Analytical computation of the optimal reference currents for MTPC/MTPA, MTPV and MTPF operation of anisotropic synchronous machines considering stator resistance and mutual inductance

Christoph M. Hackl, Julian Kullick, Hisham Eldeeb and Lorenz Horlbeck

Technische Universität München (TUM)
Munich School of Engineering (MSE)
MSE Research group “Control of Renewable Energy Systems (CRES)”

www.cres.mse.tum.de

13.09.2017

EPE’17 ECCE Europe – 19th European Conference on Power Electronics and Applications

Motivation

Synchronous machines: Widely-used, compact and efficient actuators

www.bmw.de

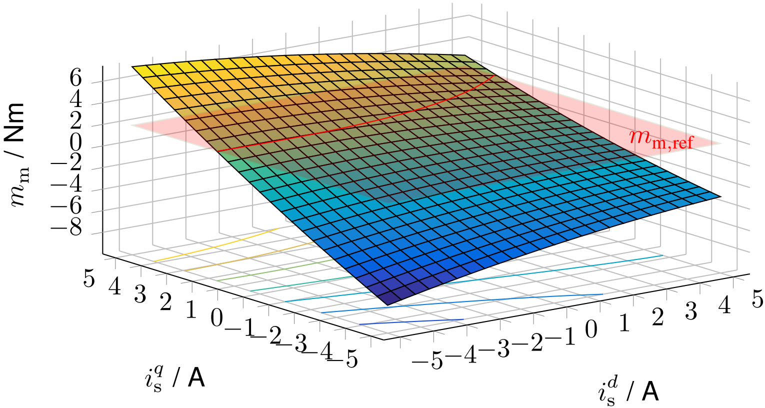
www.farmwood.co.uk

www.abb.com (from video)

- traction drives (e.g. electric vehicles or trains)
- air conditioning/fans
- pumps
- conventional power plants (e.g. hydro power plants)
- renewable energy systems (e.g. direct-drive wind turbine systems)

Motivation

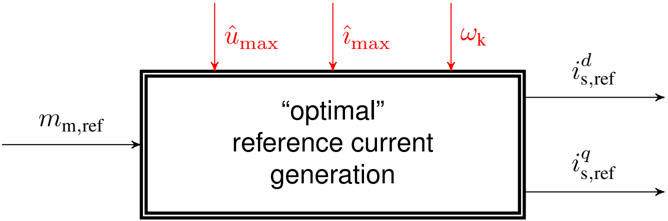
Optimal feedforward torque control problem: **Anisotropic machine**



$$m_{\mathrm{m}}\left(i_{\mathrm{s}}^{d},\, i_{\mathrm{s}}^{q}\right) = \tfrac{3}{2}\, n_{\mathrm{p}}\Big[\underbrace{\psi_{\mathrm{pm}} i_{\mathrm{s}}^{q}}_{\text{magnetic torque}} + \underbrace{\left(L_{\mathrm{s}}^{d} - L_{\mathrm{s}}^{q}\right) i_{\mathrm{s}}^{d} i_{\mathrm{s}}^{q}}_{\text{reluctance torque}} + \underbrace{L_{\mathrm{m}}\left((i_{\mathrm{s}}^{q})^2 - (i_{\mathrm{s}}^{d})^2\right)}_{\text{cross-coupling torque}}\Big]$$

