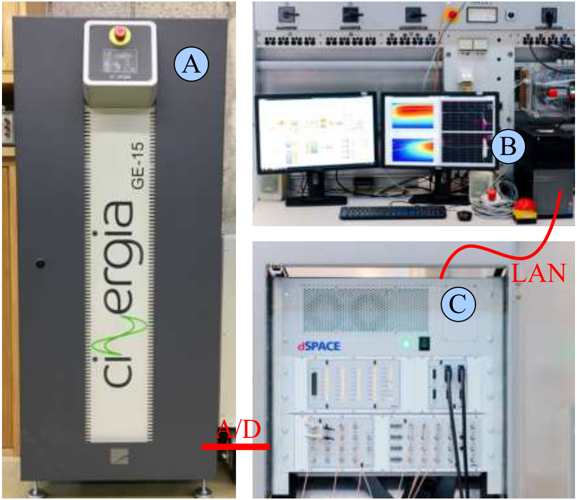
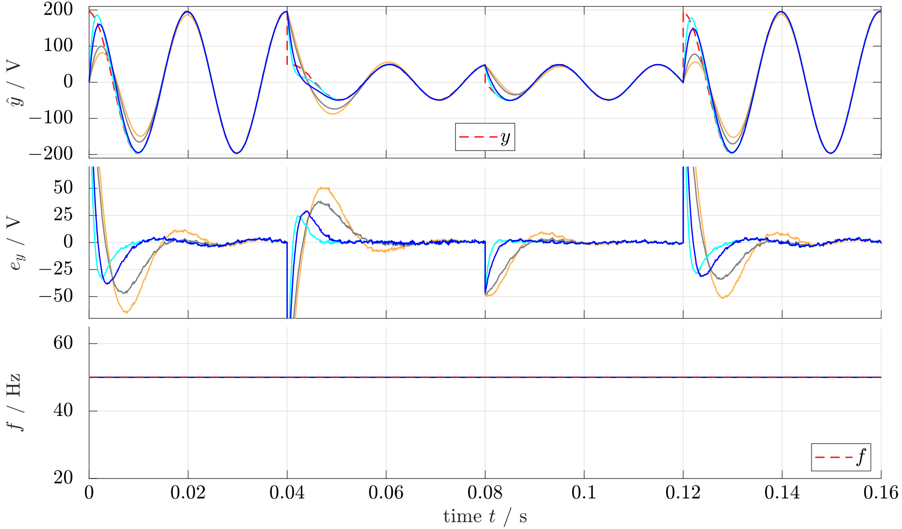
Implementation

Laboratory setup: (A) Grid emulator, (B) Host-PC and (C) Real-time system



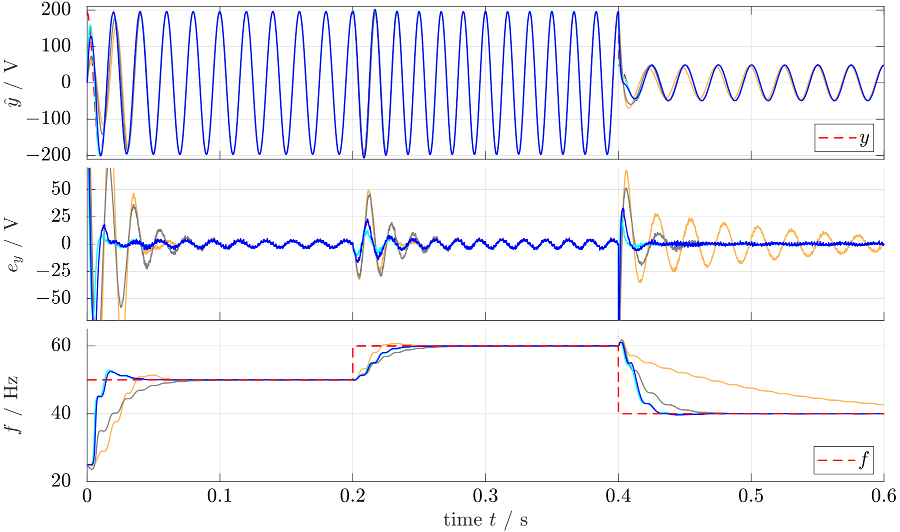
Implementation

Measurements **w/o** FLL ($\nu = 1$): mSOGI, mSOGI$_{\text{fast}}$, sSOGI & ANF



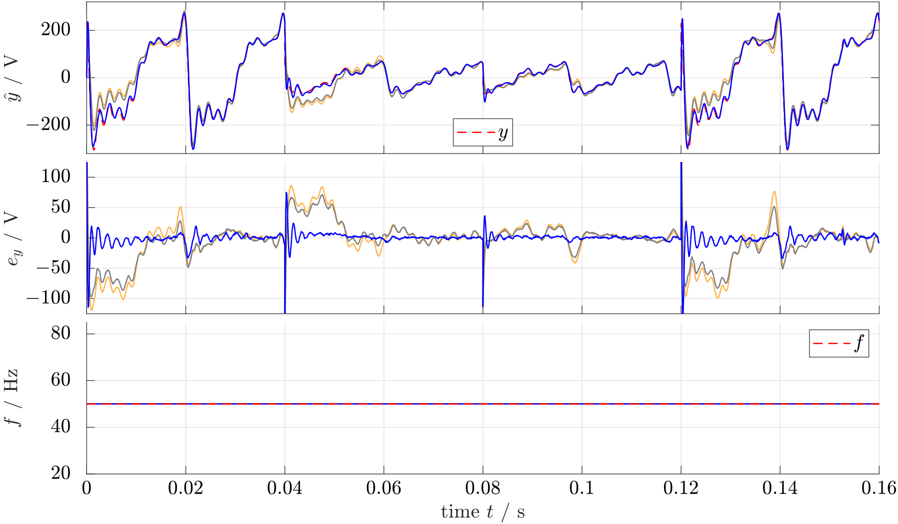
Implementation

Measurements **with** FLL ($\nu = 1$): mSOGI, $\text{mSOGI}_{\text{fast}}$, sSOGI & ANF



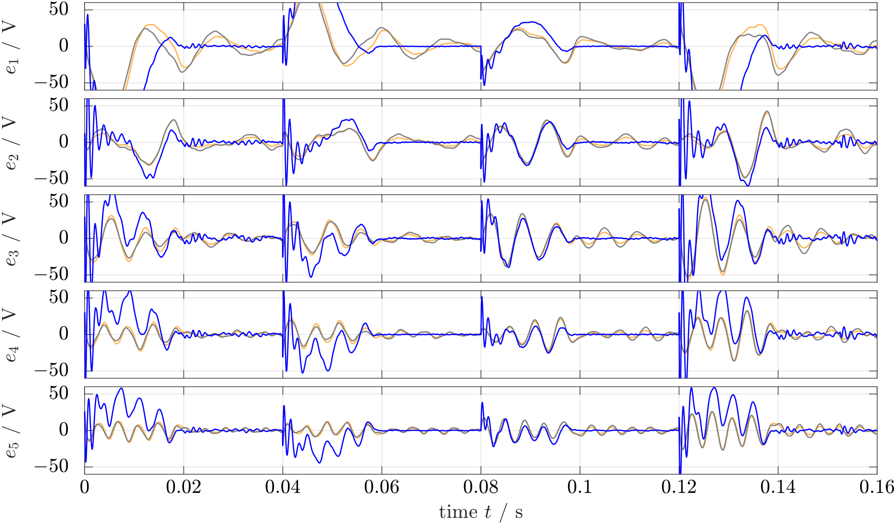
Implementation

Measurements **w/o** FLL ($\nu = 10$): mSOGI, sSOGI & ANF



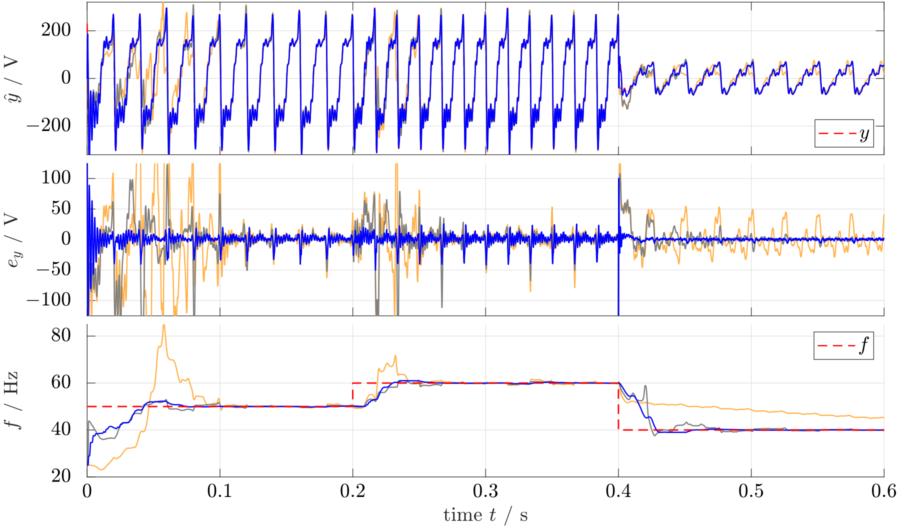
Implementation

Measurements **w/o** FLL ($\nu = 10$): $e_\nu = y_\nu - \widehat{y}_\nu$ for mSOGI, sSOGI & ANF



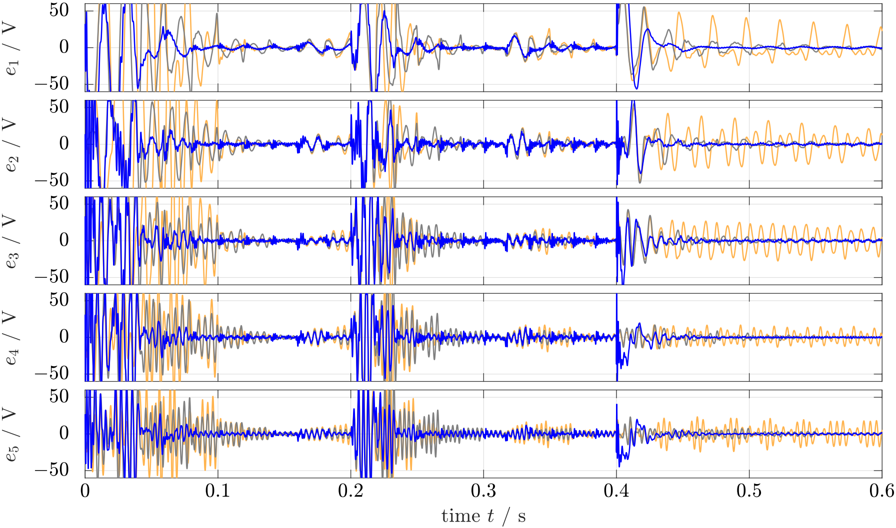
Implementation

Measurements **with** FLL ($\nu = 10$): mSOGI, sSOGI & ANF



Implementation

Measurements **with** FLL ($\nu = 10$): $e_\nu = y_\nu - \widehat{y}_\nu$ for mSOGI, sSOGI & ANF



Outline I

5 Conclusion

Conclusion

Summary and outlook

To take home

- **unified theory** for grid state estimation, based on

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- Frequency Locked-Loop weakest component $\Longrightarrow$ further improvement
- Reduction of noise sensitivity $\Longrightarrow$ (extended) Kalman filter design or pre-filtering?

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Future work

- Frequency Locked-Loop weakest component $\Longrightarrow$ further improvement
- Reduction of noise sensitivity $\Longrightarrow$ (extended) Kalman filter design or pre-filtering?
- three-phase implementation, validation and test in laboratory

Outline I

6 Appendix

- Internal model principle
- Observer design
- Frequency estimation
- Implementation
- Harmonic sequence extraction

Outline I

6 Appendix

- Internal model principle
  - Motivation
  - Signal reduplication
  - Parallelization of internal models (IMs)
  - Observability of parallelized IMs
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Internal model principle

Motivation

Walter M. Wonham (1985):
"Every good regulator [or observer] must incorporate a model of the outside world being capable to reduplicate the dynamic structure of the exogenous signals which the regulator [or observer] is required to process." [2]

Internal model principle

Constant signal reduplication

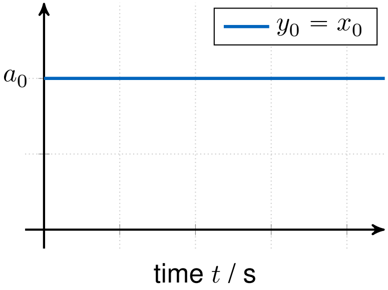
constant IM (with initial value)

Internal model principle

Constant signal reduplication

constant IM (with initial value)

$$x_{0,0} = a_0 \longrightarrow \boxed{\mathrm{IM}_{\mathrm{dc}}} \longrightarrow x_0$$



Internal model principle

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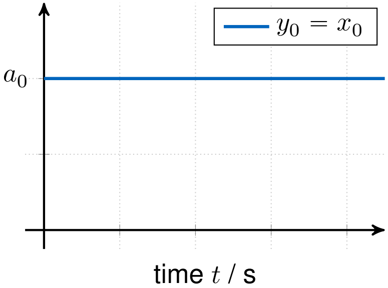

$$\frac{\mathrm{d}}{\mathrm{d}t} x_0 = 0, \quad x_0(0) = x_{0,0}$$

$$y_0 = x_0$$

Internal model principle

Sinusoidal signal reduplication of the ν-th harmonic where $\nu \in \mathbb{H}_n$

sinusoidal IM (with initial values)

$$\boldsymbol{x}_{\nu,0} = a_\nu \begin{pmatrix} \cos(\phi_{\nu,0}) \\ \sin(\phi_{\nu,0}) \end{pmatrix} \longrightarrow \boxed{\nu\text{-th IM}_{\sin}} \xrightarrow{} \boldsymbol{x}_\nu = \begin{pmatrix} x_\nu^\alpha \\ x_\nu^\beta \end{pmatrix}$$

ω

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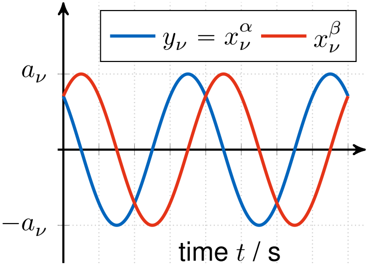

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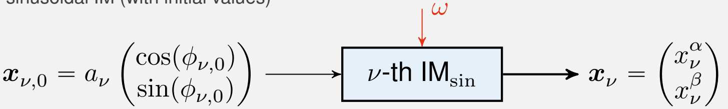
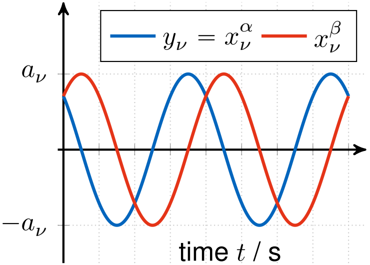


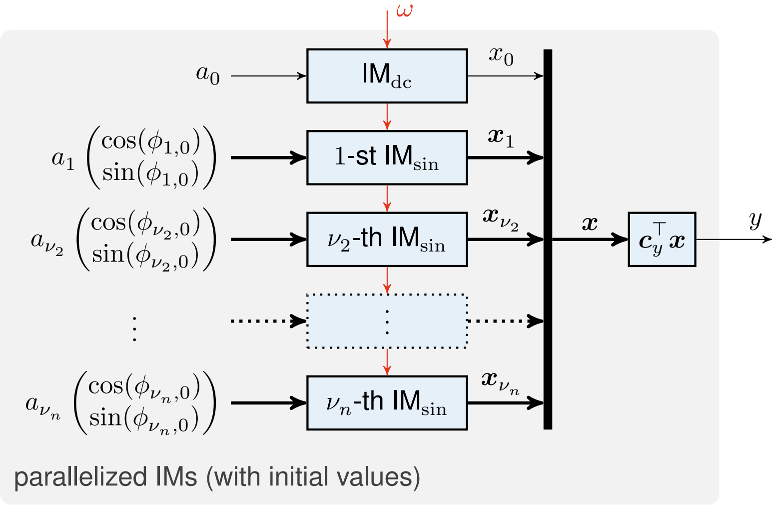


$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x}_\nu = \omega(t)\begin{bmatrix} 0 & -\nu \\ \nu & 0 \end{bmatrix}\boldsymbol{x}_\nu, \quad \boldsymbol{x}_\nu(0) = \boldsymbol{x}_{\nu,0}$$

$$y_\nu = \begin{pmatrix} 1, & 0 \end{pmatrix}\boldsymbol{x}_\nu$$

Internal model principle

Overall signal reduplication by parallelization of IMs



Internal model principle

Dynamics of parallelized IMs

$$
\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\begin{pmatrix} x_0 \\ \begin{pmatrix} x_1^\alpha \\ x_1^\beta \end{pmatrix} \\ \begin{pmatrix} x_{\nu_2}^\alpha \\ x_{\nu_2}^\beta \end{pmatrix} \\ \vdots \\ \begin{pmatrix} x_{\nu_n}^\alpha \\ x_{\nu_n}^\beta \end{pmatrix} \end{pmatrix}}_{=:\boldsymbol{x}\in\mathbb{R}^{2n+1}} = \omega(t) \underbrace{\begin{bmatrix} 0 & & & & \\ & \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} & & & \\ & & \nu_2 \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} & & \\ & & & \ddots & \\ & & & & \nu_n \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \end{bmatrix}}_{=:\boldsymbol{A}\in\mathbb{R}^{(2n+1)\times(2n+1)}} \boldsymbol{x}, \quad \boldsymbol{x}(0) = \boldsymbol{x}_0
$$

$$
y = \underbrace{\begin{pmatrix} 1, & (1, & 0), & (1, & 0), & \ldots & (1, & 0) \end{pmatrix}}_{=:\boldsymbol{c}_y^\top\in\mathbb{R}^{1\times(2n+1)}} \boldsymbol{x}
$$

Internal model principle

Dynamics of parallelized IMs

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\begin{pmatrix} x_0 \\ \begin{pmatrix} x_1^\alpha \\ x_1^\beta \end{pmatrix} \\ \begin{pmatrix} x_{\nu_2}^\alpha \\ x_{\nu_2}^\beta \end{pmatrix} \\ \vdots \\ \begin{pmatrix} x_{\nu_n}^\alpha \\ x_{\nu_n}^\beta \end{pmatrix} \end{pmatrix}}_{=:\boldsymbol{x}\in\mathbb{R}^{2n+1}} = \omega(t) \overbrace{\begin{bmatrix} 0 & & & & \\ & \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} & & & \\ & & \nu_2 \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} & & \\ & & & \ddots & \\ & & & & \nu_n \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \end{bmatrix}}^{=:\boldsymbol{A}\in\mathbb{R}^{(2n+1)\times(2n+1)}} \boldsymbol{x}, \quad \boldsymbol{x}(0) = \boldsymbol{x}_0$$

$$y = \underbrace{\big(1, \quad (1, \quad 0), \quad (1, \quad 0), \quad \ldots \quad (1, \quad 0)\big)}_{=:\boldsymbol{c}_y^\top\in\mathbb{R}^{1\times(2n+1)}} \boldsymbol{x}$$

$\Longrightarrow$ Observation problem of a dynamical system ... if observable!

Internal model principle

Observability of parallelized IMs (for constant $\omega > 0$)

$$\operatorname{rank}\begin{bmatrix} \boldsymbol{c}_y^\top \\ \boldsymbol{c}_y^\top \boldsymbol{A} \\ \boldsymbol{c}_y^\top \boldsymbol{A}^2 \\ \boldsymbol{c}_y^\top \boldsymbol{A}^3 \\ \boldsymbol{c}_y^\top \boldsymbol{A}^4 \\ \vdots \\ \boldsymbol{c}_y^\top \boldsymbol{A}^{2n-1} \\ \boldsymbol{c}_y^\top \boldsymbol{A}^{2n} \end{bmatrix} =$$

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$$\operatorname{rank}\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & \cdots & 1 & 0 \\ 0 & 0 & -1 & 0 & -\nu_2 & \cdots & 0 & -\nu_n \\ 0 & -1 & 0 & -\nu_2^2 & 0 & \cdots & -\nu_n^2 & 0 \\ 0 & 0 & 1 & 0 & \nu_2^3 & \cdots & 0 & \nu_n^3 \\ 0 & 1 & 0 & \nu_2^4 & 0 & \cdots & \nu_n^4 & 0 \\ 0 & \vdots & & & \vdots & & & \vdots \\ 0 & 0 & (-1)^n & 0 & (-1)^n \nu_2^{2n-1} & \cdots & 0 & (-1)^n \nu_n^{2n-1} \\ 0 & (-1)^n & 0 & (-1)^n \nu_2^{2n} & 0 & \cdots & (-1)^n \nu_n^{2n} & 0 \end{bmatrix}$$

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$$= \boxed{2n+1 \quad \Longleftarrow \quad \nu_i \neq \nu_j \text{ for all } i, j \in \{1, \ldots, n\}}$$

Internal model principle

Observability of linear time-varying systems [3, 9.10 Theorem]

Theorem (Observability of linear time-varying systems)

For $n, p, q \in \mathbb{N}$, let $\boldsymbol{A}(\cdot) \in \mathcal{C}^{q-1}([t_0, t_1]; \mathbb{R}^{n \times n})$, $\boldsymbol{C}(\cdot) \in \mathcal{C}^{q}([t_0, t_1]; \mathbb{R}^{p \times n})$ and

$$\left.\begin{array}{rcl} \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} & = & \boldsymbol{A}(t)\boldsymbol{x}, \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0 \\ y & = & \boldsymbol{C}(t)\boldsymbol{x}. \end{array}\right\} \tag{3}$$

and define

$$\boldsymbol{L}_0 := \boldsymbol{C}(t) \quad \text{and} \quad \forall j \in \{0, 1, 2, \dots\} \colon \boldsymbol{L}_j := \boldsymbol{L}_{j-1}\boldsymbol{A}(t) + \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{L}_{j-1}(t).$$

Then, system (3) is observable on $[t_0, t_1] \subset \mathbb{R}_{\geqslant 0}$ if, for some $\tau \in [t_0, t_1]$,

$$\operatorname{rank} \begin{bmatrix} \boldsymbol{L}_0(\tau) \\ \boldsymbol{L}_1(\tau) \\ \vdots \\ \boldsymbol{L}_q(\tau) \end{bmatrix} = n.$$

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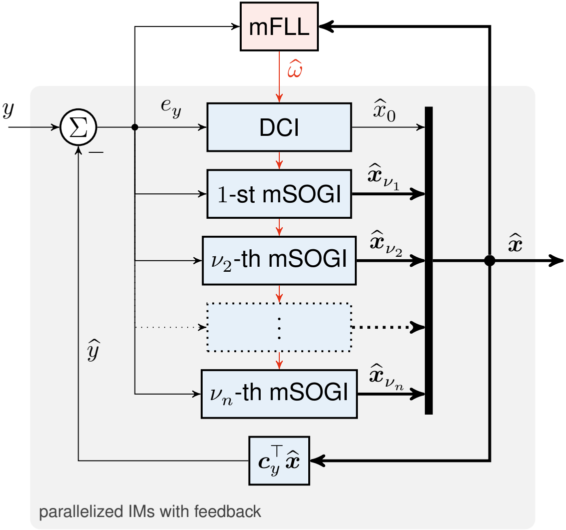
Overall observer structure



Observer design

DC-Integrator (DCI) for constant signal estimation: $\mathrm{IM}_{\mathrm{dc}}$ with **feedback**

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\widehat{x}_0 &= 0 \; \widehat{x}_0 \quad , \quad \widehat{x}_0(0) = 0 \\
\widehat{y}_0 &= \widehat{x}_0
\end{aligned}
$$

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DC-Integrator (DCI) for constant signal estimation: $\mathsf{IM}_{\mathrm{dc}}$ with **feedback**

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\widehat{x}_0 &= -\widehat{\omega}(t) l_0 \, \widehat{x}_0 + \widehat{\omega}(t) l_0 y_0, \qquad \widehat{x}_0(0) = 0 \\
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\end{aligned}
$$

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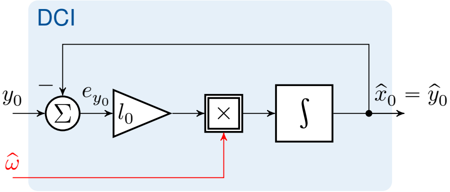

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\widehat{y}_0 &= \widehat{x}_0
\end{aligned}
$$

Observer design

Standard SOGI (sSOGI) for ν-th harmonic estimation: ν-th IM$_{\sin}$ with **feedback**

$$\frac{\mathrm{d}}{\mathrm{d}t}\overbrace{\begin{pmatrix}\widehat{x}_\nu^\alpha(t)\\ \widehat{x}_\nu^\beta(t)\end{pmatrix}}^{=:\widehat{\boldsymbol{x}}_\nu(t)\in\mathbb{R}^2} = \widehat{\omega}(t)\overbrace{\begin{bmatrix}0 & -\nu\\ \nu & 0\end{bmatrix}}^{=:\boldsymbol{A}_\nu}\widehat{\boldsymbol{x}}_\nu(t) \qquad , \quad \widehat{\boldsymbol{x}}_\nu(0)=\widehat{\boldsymbol{x}}_{\nu,0}$$

$$\widehat{y}_\nu(t) = \underbrace{\begin{pmatrix}1, & 0\end{pmatrix}}_{=:\boldsymbol{c}_\nu^\top\in\mathbb{R}^{1\times 2}}\widehat{\boldsymbol{x}}_\nu(t)$$

Observer design

Standard SOGI (sSOGI) for ν-th harmonic estimation: ν-th $\mathrm{IM}_{\mathrm{sin}}$ with **feedback**

$$\frac{\mathrm{d}}{\mathrm{d}t}\overbrace{\begin{pmatrix}\widehat{x}_\nu^\alpha(t)\\ \widehat{x}_\nu^\beta(t)\end{pmatrix}}^{=:\widehat{\boldsymbol{x}}_\nu(t)\in\mathbb{R}^2} = \widehat{\omega}(t)\overbrace{\begin{bmatrix}-\nu k_\nu & -\nu\\ \nu & 0\end{bmatrix}}^{=:\boldsymbol{A}_\nu-\boldsymbol{l}_\nu(k_\nu)\boldsymbol{c}_y^\top}\widehat{\boldsymbol{x}}_\nu(t)+\widehat{\omega}(t)\overbrace{\begin{pmatrix}\nu k_\nu\\ 0\end{pmatrix}}^{=:\boldsymbol{l}_\nu(k_\nu)}y_\nu(t), \quad \widehat{\boldsymbol{x}}_\nu(0)=\widehat{\boldsymbol{x}}_{\nu,0}$$

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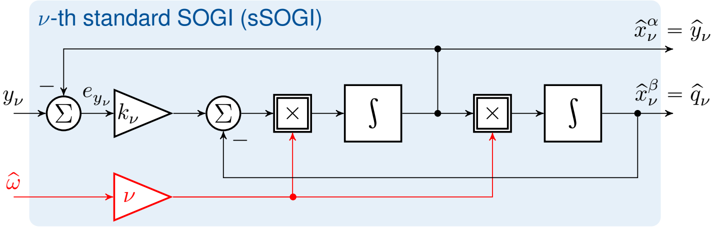

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\begin{pmatrix}\widehat{x}_\nu^\alpha(t)\\ \widehat{x}_\nu^\beta(t)\end{pmatrix}}^{=:\widehat{\boldsymbol{x}}_\nu(t)\in\mathbb{R}^2} = \widehat{\omega}(t)\overbrace{\begin{bmatrix}-\nu k_\nu & -\nu\\ \nu & 0\end{bmatrix}}^{=:\boldsymbol{A}_\nu - \boldsymbol{l}_\nu(k_\nu)\boldsymbol{c}_y^\top}\widehat{\boldsymbol{x}}_\nu(t) + \widehat{\omega}(t)\overbrace{\begin{pmatrix}\nu k_\nu\\ 0\end{pmatrix}}^{=:\boldsymbol{l}_\nu(k_\nu)} y_\nu(t), \quad \widehat{\boldsymbol{x}}_\nu(0) = \widehat{\boldsymbol{x}}_{\nu,0}$$

$$\widehat{y}_\nu(t) = \underbrace{\begin{pmatrix}1, & 0\end{pmatrix}}_{=:\boldsymbol{c}_\nu^\top\in\mathbb{R}^{1\times 2}}\widehat{\boldsymbol{x}}_\nu(t)$$

Observer design

Modified SOGI (mSOGI) for ν-th harmonic estimation: ν-th IM$_{\text{sin}}$ **with** extended **feedback**

$$\frac{\mathrm{d}}{\mathrm{d}t}\widehat{\boldsymbol{x}}_\nu(t) = \widehat{\omega}(t) \overbrace{\begin{bmatrix} -\nu k_\nu & -\nu \\ \nu & 0 \end{bmatrix}}^{=:\boldsymbol{A}_\nu - \boldsymbol{l}_\nu(k_\nu)\boldsymbol{c}_y^\top} \widehat{\boldsymbol{x}}_\nu(t) + \widehat{\omega}(t) \overbrace{\begin{pmatrix} \nu k_\nu \\ 0 \end{pmatrix}}^{=:\boldsymbol{l}_\nu(k_\nu)} y_\nu(t), \quad \widehat{\boldsymbol{x}}_\nu(0) = \widehat{\boldsymbol{x}}_{\nu,0}$$

$$\widehat{y}_\nu(t) = \boldsymbol{c}_\nu^\top \widehat{\boldsymbol{x}}_\nu(t)$$

Observer design

Modified SOGI (mSOGI) for ν-th harmonic estimation: ν-th $\mathrm{IM}_{\sin}$ **with** extended **feedback**

$$\frac{\mathrm{d}}{\mathrm{d}t}\widehat{\boldsymbol{x}}_\nu(t) = \widehat{\omega}(t)\underbrace{\begin{bmatrix} -\nu k_\nu & -\nu \\ \nu - \nu g_\nu & 0 \end{bmatrix}}_{=:\boldsymbol{A}_\nu - \boldsymbol{l}_\nu(k_\nu, g_\nu)\boldsymbol{c}_y^\top}\widehat{\boldsymbol{x}}_\nu(t) + \widehat{\omega}(t)\underbrace{\begin{pmatrix} \nu k_\nu \\ \nu g_\nu \end{pmatrix}}_{=:\boldsymbol{l}_\nu(k_\nu, g_\nu)} y_\nu(t), \quad \widehat{\boldsymbol{x}}_\nu(0) = \widehat{\boldsymbol{x}}_{\nu,0}$$

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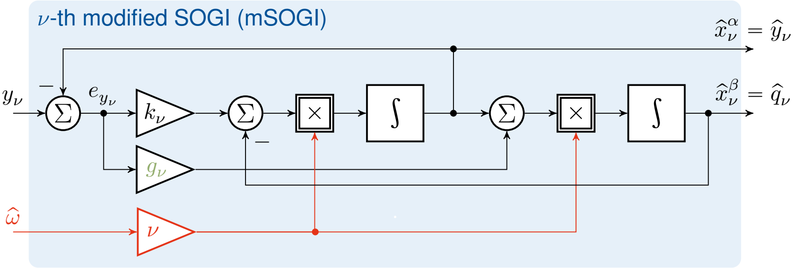