Motivation

Optimal feedforward torque control problem: **Goal**

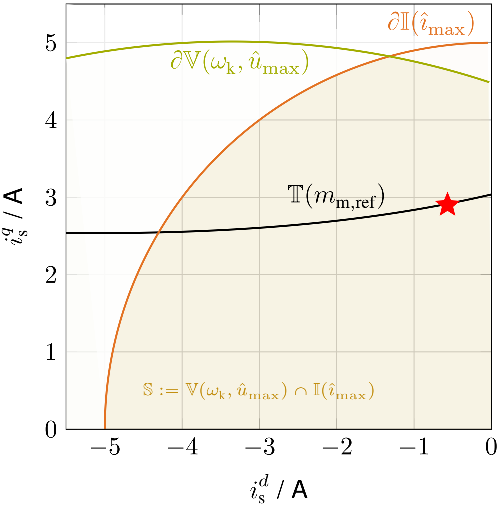


$$\boldsymbol{i}^k_\mathrm{s,ref}(m_\mathrm{m,ref}, \hat{u}_{\max}, \hat{\imath}_{\max}, \omega_\mathrm{k}) = \begin{pmatrix} i^d_\mathrm{s,ref}(m_\mathrm{m,ref}, \hat{u}_{\max}, \hat{\imath}_{\max}, \omega_\mathrm{k}) \\ i^q_\mathrm{s,ref}(m_\mathrm{m,ref}, \hat{u}_{\max}, \hat{\imath}_{\max}, \omega_\mathrm{k}) \end{pmatrix}$$

- ✓ Numerical solutions and/or look-up tables (but: limited storage, accuracy, real-time applicability)
- **?** Analytical solutions (some do exist but impose simplifying assumptions such as ~~$R_\mathrm{s} = 0$ and/or $L_\mathrm{m} = 0$~~)

Problem statement and proposed solution

Feedforward torque control: Optimization problem(s) with multiple constraints



$\max_{\boldsymbol{i}_{\mathrm{s}}^k \in \mathbb{S}} \; -f(\boldsymbol{i}_{\mathrm{s}}^k)$ subject to

$$\left\{\begin{array}{l} \|\boldsymbol{i}_{\mathrm{s}}^k\|^2 \leqslant \hat{\imath}_{\max}^2, \\ \text{(current circular area)} \\ \|\boldsymbol{u}_{\mathrm{s}}^k(\boldsymbol{i}_{\mathrm{s}}^k, \omega_{\mathrm{k}})\|^2 \leqslant \hat{u}_{\max}^2, \\ \text{(voltage elliptical area)} \\ |m_{\mathrm{m}}(\boldsymbol{i}_{\mathrm{s}}^k)| \leqslant |m_{\mathrm{m,ref}}|, \text{ and} \\ \mathrm{sign}(m_{\mathrm{m,ref}}) = \\ \mathrm{sign}(m_{\mathrm{m}}(\boldsymbol{i}_{\mathrm{s}}^k)). \end{array}\right.$$

e.g. $-f(\boldsymbol{i}_{\mathrm{s}}^k) = -\|\boldsymbol{i}_{\mathrm{s}}^k\|^2$ (minimize copper losses) $\Longrightarrow$ $\boldsymbol{i}_{\mathrm{s,ref}}^k$ at ★

Problem statement and proposed solution

Proposed solution: Implicit formulation (quadrics), optimization and intersection points

Step 1: Derivation of quadrics $Q_A(\boldsymbol{i}_\mathrm{s}^k) := (\boldsymbol{i}_\mathrm{s}^k)^\top \boldsymbol{A}\, \boldsymbol{i}_\mathrm{s}^k + 2\boldsymbol{a}^\top \boldsymbol{i}_\mathrm{s}^k + \alpha$:

- Current circular area: $(i_\mathrm{s}^d)^2 + (i_\mathrm{s}^q)^2 \leqslant \hat{\imath}_\mathrm{max}^2 \iff \|\boldsymbol{i}_\mathrm{s}^k\|^2 = (\boldsymbol{i}_\mathrm{s}^k)^\top \boldsymbol{I}_2 \boldsymbol{i}_\mathrm{s}^k \leqslant \hat{\imath}_\mathrm{max}^2$

$$\Longrightarrow \quad \mathbb{I}(\hat{\imath}_\mathrm{max}) := \left\{ \boldsymbol{i}_\mathrm{s}^k \in \mathbb{R}^2 \;\middle|\; (\boldsymbol{i}_\mathrm{s}^k)^\top \boldsymbol{I}_2 \boldsymbol{i}_\mathrm{s}^k - \hat{\imath}_\mathrm{max}^2 \leqslant 0 \right\}$$