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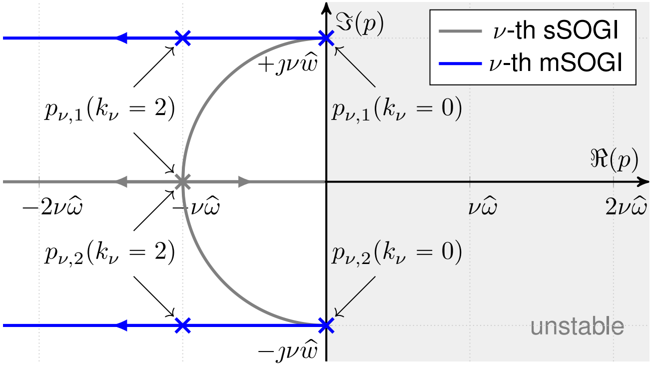
$$\widehat{y}_\nu(t) = \boldsymbol{c}_\nu^\top \widehat{\boldsymbol{x}}_\nu(t)$$

Observer design

Comparison of root loci of ν-th sSOGI and mSOGI



sSOGI

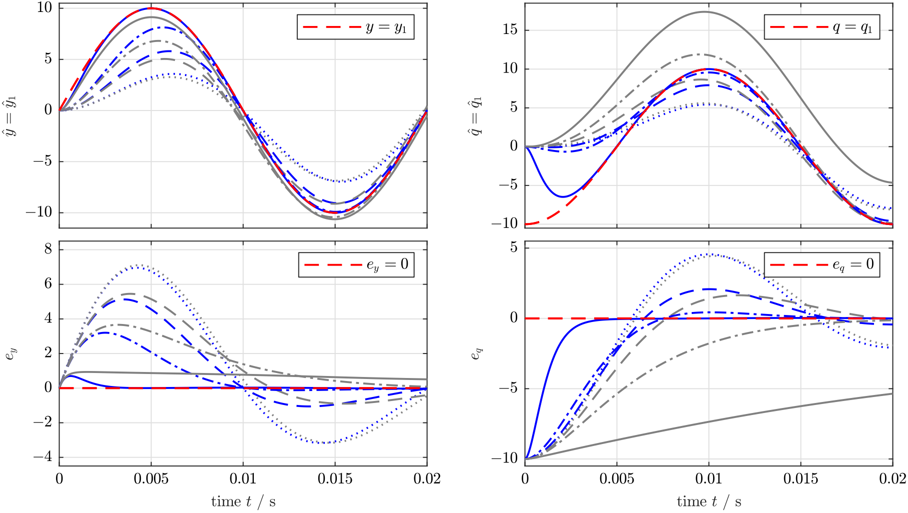
$$p_{\nu,1/2} = -\frac{\nu\widehat{\omega}k_\nu}{2}\left(1 \pm \sqrt{1 - \frac{4}{k_\nu^2}}\right)$$

mSOGI (for $g_\nu = -\frac{k_\nu^2}{4}$)

$$p_{\nu,1/2} = -\frac{\nu\widehat{\omega}k_\nu}{2} \pm \jmath\nu\widehat{\omega}.$$

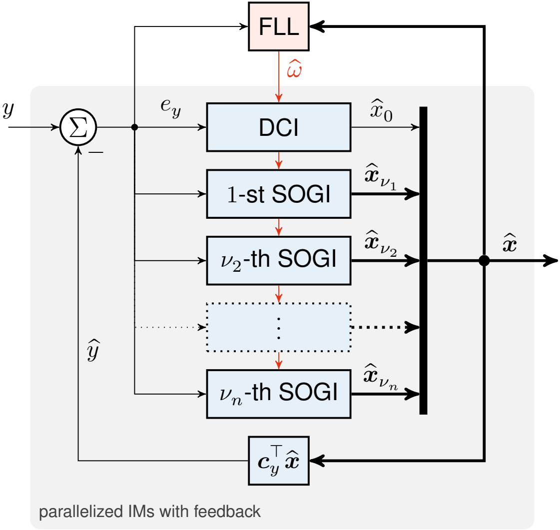
Observer design

Estimation performances of mSOGI and sSOGI for different $k_1 \in \{0.5, 1, 2, 10\}$



Observer design

Pole placement



Observer design

Pole placement: parallelized internal models with feedback

For $\boldsymbol{J} := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $\boldsymbol{l}_\nu := \nu \binom{k_\nu}{g_\nu}$, define the observer dynamics

$$\frac{\mathrm{d}}{\mathrm{d}t} \underbrace{\begin{pmatrix} \widehat{x}_0 \\ \widehat{\boldsymbol{x}}_1 \\ \widehat{\boldsymbol{x}}_{\nu_2} \\ \vdots \\ \widehat{\boldsymbol{x}}_{\nu_n} \end{pmatrix}}_{=:\widehat{\boldsymbol{x}} \in \mathbb{R}^{2n+1}} = \widehat{\omega}(t) \overbrace{\left[\begin{bmatrix} 0 & & & & \\ & \boldsymbol{J} & & & \\ & & \nu_2 \boldsymbol{J} & & \\ & & & \ddots & \\ & & & & \nu_n \boldsymbol{J} \end{bmatrix} - \begin{pmatrix} l_0 \\ \boldsymbol{l}_1 \\ \boldsymbol{l}_{\nu_2} \\ \vdots \\ \boldsymbol{l}_{\nu_n} \end{pmatrix} \boldsymbol{c}_y^\top \right]}^{= \boldsymbol{A} - \boldsymbol{l}\boldsymbol{c}_y^\top =: \boldsymbol{H}} \widehat{\boldsymbol{x}} - \widehat{\omega}(t) \boldsymbol{l}\, y, \quad \widehat{\boldsymbol{x}}(0) = \widehat{\boldsymbol{x}}_0$$

$$\widehat{y} = \underbrace{\begin{pmatrix} 1, & (1, & 0), & (1, & 0), & \ldots & (1, & 0) \end{pmatrix}}_{=:\boldsymbol{c}_y^\top \in \mathbb{R}^{1 \times (2n+1)}} \widehat{\boldsymbol{x}}$$

Observer design

Pole placement: parallelized internal models with **feedback**

For $\boldsymbol{J} := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $\boldsymbol{l}_\nu := \nu \begin{pmatrix} k_\nu \\ g_\nu \end{pmatrix}$, define the observer dynamics

$$\frac{\mathrm{d}}{\mathrm{d}t} \overbrace{\begin{pmatrix} \widehat{x}_0 \\ \widehat{\boldsymbol{x}}_1 \\ \widehat{\boldsymbol{x}}_{\nu_2} \\ \vdots \\ \widehat{\boldsymbol{x}}_{\nu_n} \end{pmatrix}}^{=:\widehat{\boldsymbol{x}} \in \mathbb{R}^{2n+1}} = \widehat{\omega}(t) \overbrace{\left[\begin{bmatrix} 0 & & & & \\ & \boldsymbol{J} & & & \\ & & \nu_2 \boldsymbol{J} & & \\ & & & \ddots & \\ & & & & \nu_n \boldsymbol{J} \end{bmatrix} - \begin{pmatrix} l_0 \\ \boldsymbol{l}_1 \\ \boldsymbol{l}_{\nu_2} \\ \vdots \\ \boldsymbol{l}_{\nu_n} \end{pmatrix} \boldsymbol{c}_y^\top \right]}^{=\boldsymbol{A} - \boldsymbol{l}\boldsymbol{c}_y^\top =: \boldsymbol{H}} \widehat{\boldsymbol{x}} - \widehat{\omega}(t) \boldsymbol{l}\, y, \quad \widehat{\boldsymbol{x}}(0) = \widehat{\boldsymbol{x}}_0$$

$$\widehat{y} = \underbrace{\begin{pmatrix} 1, & (1, & 0), & (1, & 0), & \ldots & (1, & 0) \end{pmatrix}}_{=:\boldsymbol{c}_y^\top \in \mathbb{R}^{1\times(2n+1)}} \widehat{\boldsymbol{x}}$$

- Recall: $(\boldsymbol{c}_y^\top, \boldsymbol{A})$ observable $\Longrightarrow \exists \boldsymbol{l} \in \mathbb{R}^{2n+1}$: $\boldsymbol{H} := \boldsymbol{A} - \boldsymbol{l}\boldsymbol{c}_y^\top$ Hurwitz!

Observer design

Pole placement: parallelized internal models with **feedback**

For $\boldsymbol{J} := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $\boldsymbol{l}_\nu := \nu \begin{pmatrix} k_\nu \\ g_\nu \end{pmatrix}$, define the observer dynamics

$$\frac{\mathrm{d}}{\mathrm{d}t} \overbrace{\begin{pmatrix} \hat{x}_0 \\ \hat{\boldsymbol{x}}_1 \\ \hat{\boldsymbol{x}}_{\nu_2} \\ \vdots \\ \hat{\boldsymbol{x}}_{\nu_n} \end{pmatrix}}^{=:\hat{\boldsymbol{x}} \in \mathbb{R}^{2n+1}} = \hat{\omega}(t) \overbrace{\left[\begin{bmatrix} 0 & & & & \\ & \boldsymbol{J} & & & \\ & & \nu_2 \boldsymbol{J} & & \\ & & & \ddots & \\ & & & & \nu_n \boldsymbol{J} \end{bmatrix} - \begin{pmatrix} l_0 \\ \boldsymbol{l}_1 \\ \boldsymbol{l}_{\nu_2} \\ \vdots \\ \boldsymbol{l}_{\nu_n} \end{pmatrix} \boldsymbol{c}_y^\top \right]}^{= \boldsymbol{A} - \boldsymbol{l} \boldsymbol{c}_y^\top =: \boldsymbol{H}} \hat{\boldsymbol{x}} - \hat{\omega}(t) \boldsymbol{l}\, y, \quad \hat{\boldsymbol{x}}(0) = \hat{\boldsymbol{x}}_0$$

$$\hat{y} = \underbrace{\big(1, \quad (1, \quad 0), \quad (1, \quad 0), \quad \ldots \quad (1, \quad 0)\big)}_{=: \boldsymbol{c}_y^\top \in \mathbb{R}^{1 \times (2n+1)}} \hat{\boldsymbol{x}}$$

- Recall: $(\boldsymbol{c}_y^\top, \boldsymbol{A})$ observable $\implies \exists \boldsymbol{l} \in \mathbb{R}^{2n+1}$: $\boldsymbol{H} := \boldsymbol{A} - \boldsymbol{l} \boldsymbol{c}_y^\top$ Hurwitz!
- Pole placement: $\boldsymbol{l} = \boldsymbol{S} \boldsymbol{p}_{\boldsymbol{H}}^* \implies s_0^* = -p_0^* \wedge \forall \nu \in \mathbb{H}_n : \; s_{\nu,1/2}^* = -p_\nu^* \pm \jmath \nu$

Observer design

Pole placement

A comparison with the desired polynomial $\chi_{\boldsymbol{H}}^* := \prod_{i=1}^{2n}(s - s_i^*)$, having the coefficient vector

$$\boldsymbol{p}_{\boldsymbol{H}}^* := \left(-\sum_{i=1}^{2n} s_i^*,\ \sum_{i=1}^{2n} s_i^* \sum_{j=i+1}^{2n} s_j^*,\ -\sum_{i=1}^{2n} s_i^* \sum_{j=i+1}^{2n} s_j^* \sum_{k=j+1}^{2n} s_k^*,\ \ldots,\ \prod_{i=1}^{2n} s_i^* \right)^\top,$$

leads, with $\boldsymbol{R}_i^{-1} := \begin{bmatrix} 1 & 0 \\ 0 & -\nu_i. \end{bmatrix}$, to

$$\boldsymbol{p}_{\boldsymbol{H}}^* - \begin{pmatrix} 0 \\ \sum_{i=1}^{n} \nu_i^2 \\ \vdots \\ 0 \\ \prod_{i=1}^{n} \nu_i^2 \end{pmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{R}_1^{-1} & \boldsymbol{R}_2^{-1} & \cdots & \boldsymbol{R}_n^{-1} \\ \sum_{\substack{i=1\\ i\neq 1}}^{n} \nu_i^2 \boldsymbol{R}_1^{-1} & \sum_{\substack{i=1\\ i\neq 2}}^{n} \nu_i^2 \boldsymbol{R}_2^{-1} & \cdots & \sum_{\substack{i=1\\ i\neq n}}^{n} \nu_i^2 \boldsymbol{R}_n^{-1} \\ \vdots & \vdots & \ddots & \vdots \\ \prod_{\substack{i=1\\ i\neq 1}}^{n} \nu_i^2 \boldsymbol{R}_1^{-1} & \prod_{\substack{i=1\\ i\neq 2}}^{n} \nu_i^2 \boldsymbol{R}_2^{-1} & \cdots & \prod_{\substack{i=1\\ i\neq n}}^{n} \nu_i^2 \boldsymbol{R}_n^{-1} \end{bmatrix}}_{=:\boldsymbol{S}^{-1}} \boldsymbol{l}, \quad (4)$$

Observer design

Pole placement (continued)

$$
\boldsymbol{l} = \underbrace{\begin{bmatrix}
\frac{\nu_1^{2(n-1)}}{\prod\limits_{\substack{i=1\\ i\neq 1}}^{n}(\nu_1^2-\nu_i^2)}\boldsymbol{R}_1 & -\frac{\nu_1^{2(n-2)}}{\prod\limits_{\substack{i=1\\ i\neq 1}}^{n}(\nu_1^2-\nu_i^2)}\boldsymbol{R}_1 & \cdots & \frac{(-1)^{n+1}}{\prod\limits_{\substack{i=1\\ i\neq 1}}^{n}(\nu_1^2-\nu_i^2)}\boldsymbol{R}_1 \\
\frac{\nu_2^{2(n-1)}}{\prod\limits_{\substack{i=1\\ i\neq 2}}^{n}(\nu_2^2-\nu_i^2)}\boldsymbol{R}_2 & -\frac{\nu_2^{2(n-2)}}{\prod\limits_{\substack{i=1\\ i\neq 2}}^{n}(\nu_2^2-\nu_i^2)}\boldsymbol{R}_2 & \cdots & \frac{(-1)^{n+1}}{\prod\limits_{\substack{i=1\\ i\neq 2}}^{n}(\nu_2^2-\nu_i^2)}\boldsymbol{R}_2 \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\nu_n^{2(n-1)}}{\prod\limits_{\substack{i=1\\ i\neq n}}^{n}(\nu_n^2-\nu_i^2)}\boldsymbol{R}_n & -\frac{\nu_n^{2(n-2)}}{\prod\limits_{\substack{i=1\\ i\neq n}}^{n}(\nu_n^2-\nu_i^2)}\boldsymbol{R}_n & \cdots & \frac{(-1)^{n+1}}{\prod\limits_{\substack{i=1\\ i\neq n}}^{n}(\nu_n^2-\nu_i^2)}\boldsymbol{R}_n
\end{bmatrix}}_{=:\boldsymbol{S}}
\underbrace{\left(\boldsymbol{p}_{\boldsymbol{H}}^* - \begin{pmatrix} 0 \\ \sum\limits_{i=1}^{n}\nu_i^2 \\ \vdots \\ 0 \\ \prod\limits_{i=1}^{n}\nu_i^2 \end{pmatrix}\right)}_{=:\tilde{\boldsymbol{p}}_{\boldsymbol{H}}^*}.
$$

Clearly, if and only if $\boldsymbol{p}_{\boldsymbol{H}}^* = \boldsymbol{p}_{\boldsymbol{H}}$ holds, the eigenvalues of $\boldsymbol{H} = \boldsymbol{A} - \boldsymbol{l}\boldsymbol{c}_y^\top$ are given by $s_i^* \in \mathbb{C}_{<0}$, $i \in \{1, \ldots, 2n\}$ and $\boldsymbol{H}$ is a Hurwitz matrix.