- Voltage elliptical area: $(u_\mathrm{s}^d)^2 + (u_\mathrm{s}^q)^2 \leqslant \hat{u}_\mathrm{max}^2 \iff \|\boldsymbol{u}_\mathrm{s}^k(\boldsymbol{i}_\mathrm{s}^k, \omega_\mathrm{k})\|^2 = \ldots \leqslant \hat{u}_\mathrm{max}^2$

$$\Longrightarrow \quad \mathbb{V}(\omega_\mathrm{k}, \hat{u}_\mathrm{max}) := \left\{ \boldsymbol{i}_\mathrm{s}^k \in \mathbb{R}^2 \;\middle|\; (\boldsymbol{i}_\mathrm{s}^k)^\top \boldsymbol{V}(\omega_\mathrm{k})\, \boldsymbol{i}_\mathrm{s}^k + 2\, \boldsymbol{v}(\omega_\mathrm{k})^\top \boldsymbol{i}_\mathrm{s}^k + \nu(\omega_\mathrm{k}, \hat{u}_\mathrm{max}) \leqslant 0 \right\}$$

- Reference torque hyperbola: $m_\mathrm{m}(\boldsymbol{i}_\mathrm{s}^k) = \frac{3}{2} n_\mathrm{p} (\boldsymbol{i}_\mathrm{s}^k)^\top \boldsymbol{J} \boldsymbol{\psi}_\mathrm{s}^k(\boldsymbol{i}_\mathrm{s}^k) \stackrel{!}{=} m_\mathrm{m,ref} \iff \ldots$

$$\Longrightarrow \quad \mathbb{T}(m_\mathrm{m,ref}) := \left\{ \boldsymbol{i}_\mathrm{s}^k \in \mathbb{R}^2 \;\middle|\; (\boldsymbol{i}_\mathrm{s}^k)^\top \boldsymbol{T} \boldsymbol{i}_\mathrm{s}^k + 2\, \boldsymbol{t}^\top \boldsymbol{i}_\mathrm{s}^k - m_\mathrm{m,ref} = 0 \right\}$$

- ...

Problem statement and proposed solution

Proposed solution: Implicit formulation (quadrics), optimization and intersection points (cont'd)

Step 2: Optimization problem with equality constraint

$$\max_{\boldsymbol{i}_\mathrm{s}^k} -\Big(\underbrace{(\boldsymbol{i}_\mathrm{s}^k)^\top \boldsymbol{A} \boldsymbol{i}_\mathrm{s}^k + 2\boldsymbol{a}^\top \boldsymbol{i}_\mathrm{s}^k + \alpha}_{=:Q_A(\boldsymbol{i}_\mathrm{s}^k)} \Big) \text{ s.t. } \underbrace{(\boldsymbol{i}_\mathrm{s}^k)^\top \boldsymbol{B} \boldsymbol{i}_\mathrm{s}^k + 2\boldsymbol{b}^\top \boldsymbol{i}_\mathrm{s}^k + \beta}_{=:Q_B(\boldsymbol{i}_\mathrm{s}^k)} = 0$$

$\Longrightarrow$ Hyperbolas for MTPC, MTPV and MTPF (with $R_\mathrm{s} \neq 0$ & $L_\mathrm{m} \neq 0$!)

Step 3: Intersection of two quadrics (e.g. voltage ellipse and current circle)

$$\boldsymbol{i}_{\mathrm{s,ref}}^k := \arg\min_{\|\boldsymbol{i}_\mathrm{s}^k\|} \left\{ \boldsymbol{i}_\mathrm{s}^k \in \mathbb{R}^2 \;\middle|\; Q_A(\boldsymbol{i}_\mathrm{s}^k) = 0 \;\wedge\; Q_B(\boldsymbol{i}_\mathrm{s}^k) = 0 \right\}$$

$\Longrightarrow$ Optimal operation point ★ (reference current)