Observer design

Stability analysis for $\omega(\cdot) \in \mathcal{C}^{\mathrm{pw}} \cap \mathcal{L}^{\infty}(\mathbb{R}_{\geqslant 0}; \mathbb{R}_{\geqslant \varepsilon})$ and $\widehat{\omega}(\cdot) \in \mathcal{C}^{1} \cap \mathcal{L}^{\infty}(\mathbb{R}_{\geqslant 0}; \mathbb{R}_{\geqslant \varepsilon})$

- **Error dynamics** $\boldsymbol{e}_x := \boldsymbol{x} - \widehat{\boldsymbol{x}}$ and $e_\omega := \omega - \widehat{\omega}$ $\implies$ $\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{e}_x = \boldsymbol{f}(\boldsymbol{e}_x, e_\omega, \dots)$

Observer design

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- **Lyapunov candidate** $V(\boldsymbol{e}_x) := \boldsymbol{e}_x^\top \boldsymbol{P} \boldsymbol{e}_x$ and using

Observer design

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  - $\boldsymbol{H} := \boldsymbol{A} - \boldsymbol{l}\boldsymbol{c}_y^\top$ Hurwitz $\Longrightarrow$
    $\forall \boldsymbol{Q} = \boldsymbol{Q}^\top > 0 \, \exists \boldsymbol{P} = \boldsymbol{P}^\top > 0: \; \boldsymbol{H}^\top \boldsymbol{P} + \boldsymbol{P}\boldsymbol{H} = -\boldsymbol{Q} < 0$ [4, Cor. 3.3.47]

Observer design

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  - $\forall a, b, m \in \mathbb{R}: \; 2\,a\,b = \frac{a^2}{m} + mb^2 - \left(\frac{a}{\sqrt{m}} - \sqrt{m}\,b\right)^2 \leqslant \frac{a^2}{m} + m\,b^2$

Observer design

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$\Longrightarrow$ $\frac{\mathrm{d}}{\mathrm{d}t}V(\boldsymbol{e}_x) = \cdots \leqslant -\mu_V V(\boldsymbol{e}_x) + c_\omega e_\omega^2$ with $e_\omega(\cdot) \in \mathcal{L}^{\infty}(\mathbb{R}_{\geqslant 0};\mathbb{R})$

Observer design

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$\Longrightarrow$ $\frac{\mathrm{d}}{\mathrm{d}t}V(\boldsymbol{e}_x) = \cdots \leqslant -\mu_V V(\boldsymbol{e}_x) + c_\omega e_\omega^2$ with $e_\omega(\cdot) \in \mathcal{L}^{\infty}(\mathbb{R}_{\geqslant 0};\mathbb{R})$

- Bellmann-Gronwall Lemma [5, Lemma 5.50]

Observer design

Stability analysis for $\omega(\cdot) \in \mathcal{C}^{\mathrm{pw}} \cap \mathcal{L}^{\infty}(\mathbb{R}_{\geqslant 0}; \mathbb{R}_{\geqslant \varepsilon})$ and $\widehat{\omega}(\cdot) \in \mathcal{C}^{1} \cap \mathcal{L}^{\infty}(\mathbb{R}_{\geqslant 0}; \mathbb{R}_{\geqslant \varepsilon})$

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  - $\forall a, b, m \in \mathbb{R}\colon \; 2\,a\,b = \frac{a^2}{m} + mb^2 - \left(\frac{a}{\sqrt{m}} - \sqrt{m}\,b\right)^2 \leqslant \frac{a^2}{m} + m\,b^2$

$\implies$ $\frac{\mathrm{d}}{\mathrm{d}t} V(\boldsymbol{e}_x) = \cdots \leqslant -\mu_V V(\boldsymbol{e}_x) + c_\omega e_\omega^2$ with $e_\omega(\cdot) \in \mathcal{L}^{\infty}(\mathbb{R}_{\geqslant 0}; \mathbb{R})$

  - Bellmann-Gronwall Lemma [5, Lemma 5.50]

$\implies$ $\forall t \geqslant t_i\colon \; V(\boldsymbol{e}_x(t)) \leqslant V(\boldsymbol{e}_x(t_i))\mathrm{e}^{-\mu_V\,(t - t_i)} + c_\omega \int_{t_i}^{t} e_\omega(\tau)^2 \mathrm{e}^{-\mu_V\,(t-\tau)}\, \mathrm{d}\tau$

Observer design

Stability analysis for $\omega(\cdot) \in \mathcal{C}^{\mathrm{pw}} \cap \mathcal{L}^{\infty}(\mathbb{R}_{\geqslant 0}; \mathbb{R}_{\geqslant \varepsilon})$ and $\widehat{\omega}(\cdot) \in \mathcal{C}^{1} \cap \mathcal{L}^{\infty}(\mathbb{R}_{\geqslant 0}; \mathbb{R}_{\geqslant \varepsilon})$

- **Error dynamics** $\boldsymbol{e}_x := \boldsymbol{x} - \widehat{\boldsymbol{x}}$ and $e_\omega := \omega - \widehat{\omega} \implies \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{e}_x = \boldsymbol{f}(\boldsymbol{e}_x, e_\omega, \dots)$
- **Lyapunov candidate** $V(\boldsymbol{e}_x) := \boldsymbol{e}_x^\top \boldsymbol{P} \boldsymbol{e}_x$ and using
  - $\boldsymbol{H} := \boldsymbol{A} - \boldsymbol{l}\boldsymbol{c}_y^\top$ Hurwitz $\implies$ $\forall \boldsymbol{Q} = \boldsymbol{Q}^\top > 0 \, \exists \boldsymbol{P} = \boldsymbol{P}^\top > 0 \colon \; \boldsymbol{H}^\top \boldsymbol{P} + \boldsymbol{P}\boldsymbol{H} = -\boldsymbol{Q} < 0$ [4, Cor. 3.3.47]
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$\implies \frac{\mathrm{d}}{\mathrm{d}t} V(\boldsymbol{e}_x) = \cdots \leqslant -\mu_V V(\boldsymbol{e}_x) + c_\omega e_\omega^2$ with $e_\omega(\cdot) \in \mathcal{L}^{\infty}(\mathbb{R}_{\geqslant 0}; \mathbb{R})$

  - Bellmann-Gronwall Lemma [5, Lemma 5.50]

$$\implies \forall t \geqslant t_i \colon \; V(\boldsymbol{e}_x(t)) \leqslant V(\boldsymbol{e}_x(t_i))\mathrm{e}^{-\mu_V (t - t_i)} + c_\omega \int_{t_i}^{t} e_\omega(\tau)^2 \mathrm{e}^{-\mu_V (t-\tau)} \, \mathrm{d}\tau$$

- **Error bound** by invoking $\lambda_{\min}(\boldsymbol{P}) \|\boldsymbol{e}_x\|^2 \leqslant V(\boldsymbol{e}_x) = \boldsymbol{e}_x^\top \boldsymbol{P} \boldsymbol{e}_x \leqslant \lambda_{\max}(\boldsymbol{P}) \|\boldsymbol{e}_x\|^2$

$$\implies \forall t \geqslant t_i \colon \; \|\boldsymbol{e}_x(t)\|^2 \leqslant c_{\boldsymbol{P}} \|\boldsymbol{e}_x(t_i)\|^2 \mathrm{e}^{-\mu_V (t-t_i)} + c_\omega' \int_{t_i}^{t} e_\omega(\tau)^2 \mathrm{e}^{-\mu_V (t-\tau)} \, \mathrm{d}\tau$$

Observer design

Bellmann-Gronwall Lemma or ‘Comparison Lemma’ [5, Lemma 5.50]

Lemma (Bellman-Gronwall Lemma (differential form))

Let $\alpha \in \mathbb{R}$, $I \subseteq \mathbb{R}$ an interval and $t_0, t \in I$ with $t \geqslant t_0$. If, for $\beta(\cdot)$, $\gamma(\cdot) \in \mathcal{C}(I;\mathbb{R})$, $\xi(\cdot) \in \mathcal{C}^1(I;\mathbb{R})$, the following holds

$$\forall\, t \geqslant t_0: \qquad \frac{\mathrm{d}}{\mathrm{d}t}\xi(t) \leqslant \beta(t)\xi(t) + \gamma(t),$$

then

$$\forall\, t \geqslant t_0: \qquad \xi(t) \leqslant \xi(t_0)\exp\left(\int_{t_0}^{t} \beta(\tau)\,\mathrm{d}\tau\right) + \int_{t_0}^{t} \gamma(\tau)\exp\left(\int_{\tau}^{t} \beta(s)\,\mathrm{d}s\right)\mathrm{d}\tau\,.$$

For proofs, see e.g.:

- [6, Lemma 1.1] (for differential form) or
- [7, Lemma 2.1.18] and [8, Lemma A.1] (for integral form).

Observer design

Stability of slowly time-varying linear systems [9, Proposition 1.4]

Proposition (Slowly time-varying linear systems)

Suppose $\boldsymbol{A}(\cdot) \in \mathcal{C}^{\mathrm{pw}}(\mathbb{R}_{\geqslant 0}; \mathbb{C}^{n \times n})$ satisfies for some $\alpha, \beta > 0$ and all $t \geqslant 0$:

$$\|\boldsymbol{A}(t)\| \leqslant \beta \quad \text{and} \quad \mathrm{spec}\big(\boldsymbol{A}(t)\big) \subset \mathbb{C}_{<-\alpha} := \{\, s \in \mathbb{C} \mid \Re(s) < -\alpha \,\}$$

Then the system

$$\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x}(t) = \boldsymbol{A}(t)\boldsymbol{x}(t), \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0$$

is exponentially stable if one of the following holds true for all $t \geqslant 0$:

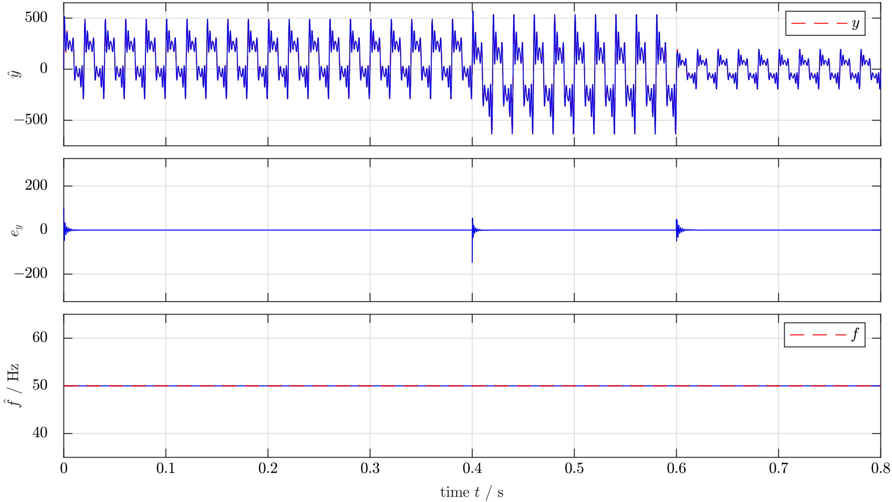
(i) $-\alpha < -4\beta$;

(ii) $\boldsymbol{A}(\cdot)$ is piecewise differentiable and $\|\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{A}(t)\| \leqslant \delta < \frac{2}{2n-1}\frac{\alpha^{4n-2}}{2\beta^{4n-4}}$

(iii) $\cdots$

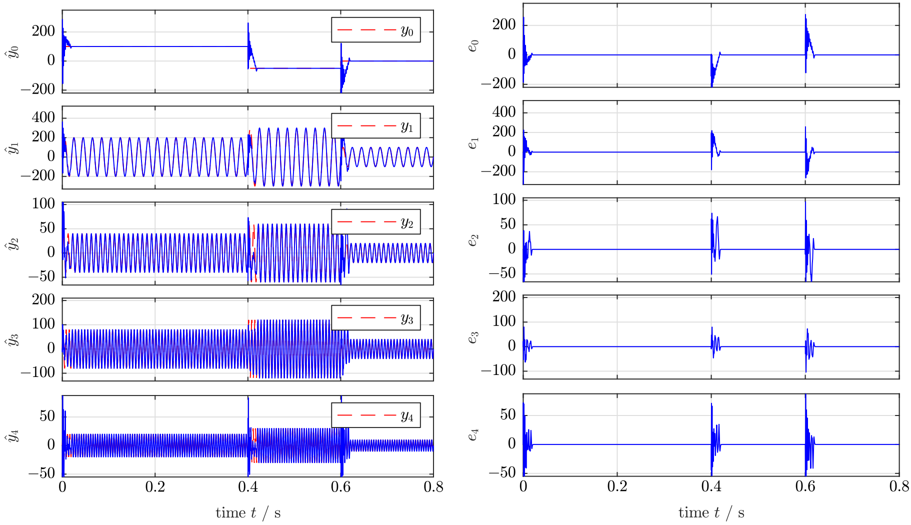
Observer design

Simulation results: Estimate $\widehat{y}$, estimation error $e_y = y - \widehat{y}$ and estimated frequency $\widehat{f} = \frac{\widehat{\omega}}{2\pi}$



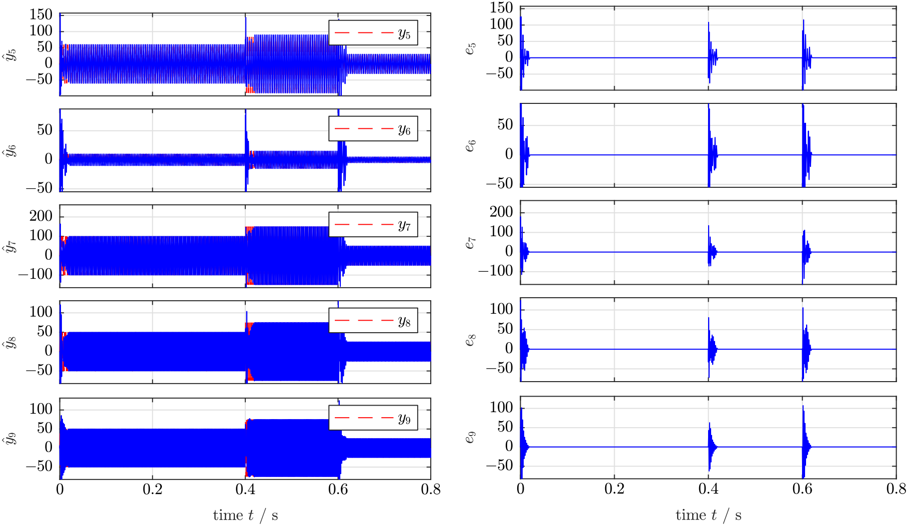
Observer design

Simulation results: Estimate $\hat{y}_\nu$ (left) and estimation error $e_\nu = y_\nu - \hat{y}_\nu$ (right)



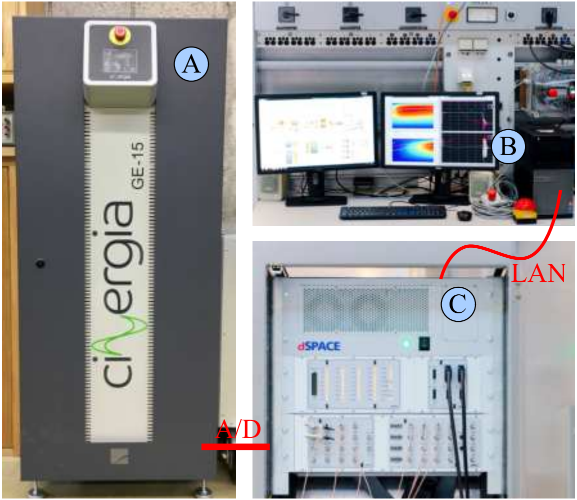
Observer design

Simulation results: Estimate $\widehat{y}_\nu$ (left) and estimation error $e_\nu = y_\nu - \widehat{y}_\nu$ (right)



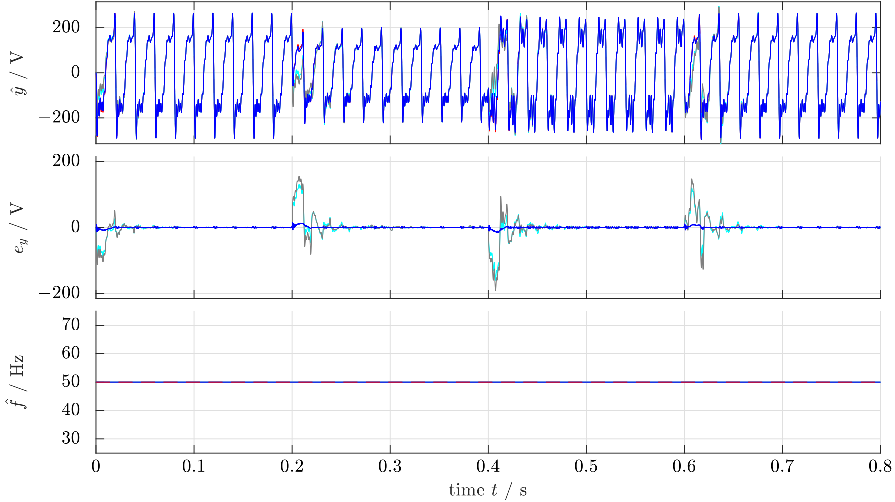
Observer design

Measurement results in laboratory: (A) Grid emulator, (B) Host-PC and (C) Real-time system



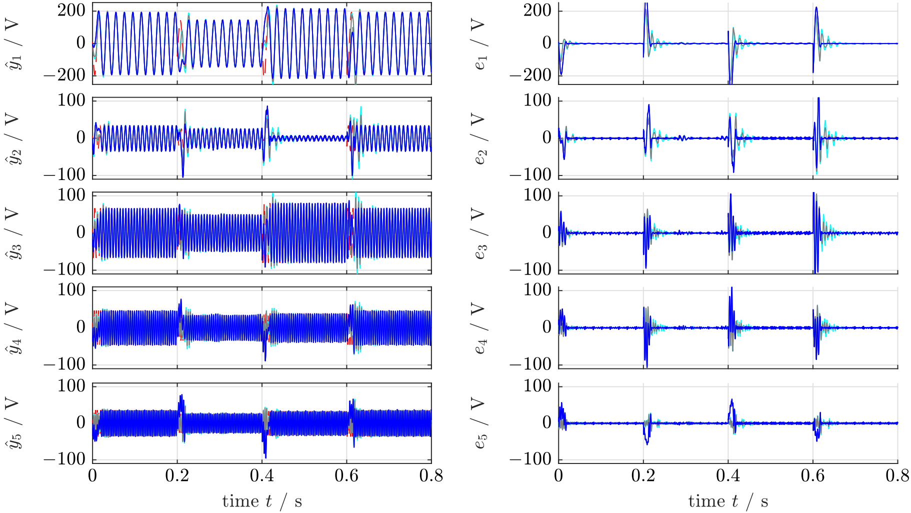
Observer design

Measurement results: mSOGI, sSOGI, ANF, y, f



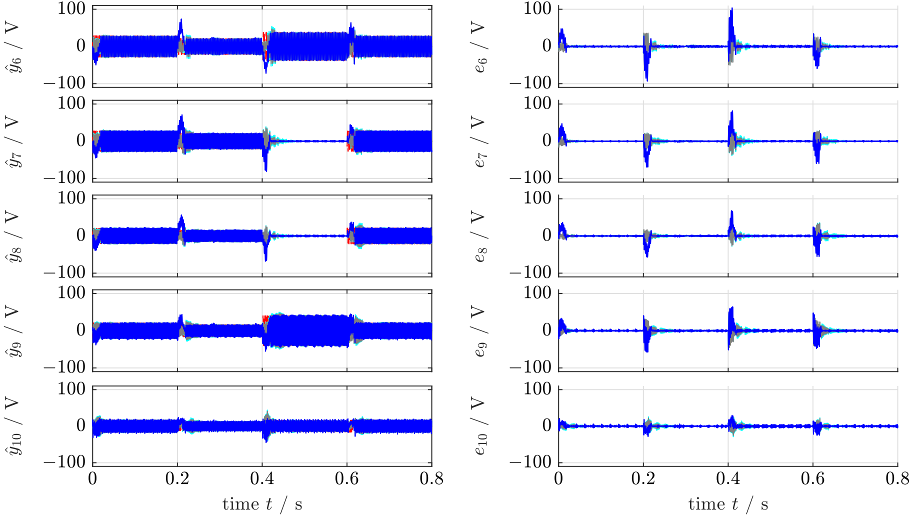
Observer design

Measurement results: mSOGI, sSOGI, ANF, y_ν (left)



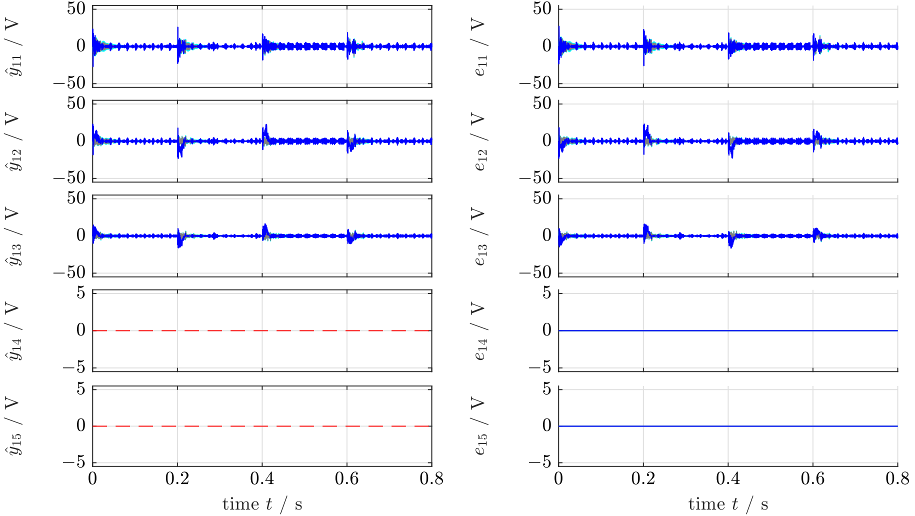
Observer design

Measurement results: mSOGI, sSOGI, ANF, y_ν (left)



Observer design

Measurement results: mSOGI, sSOGI, ANF, y_ν (left)



Outline I

6 Appendix

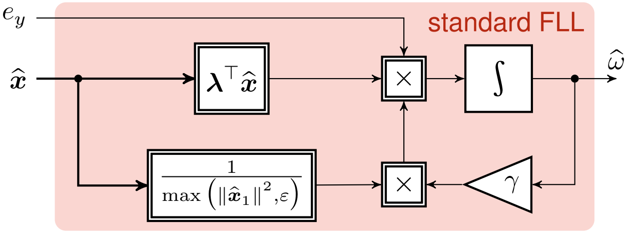
- Internal model principle
  - Motivation
  - Signal reduplication
  - Parallelization of internal models (IMs)
  - Observability of parallelized IMs
  - Observability of linear time-varying systems
- Observer design
  - Overall observer structure
  - DC-integrator (for dc offset estimation)
  - Second-order generalized integrators (SOGIs)
  - Pole placement
  - Stability analysis
  - Bellmann-Gronwall Lemma
  - Stability of slowly time-varying linear systems
  - Simulation results
  - Measurement results

Outline II

- Frequency estimation
  - Standard Frequency Locked Loop
  - Modified Frequency Locked Loop
  - Sign-correct adaption law for the FLL
  - Modification for mFLL
  - Comparative simulation results with FLL
- Implementation
  - Laboratory setup
  - Measurement results for signal with fundamental only
  - Measurement results for signal with harmonics
- Harmonic sequence extraction
  - Overall implementation
  - Positive, negative and zero sequence estimation
  - In-phase and quadrature signal vector of ν-th harmonic
  - Fortescue transformation
  - Simulation results

Frequency estimation

Standard Frequency Locked Loop (sFLL)

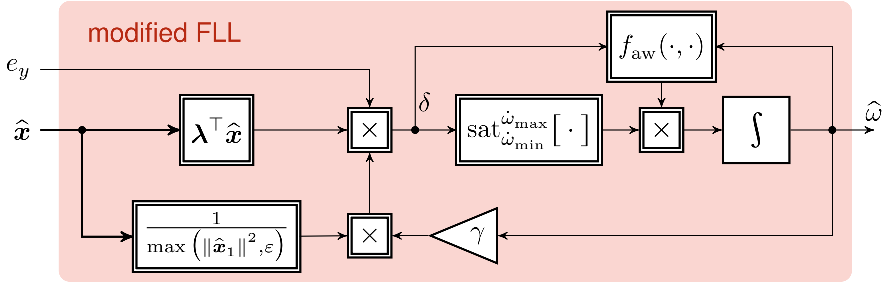


$$\frac{\mathrm{d}}{\mathrm{d}t}\widehat{\omega} = \frac{\gamma\,\widehat{\omega}\,e_y\,\boldsymbol{\lambda}^\top\widehat{\boldsymbol{x}}}{\max\left(\|\widehat{\boldsymbol{x}}_1\|^2, \varepsilon\right)} = \begin{cases} > 0, & \widehat{\omega} < \omega \wedge e_y \neq 0 \\ < 0, & ,\widehat{\omega} > \omega \wedge e_y \neq 0 \end{cases}$$

$$\Longleftarrow \quad \boldsymbol{\lambda} := \mathrm{blockdiag}\left(0, \boldsymbol{J}^{-1}, \boldsymbol{O}_{2\times 2}, \ldots, \boldsymbol{O}_{2\times 2}\right)\boldsymbol{l} \qquad \text{(sFLL)}$$

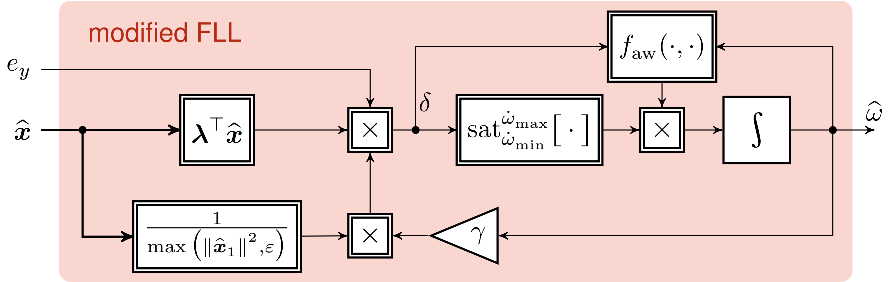
Frequency estimation

Modified Frequency Locked Loop (mFLL) with **anti-windup (AWU)** & **rate limitation (RL)**



Frequency estimation

Modified Frequency Locked Loop (mFLL) with **anti-windup (AWU)** & **rate limitation (RL)**

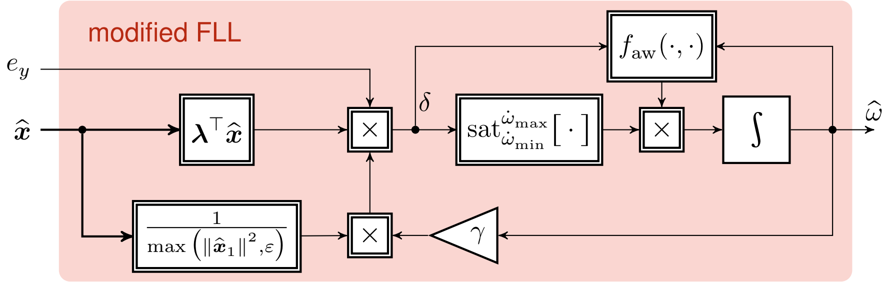


$$\frac{\mathrm{d}}{\mathrm{d}t}\widehat{\omega} = \underbrace{f_{\mathrm{aw}}(\widehat{\omega}, \delta)}_{\in\{0,1\}} \cdot \mathrm{sat}_{\dot\omega_{\min}}^{\dot\omega_{\max}}\left[\underbrace{\frac{\gamma\,\widehat{\omega}\,e_y\,\boldsymbol{\lambda}^\top \widehat{\boldsymbol{x}}}{\max\left(\|\widehat{\boldsymbol{x}}_1\|^2, \varepsilon\right)}}_{=:\delta}\right]$$ with $\boldsymbol{\lambda} := (0, -g_1, k_1, 0, \ldots, 0)^\top$

and $$f_{\mathrm{aw}}(\widehat{\omega}, \delta) := \begin{cases} 0, & \text{for } (\widehat{\omega} \geqslant \omega_{\max} \wedge \delta > 0) \vee (\widehat{\omega} \leqslant \omega_{\min} \wedge \delta < 0) \\ 1, & \text{else} \end{cases}$$

Frequency estimation

Modified Frequency Locked Loop (mFLL) with sign-correct anti-windup and rate limitation



$$\frac{\mathrm{d}}{\mathrm{d}t}\widehat{\omega} = f_{\mathrm{aw}}(\widehat{\omega}, \delta) \cdot \mathrm{sat}_{\dot{\omega}_{\min}}^{\dot{\omega}_{\max}}\left[\underbrace{\frac{\gamma\, \widehat{\omega}\, e_y\, \boldsymbol{\lambda}^\top \widehat{\boldsymbol{x}}}{\max\left(\|\widehat{\boldsymbol{x}_1}\|^2, \varepsilon\right)}}_{=:\delta} \right] = \begin{cases} > 0, & \widehat{\omega} < \omega \wedge e_y \neq 0 \\ < 0, & ,\widehat{\omega} > \omega \wedge e_y \neq 0 \end{cases}$$

$$\Longleftarrow \quad \boldsymbol{\lambda} := \mathrm{blockdiag}\left(0, \boldsymbol{J}^{-1}, \boldsymbol{O}_{2\times 2}, \ldots, \boldsymbol{O}_{2\times 2}\right) \boldsymbol{l} \qquad \text{(mFLL)}$$

Frequency estimation

Sign-correct adaption law for the FLL

The choice

$$\boldsymbol{\lambda} := \mathrm{blockdiag}\left(0, \boldsymbol{J}^{-1}, \boldsymbol{O}_{2\times 2}, \ldots, \boldsymbol{O}_{2\times 2}\right) \boldsymbol{l} \tag{5}$$

leads (in steady state) to

$$\begin{aligned}
\boldsymbol{\lambda}^\top \widehat{\boldsymbol{x}}\, e_{y_\nu} &\approx \frac{\widehat{\omega}^2 \left(-g_\nu, \quad k_\nu\right) \boldsymbol{\lambda}}{\widehat{\omega}^2 - \omega^2} e_{y_\nu}^2 && (6)\\
&\overset{(5)}{=} \frac{\widehat{\omega}^2 (k_\nu^2 + g_\nu^2)}{\omega^2 - \widehat{\omega}^2} e_{y_\nu}^2 \begin{cases} \geqslant 0, & \widehat{\omega} < \omega \wedge e_y \neq 0 \\ < 0, & \widehat{\omega} > \omega \wedge e_y \neq 0 \end{cases}, && (7)
\end{aligned}$$

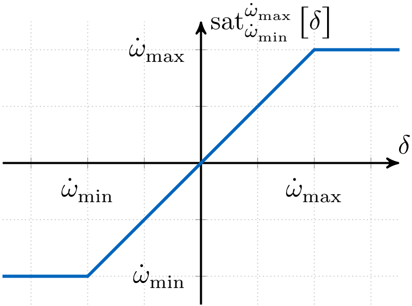
which gives a sign-correct adaption, since

$$\frac{\mathrm{d}}{\mathrm{d}t}\widehat{\omega} \propto \boldsymbol{\lambda}^\top \widehat{\boldsymbol{x}}\, e_{y_\nu} = \begin{cases} > 0, & \widehat{\omega} < \omega \wedge e_y \neq 0 \\ < 0, & \widehat{\omega} > \omega \wedge e_y \neq 0 \end{cases}.$$

□

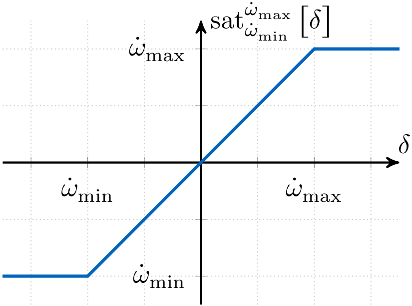
Frequency estimation

Modified FLL: Saturation of derivative of angular frequency estimate (rate limitation)