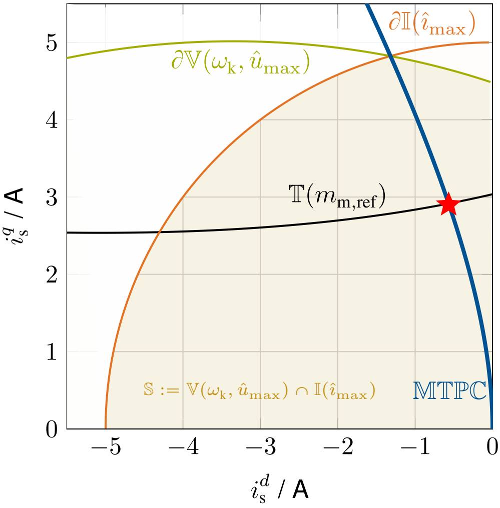
Both lead to subproblem of solving a fourth-order (quartic) polynomial

$$\chi(\lambda) := c_4\,\lambda^4 + c_3\,\lambda^3 + c_2\,\lambda^2 + c_1\,\lambda + c_0 \overset{!}{=} 0$$

$\Longrightarrow$ Analytical solutions exist (e.g. Euler's solution, see [1])

Operation management and operation strategies

Maximum Torque per Current (**MTPC**): Reference torque feasible



Optimization problem

$$\max_{\boldsymbol{i}_{\mathrm{s}}^{k} \in \mathbb{S}} -\|\boldsymbol{i}_{\mathrm{s}}^{k}\|^2 \quad \text{s.t.}$$

$$\underbrace{(\boldsymbol{i}_{\mathrm{s}}^{k})^\top \boldsymbol{T} \boldsymbol{i}_{\mathrm{s}}^{k} + 2\,\boldsymbol{t}^\top \boldsymbol{i}_{\mathrm{s}}^{k}}_{=m_{\mathrm{m}}(\boldsymbol{i}_{\mathrm{s}}^{k})} - m_{\mathrm{m,ref}} = 0$$

Solution set

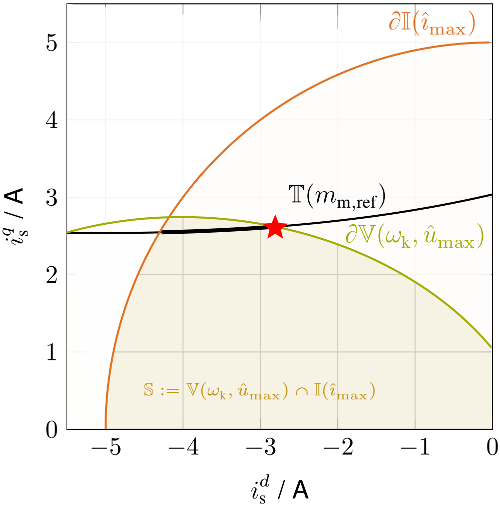
$$\mathbb{MTPC} := \left\{ \boldsymbol{i}_{\mathrm{s}}^{k} \in \mathbb{R}^2 \;\middle|\; (\boldsymbol{i}_{\mathrm{s}}^{k})^\top \boldsymbol{M}_{\mathrm{C}}\, \boldsymbol{i}_{\mathrm{s}}^{k} + 2\,\boldsymbol{m}_{\mathrm{C}}^\top \boldsymbol{i}_{\mathrm{s}}^{k} = 0 \right\}$$

Optimal current reference (★)

$$\boldsymbol{i}_{\mathrm{s,ref}}^{k} = \mathbb{MTPC} \cap \mathbb{T}(m_{\mathrm{m,ref}})$$

Operation management and operation strategies

Field Weakening (**FW**): Reference torque feasible



Optimization problem

$$\max_{\boldsymbol{i}_{\mathrm{s}}^{k} \in \mathbb{S}} -\|\boldsymbol{i}_{\mathrm{s}}^{k}\|^2 \quad \text{s.t.}$$

$$\underbrace{(\boldsymbol{i}_{\mathrm{s}}^{k})^\top \boldsymbol{T} \boldsymbol{i}_{\mathrm{s}}^{k} + 2\,\boldsymbol{t}^\top \boldsymbol{i}_{\mathrm{s}}^{k}}_{=m_{\mathrm{m}}(\boldsymbol{i}_{\mathrm{s}}^{k})} - m_{\mathrm{m,ref}} = 0$$

Feasible set

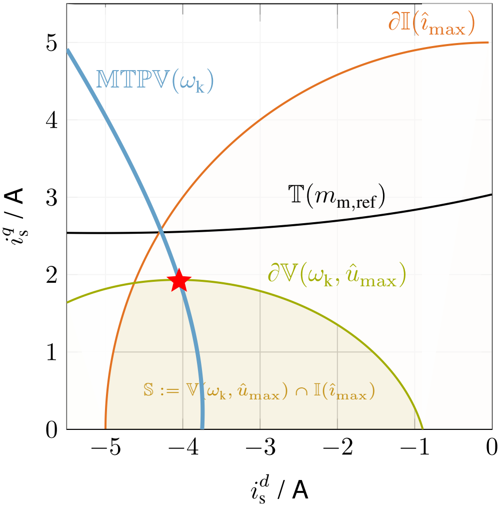
$$\mathbb{FW}(m_{\mathrm{m,ref}}, \omega_{\mathrm{k}}, \hat{u}_{\max}) := \mathbb{S}(\omega_{\mathrm{k}}, \hat{u}_{\max}, \hat{\imath}_{\max}) \cap \mathbb{T}(m_{\mathrm{m,ref}})$$

Optimal current reference (★)

$$\boldsymbol{i}_{\mathrm{s,ref}}^{k} = \partial\mathbb{V}(\omega_{\mathrm{k}}, \hat{u}_{\max}) \cap \mathbb{T}(m_{\mathrm{m,ref}})$$

Operation management and operation strategies

Maximum Torque per Voltage (**MTPV**): Reference torque **not** feasible



Optimization problem

$$\max_{\boldsymbol{i}_{\mathrm{s}}^{k} \in \mathbb{S}} -\|\boldsymbol{u}_{\mathrm{s}}^{k}(\boldsymbol{i}_{\mathrm{s}}^{k})\|^2 \quad \text{s.t.}$$

$$\underbrace{(\boldsymbol{i}_{\mathrm{s}}^{k})^\top \boldsymbol{T} \boldsymbol{i}_{\mathrm{s}}^{k} + 2\,\boldsymbol{t}^\top \boldsymbol{i}_{\mathrm{s}}^{k}}_{=m_{\mathrm{m}}(\boldsymbol{i}_{\mathrm{s}}^{k})} - m_{\mathrm{m,ref}} = 0$$

Solution set

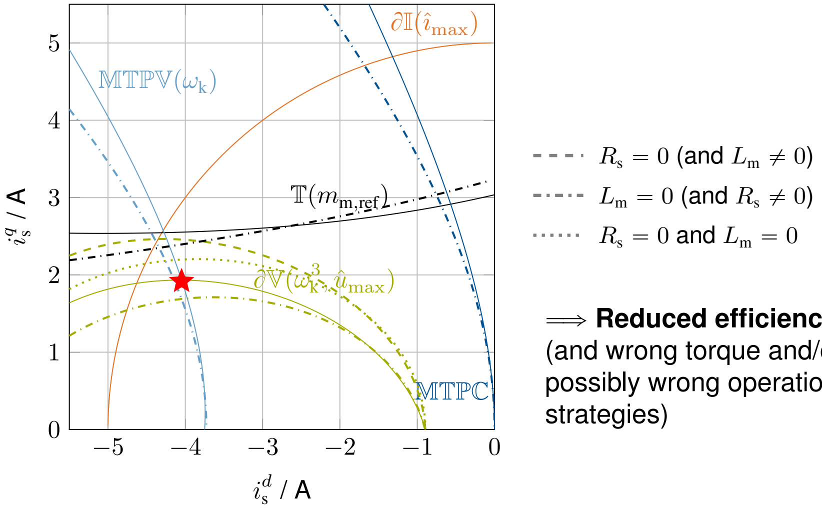
$$\mathrm{MTPV}(\omega_{\mathrm{k}}) := \left\{ \boldsymbol{i}_{\mathrm{s}}^{k} \in \mathbb{R}^2 \,\middle|\, (\boldsymbol{i}_{\mathrm{s}}^{k})^\top \boldsymbol{M}_{\mathrm{V}}\, \boldsymbol{i}_{\mathrm{s}}^{k} + 2\,\boldsymbol{m}_{\mathrm{V}}^\top \boldsymbol{i}_{\mathrm{s}}^{k} + \mu_{\mathrm{V}} = 0 \right\}$$