Frequency estimation

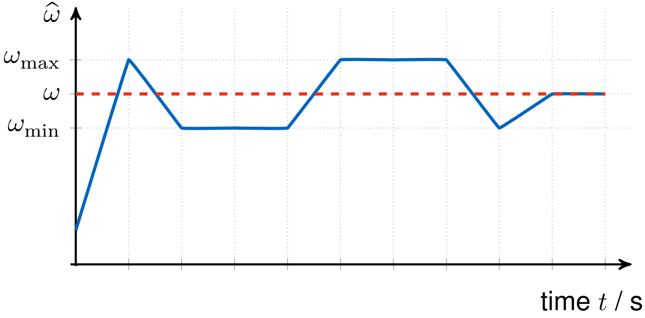
Modified FLL: Saturation of derivative of angular frequency estimate (rate limitation)



$$\mathrm{sat}_{\dot\omega_{\min}}^{\dot\omega_{\max}}[\delta] = \begin{cases} \dot\omega_{\max} & , \delta > \dot\omega_{\max} \\ \delta & , \dot\omega_{\min} \leqslant \delta \leqslant \dot\omega_{\max} \\ \dot\omega_{\min} & , \delta < \dot\omega_{\min} \end{cases}$$

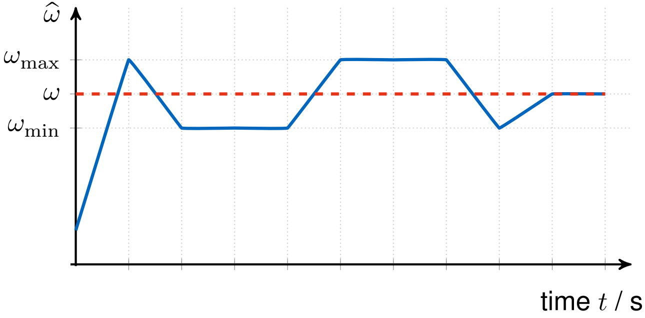
Frequency estimation

Modified FLL: Sign-correct anti-windup



Frequency estimation

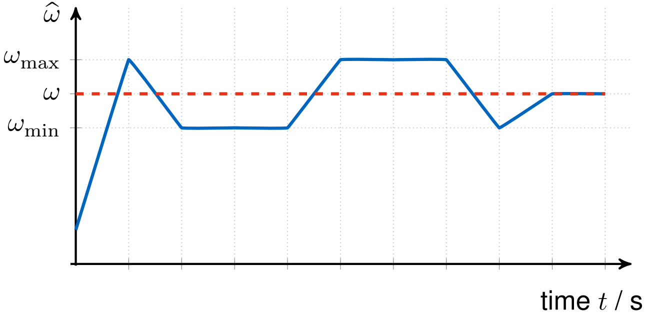
Modified FLL: Sign-correct anti-windup



$$f_{\mathrm{aw}}\left(\widehat{\omega}, \tfrac{\mathrm{d}}{\mathrm{d}t}\widehat{\omega}\right) := \begin{cases} 0, & \text{for } \left(\widehat{\omega} \geqslant \omega_{\max} \wedge \tfrac{\mathrm{d}}{\mathrm{d}t}\widehat{\omega} > 0\right) \vee \left(\widehat{\omega} \leqslant \omega_{\min} \wedge \tfrac{\mathrm{d}}{\mathrm{d}t}\widehat{\omega} < 0\right) \\ 1, & \text{else} \end{cases}$$

Frequency estimation

Modified FLL: Sign-correct anti-windup

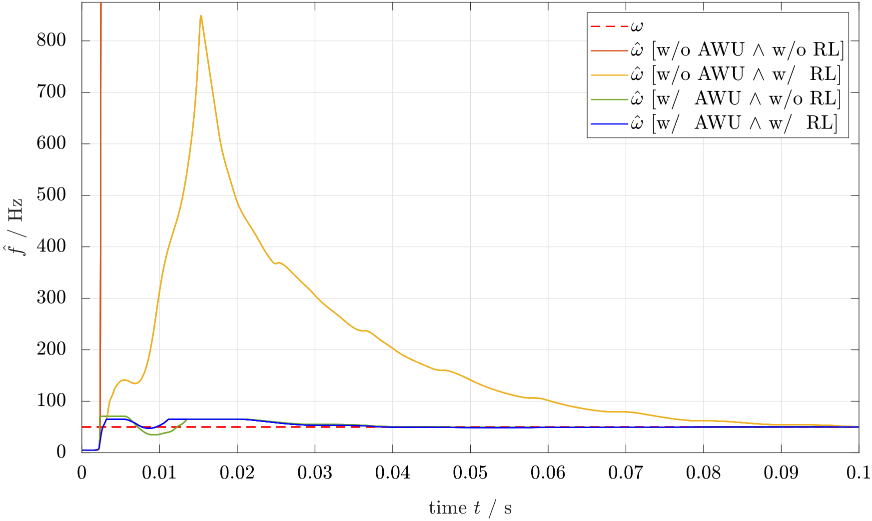


$$f_{\mathrm{aw}}\left(\widehat{\omega}, \tfrac{\mathrm{d}}{\mathrm{d}t}\widehat{\omega}\right) := \begin{cases} 0, & \text{for } \left(\widehat{\omega} \geqslant \omega_{\max} \wedge \tfrac{\mathrm{d}}{\mathrm{d}t}\widehat{\omega} > 0\right) \vee \left(\widehat{\omega} \leqslant \omega_{\min} \wedge \tfrac{\mathrm{d}}{\mathrm{d}t}\widehat{\omega} < 0\right) \\ 1, & \text{else} \end{cases}$$

$$\Longrightarrow \ \forall t \geqslant 0\colon \ \widehat{\omega}(t) \in \left[\min(\omega_{\min}, \widehat{\omega}_0), \min(\omega_{\max}, \widehat{\omega}_0)\right] \ \Longrightarrow \ \widehat{\omega}(\cdot) \in \mathcal{L}^{\infty}(\mathbb{R}_{\geqslant 0}; \mathbb{R}_{>0})$$

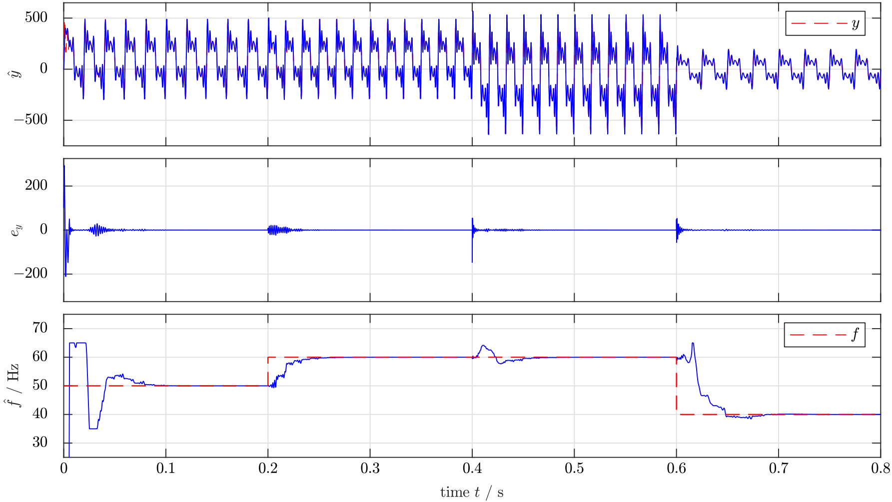
Frequency estimation

Simulation results with mFLL: Impact of modifications on frequency estimation



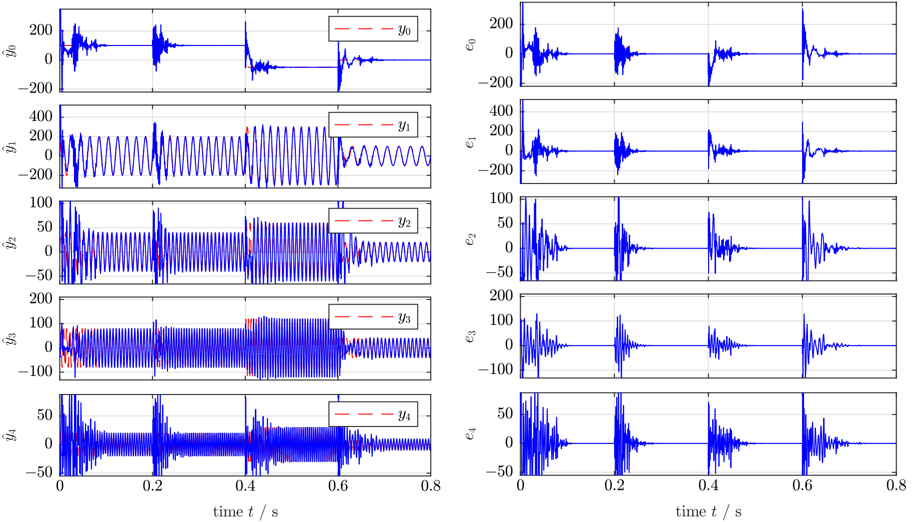
Frequency estimation

Simulation results with FLL: Estimate $\hat{y}$, error $e_y = y - \hat{y}$, estimated frequency $\hat{f} = \frac{\hat{\omega}}{2\pi}$



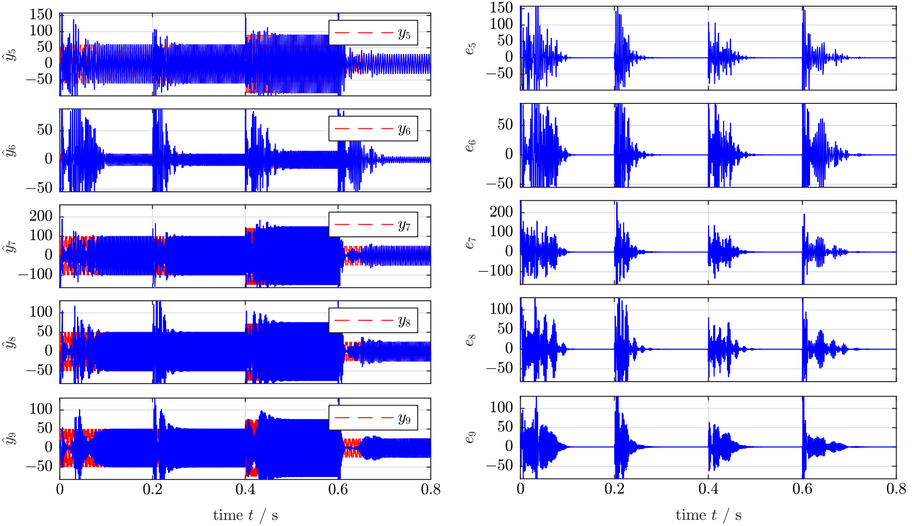
Frequency estimation

Simulation results with FLL: Estimate $\hat{y}_\nu$ (left) and estimation error $e_\nu = y_\nu - \hat{y}_\nu$ (right)



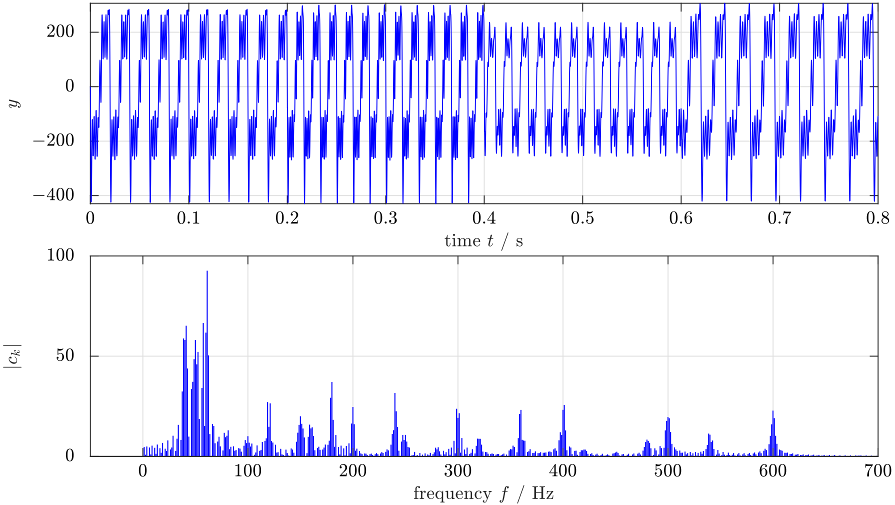
Frequency estimation

Simulation results with FLL: Estimate $\widehat{y}_\nu$ (left) and estimation error $e_\nu = y_\nu - \widehat{y}_\nu$ (right)



Frequency estimation

Measurement results with FLL: Signal and Fast Fourier Transformation



Outline I

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  - Laboratory setup
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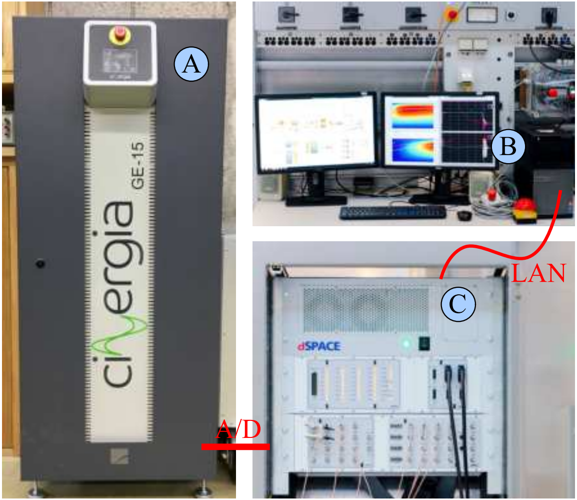
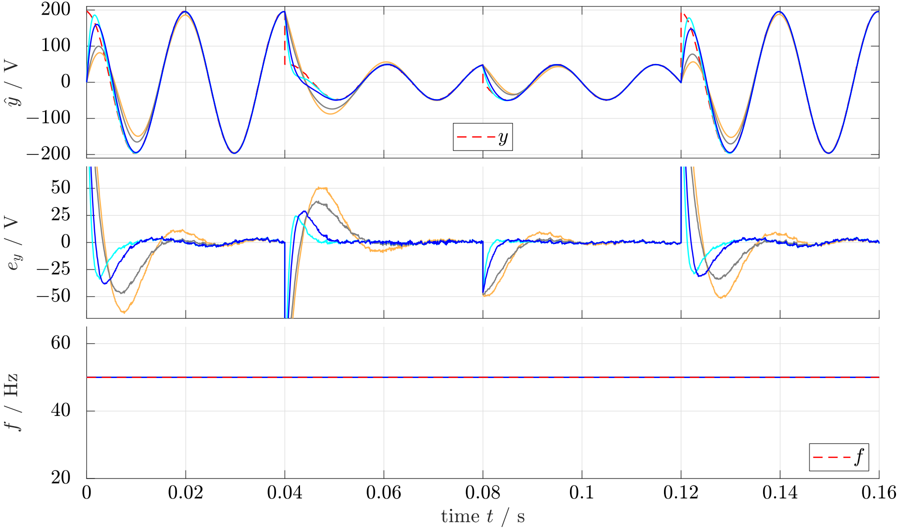
Implementation

Laboratory setup: (A) Grid emulator, (B) Host-PC and (C) Real-time system



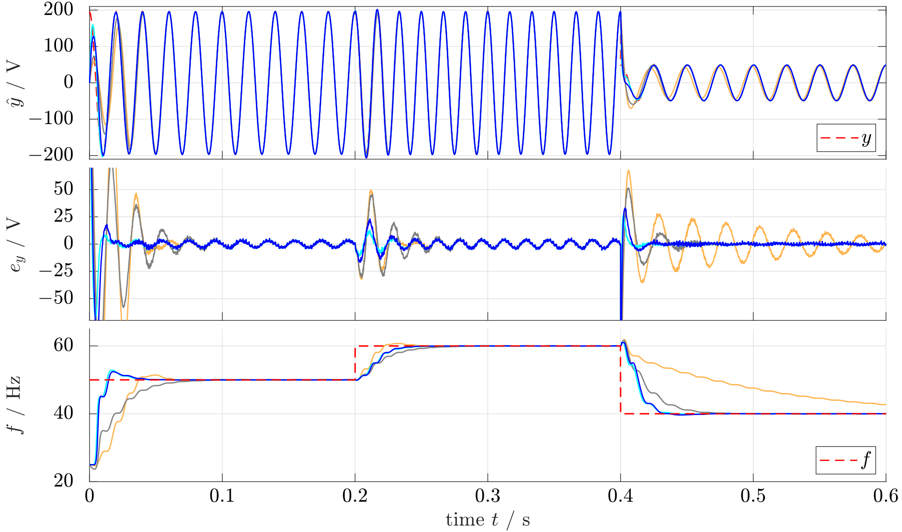
Implementation

Measurements **w/o** FLL ($\nu = 1$): mSOGI, $\text{mSOGI}_{\text{fast}}$, sSOGI & ANF



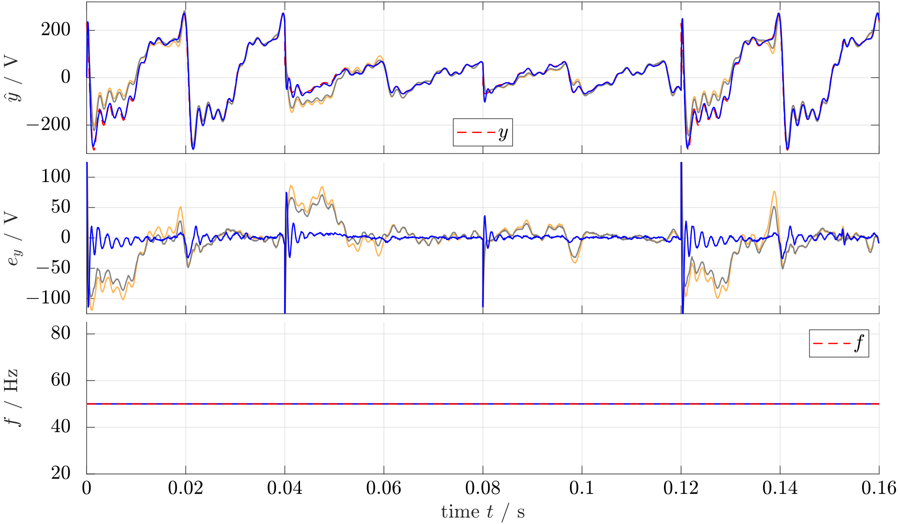
Implementation

Measurements **with** FLL ($\nu = 1$): mSOGI, $\text{mSOGI}_{\text{fast}}$, sSOGI & ANF



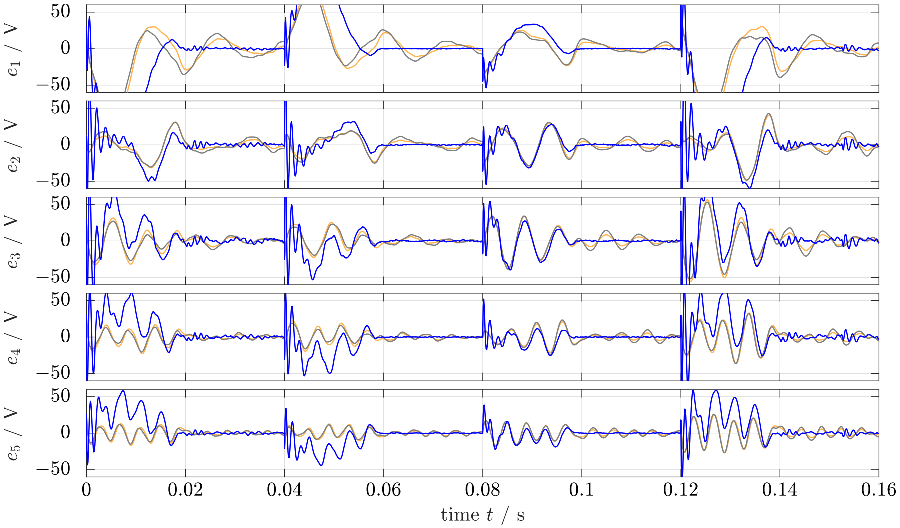
Implementation

Measurements **w/o** FLL ($\nu = 10$): mSOGI, sSOGI & ANF



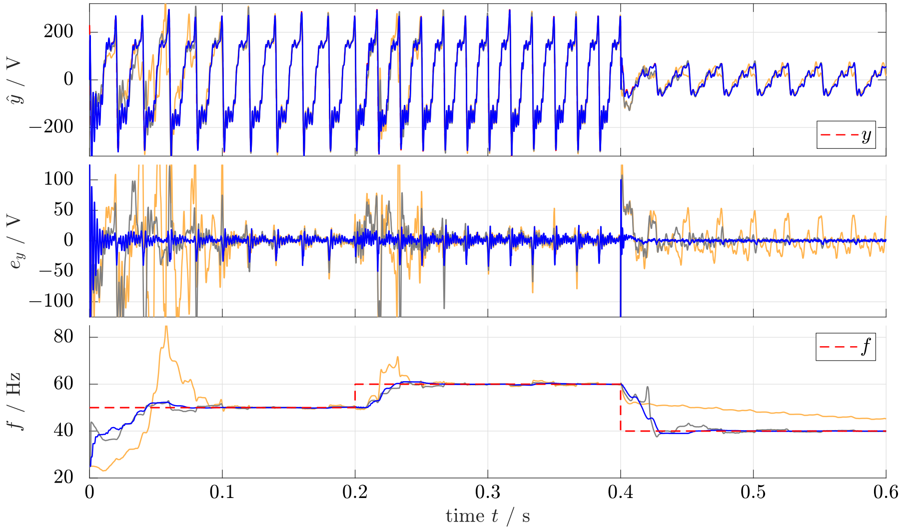
Implementation

Measurements **w/o** FLL ($\nu = 10$): $e_\nu = y_\nu - \widehat{y}_\nu$ for mSOGI, sSOGI & ANF



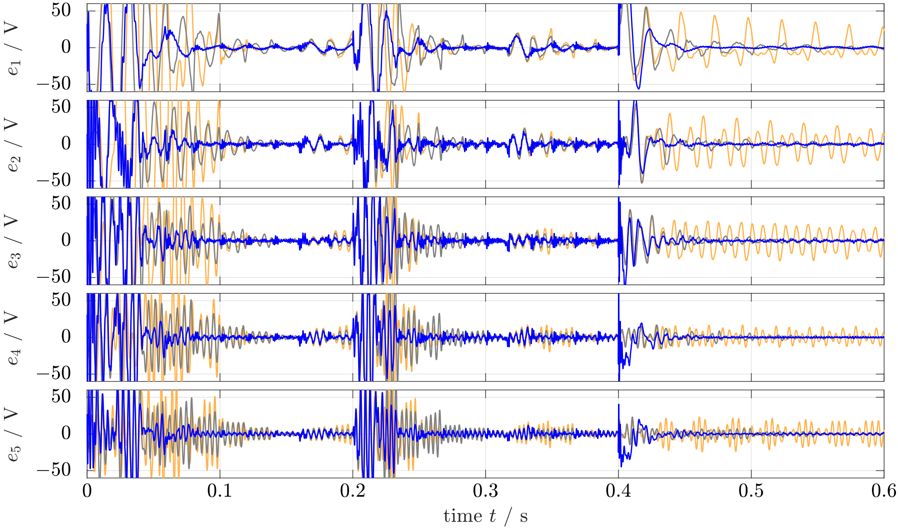
Implementation

Measurements **with** FLL ($\nu = 10$): mSOGI, sSOGI & ANF



Implementation

Measurements **with** FLL ($\nu = 10$): $e_\nu = y_\nu - \widehat{y}_\nu$ for mSOGI, sSOGI & ANF



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  - Stability analysis
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  - Stability of slowly time-varying linear systems
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  - Sign-correct adaption law for the FLL
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  - Laboratory setup
  - Measurement results for signal with fundamental only
  - Measurement results for signal with harmonics
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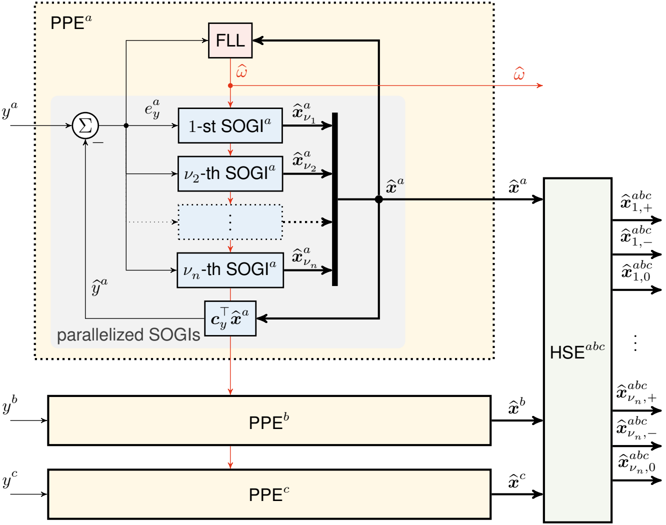
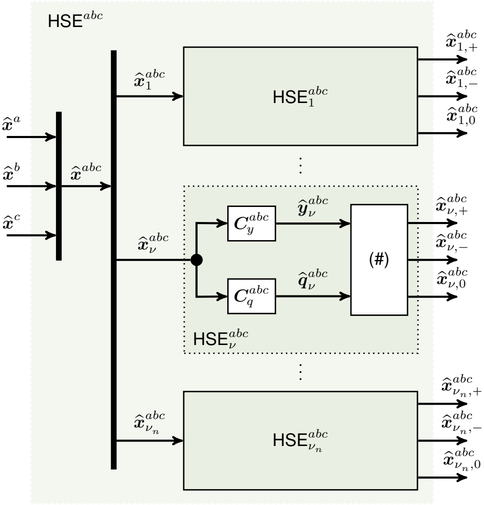
Harmonic sequence extraction

Overview: Phase parameter estimation (PPE) and Harmonic sequence extraction (HSE)



Harmonic sequence extraction

Harmonic sequence extraction (HSE) for the three phases a, b, c



Harmonic sequence extraction

Positive, negative and zero sequence estimation of ν-th harmonic

Let $\nu \in \mathbb{H}_n$, $\boldsymbol{J}_1 := \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\boldsymbol{T}_\mathrm{c} := \kappa \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$, $\kappa \in \{\frac{2}{3}, \sqrt{\frac{2}{3}}\}$,

define

$$\left.\begin{aligned} \boldsymbol{y}_\nu^{abc}(t) = & \underbrace{\tfrac{1}{2}\boldsymbol{T}_\mathrm{c}^{-1} \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}^{=:\boldsymbol{D}_{12}} \boldsymbol{T}_\mathrm{c} \widehat{\boldsymbol{y}}_\nu^{abc}(t) + \tfrac{1}{2}\boldsymbol{T}_\mathrm{c}^{-1} \boldsymbol{J}_1 \boldsymbol{D}_{12} \boldsymbol{T}_\mathrm{c} \widehat{\boldsymbol{q}}_\nu^{abc}(t)}_{=:\widehat{\boldsymbol{x}}_{\nu,+}^{abc}(t)\ \text{[positive sequence]}} \\ & + \underbrace{\tfrac{1}{2}\boldsymbol{T}_\mathrm{c}^{-1} \boldsymbol{D}_{12} \boldsymbol{T}_\mathrm{c} \widehat{\boldsymbol{y}}_\nu^{abc}(t) - \tfrac{1}{2}\boldsymbol{T}_\mathrm{c}^{-1} \boldsymbol{J}_1 \boldsymbol{D}_{12} \boldsymbol{T}_\mathrm{c} \widehat{\boldsymbol{q}}_\nu^{abc}(t)}_{=:\widehat{\boldsymbol{x}}_{\nu,-}^{abc}(t)\ \text{[negative sequence]}} \\ & + \underbrace{\boldsymbol{T}_\mathrm{c}^{-1} \overbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{=:\boldsymbol{D}_3} \boldsymbol{T}_\mathrm{c} \widehat{\boldsymbol{y}}_\nu^{abc}(t)}_{=:\widehat{\boldsymbol{x}}_{\nu,0}^{abc}(t)\ \text{[zero sequence]}} \end{aligned}\right\} \qquad (\#)$$

Harmonic sequence extraction

In-phase and quadrature signal vector of ν-th harmonic

For all $\nu \in \mathbb{H}_n$ and $t \geqslant 0$, define

$$
\left.
\begin{aligned}
\widehat{\boldsymbol{y}}_\nu^{abc}(t) &:= \begin{pmatrix} \widehat{y}_\nu^a(t) \\ \widehat{y}_\nu^b(t) \\ \widehat{y}_\nu^c(t) \end{pmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}}_{=:\boldsymbol{C}_y^{abc} \in \mathbb{R}^{3\times 6}} \underbrace{\begin{pmatrix} \widehat{\boldsymbol{x}}_\nu^a(t) \\ \widehat{\boldsymbol{x}}_\nu^b(t) \\ \widehat{\boldsymbol{x}}_\nu^c(t) \end{pmatrix}}_{=:\widehat{\boldsymbol{x}}_\nu^{abc}(t) \in \mathbb{R}^6} \in \mathbb{R}^3 \text{ and} \\
\widehat{\boldsymbol{q}}_\nu^{abc}(t) &:= \begin{pmatrix} \widehat{q}_\nu^a(t) \\ \widehat{q}_\nu^b(t) \\ \widehat{q}_\nu^c(t) \end{pmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{=:\boldsymbol{C}_q^{abc} \in \mathbb{R}^{3\times 6}} \widehat{\boldsymbol{x}}_\nu^{abc}(t) \in \mathbb{R}^3,
\end{aligned}
\right\} \quad (8)
$$

Harmonic sequence extraction

Fortescue transformation (complex; in steady state)

For $\underline{\alpha} := \mathrm{e}^{\jmath \frac{2\pi}{3}}$, define the Fortescue matrix

$$\underline{\boldsymbol{T}}_{\mathrm{Fortescue}} := \tfrac{1}{3} \begin{bmatrix} 1 & \underline{\alpha} & \underline{\alpha}^2 \\ 1 & \underline{\alpha}^2 & \underline{\alpha} \\ 1 & 1 & 1 \end{bmatrix} \in \mathbb{C}^{3\times 3}$$

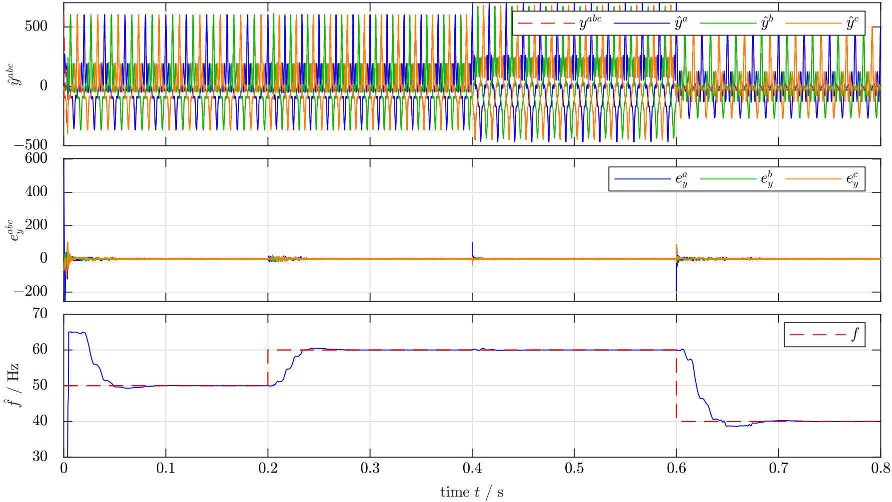
then, in steady state (as phasor!), the positive, negative and zero sequence of phase a are given by [10, Appendix A]

$$\underbrace{\begin{pmatrix} \underline{u}_{+}^{a} \\ \underline{u}_{-}^{a} \\ \underline{u}_{0}^{a} \end{pmatrix}}_{=:\underline{\boldsymbol{u}}_{+-0}^{a}} := \underline{\boldsymbol{T}}_{\mathrm{Fortescue}} \underbrace{\begin{pmatrix} \underline{u}^{c} \\ \underline{u}^{b} \\ \underline{u}^{c} \end{pmatrix}}_{=:\underline{\boldsymbol{u}}^{abc}}$$

and $\underline{u}_{+}^{b} := \underline{\alpha}^2 \, \underline{u}_{+}^{a}$, $\underline{u}_{-}^{b} := \underline{\alpha} \, \underline{u}_{-}^{a}$, $\underline{u}_{+}^{c} := \underline{\alpha} \, \underline{u}_{+}^{a}$ and $\underline{u}_{-}^{c} := \underline{\alpha}^2 \, \underline{u}_{-}^{a}$.

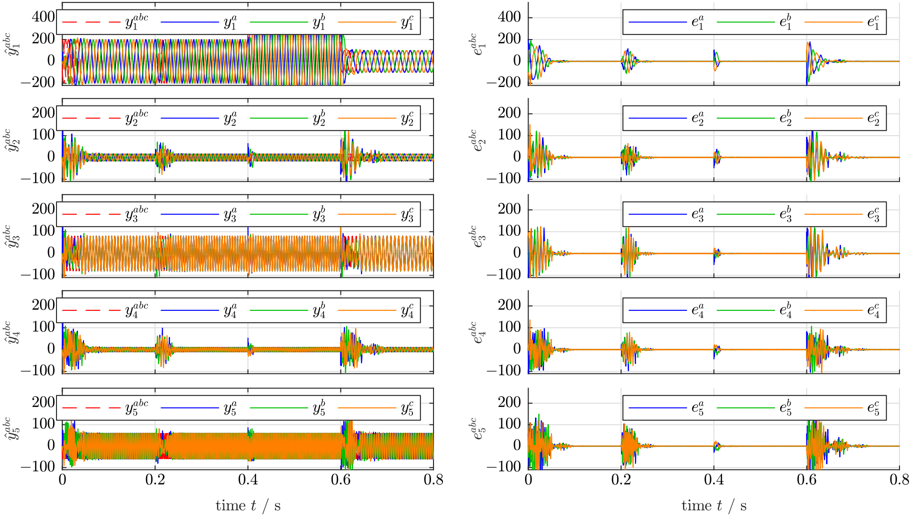
Harmonic sequence extraction

Simulation results: Estimate $\widehat{\boldsymbol{y}}^{abc}$, estimation error $\boldsymbol{e}^{abc} = \boldsymbol{y}^{abc} - \widehat{\boldsymbol{y}}^{abc}$ & estimated frequency $\widehat{f}$



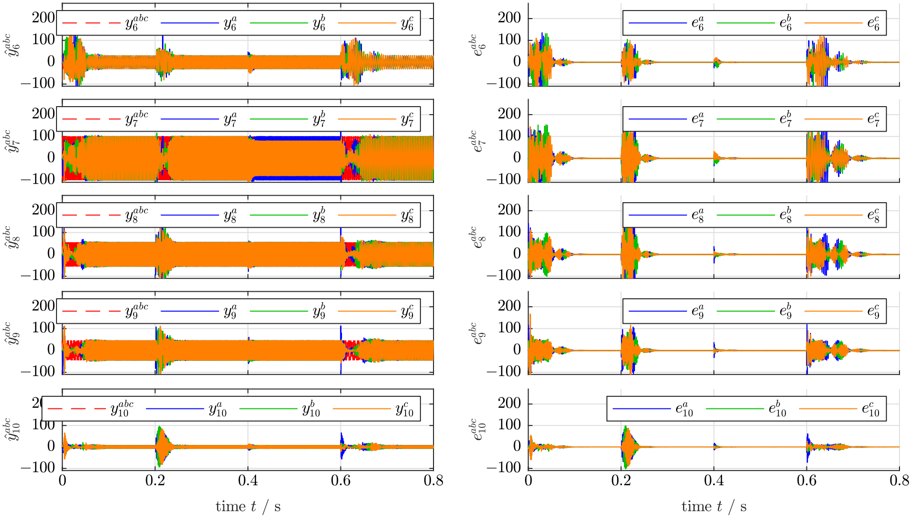
Harmonic sequence extraction

Simulation results: ν-th estimate $\widehat{y}^p_\nu$ (left) & ν-th estimation error $e^p_\nu = y^p_\nu - \widehat{y}^p_\nu$ (right), $p \in \{a, b, c\}$



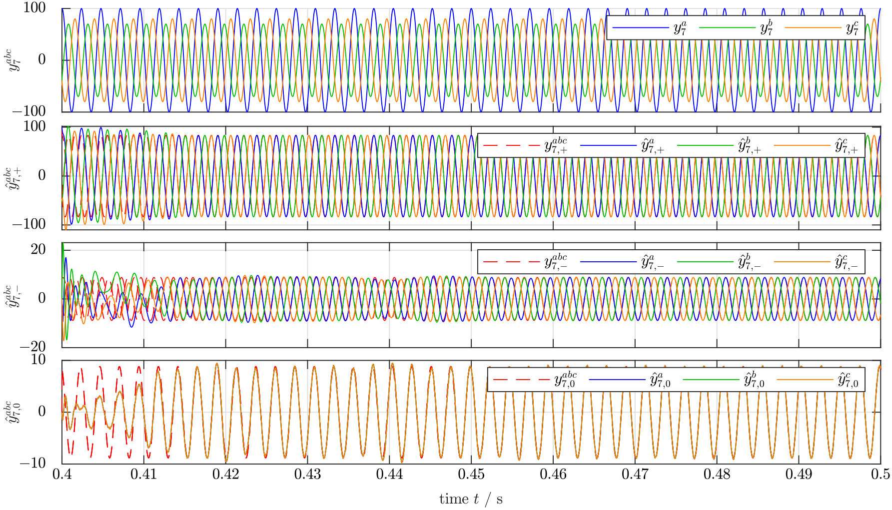
Harmonic sequence extraction

Simulation results: Estimate $\hat{y}_\nu$ (left) and estimation error $e_\nu = y_\nu - \hat{y}_\nu$ (right)



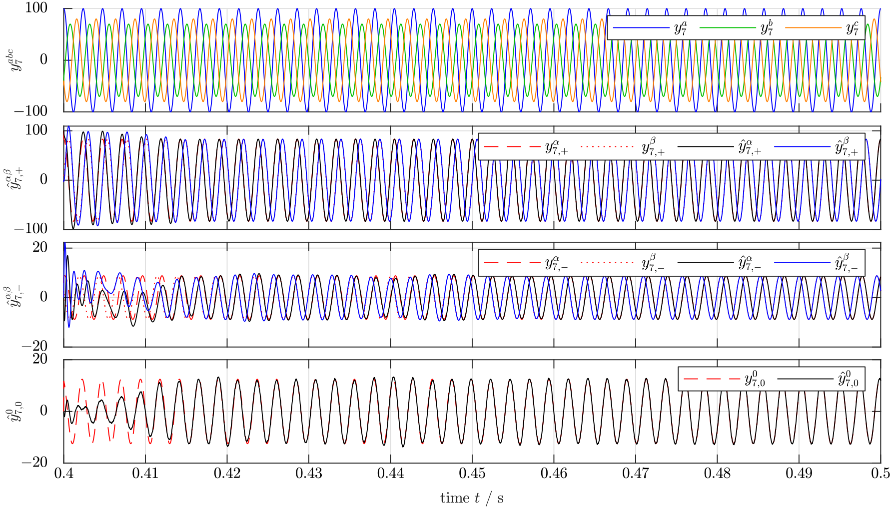
Harmonic sequence extraction

Simulation results: Harmonic $\boldsymbol{y}_7^{abc}$, positive $\widehat{\boldsymbol{y}}_{7,+}^{abc}$, negative $\widehat{\boldsymbol{y}}_{7,-}^{abc}$ & zero $\widehat{\boldsymbol{y}}_{7,0}^{abc}$ sequence



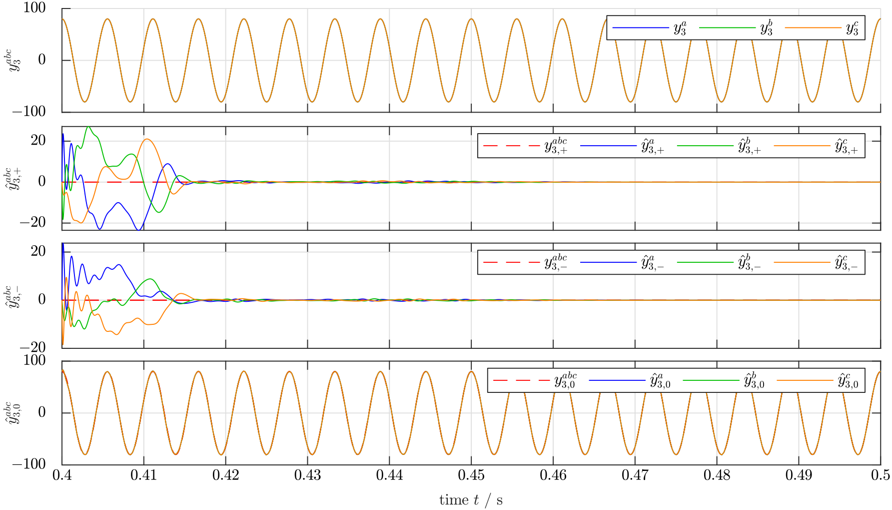
Harmonic sequence extraction

Simulation results: Harmonic $\boldsymbol{y}_7^{abc}$, positive $\widehat{\boldsymbol{y}}_{7,+}^{\alpha\beta}$, negative $\widehat{\boldsymbol{y}}_{7,-}^{\alpha\beta}$ & zero $\widehat{y}_{7,0}^{0}$ sequence



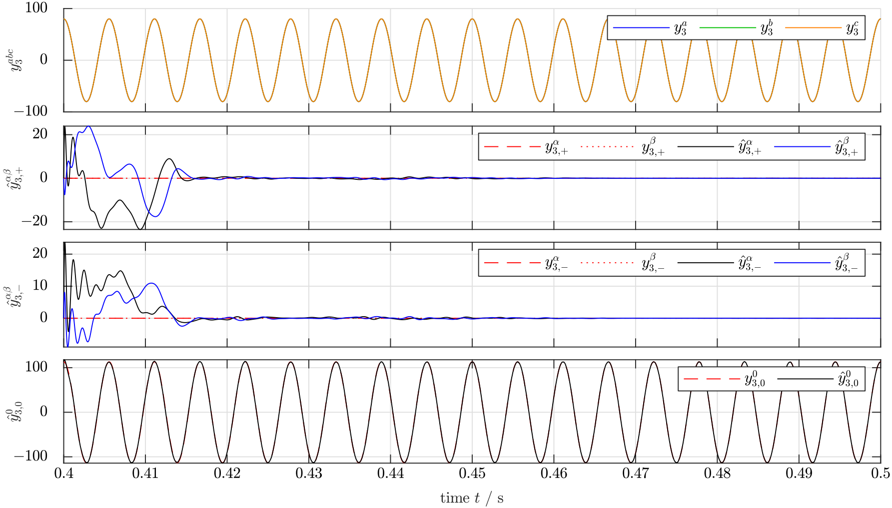
Harmonic sequence extraction

Simulation results: Harmonic $\boldsymbol{y}_3^{abc}$, positive $\widehat{\boldsymbol{y}}_{3,+}^{abc}$, negative $\widehat{\boldsymbol{y}}_{3,-}^{abc}$ & zero $\widehat{\boldsymbol{y}}_{3,0}^{abc}$ sequence



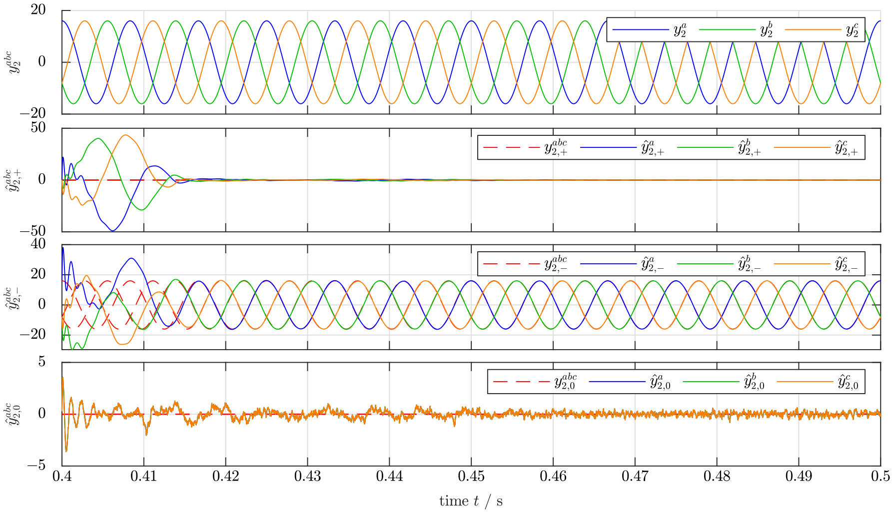
Harmonic sequence extraction

Simulation results: Harmonic $\boldsymbol{y}_3^{abc}$, positive $\widehat{\boldsymbol{y}}_{3,+}^{\alpha\beta}$, negative $\widehat{\boldsymbol{y}}_{3,-}^{\alpha\beta}$ & zero $\widehat{y}_{3,0}^{0}$ sequence



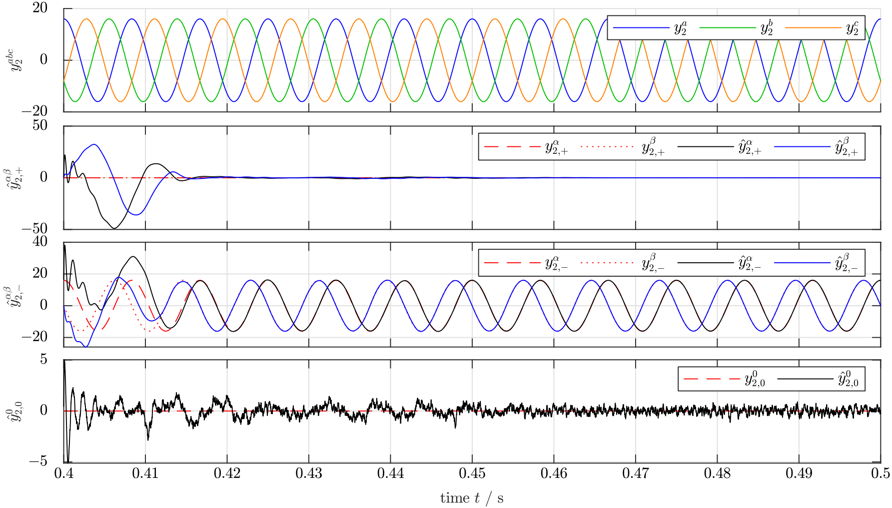
Harmonic sequence extraction

Simulation results: Harmonic $\boldsymbol{y}_2^{abc}$, positive $\widehat{\boldsymbol{y}}_{2,+}^{abc}$, negative $\widehat{\boldsymbol{y}}_{2,-}^{abc}$ & zero $\widehat{\boldsymbol{y}}_{2,0}^{abc}$ sequence



Harmonic sequence extraction

Simulation results: Harmonic $\boldsymbol{y}_2^{abc}$, positive $\widehat{\boldsymbol{y}}_{2,+}^{\alpha\beta}$, negative $\widehat{\boldsymbol{y}}_{2,-}^{\alpha\beta}$ & zero $\widehat{y}_{2,0}^{0}$ sequence



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