Optimal current reference (★)

$$\boldsymbol{i}_{\mathrm{s,ref}}^{k} = \mathrm{MTPV}(\omega_{\mathrm{k}}) \cap \partial\mathbb{V}(\omega_{\mathrm{k}}, \hat{u}_{\max})$$

Operation management and operation strategies

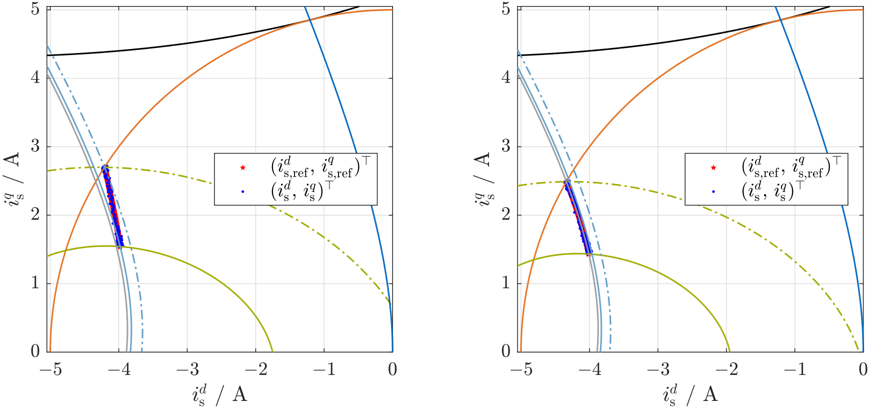
Illustration of the effect of neglecting R_s and/or L_m



$\Longrightarrow$ **Reduced efficiency**!
(and wrong torque and/or possibly wrong operation strategies)

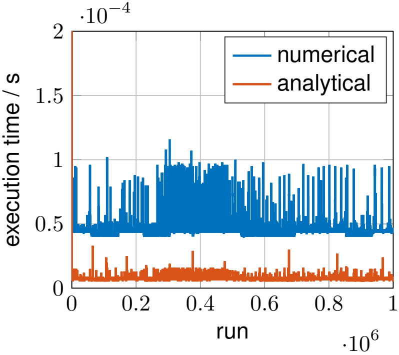
Implementation results

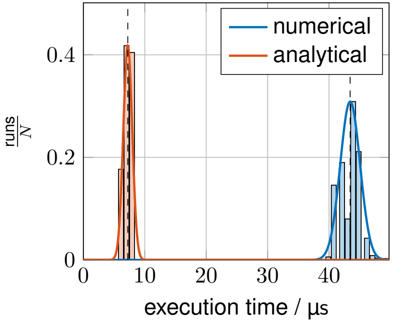
Simulation (including switching behavior): MTPV and MPTF



Implementation results

Experiments: Comparison of the computational load (Euler's solution for fourth-order polynomials)

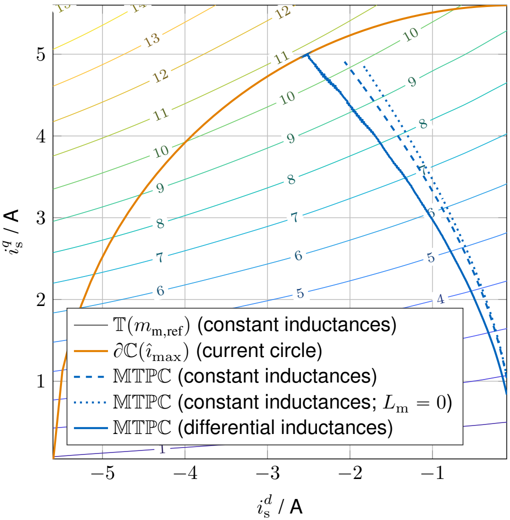




- Execution times for $N = 10^6$ runs (downsampled by factor 100);
- Average execution times $\mu_n = 43{,}4 \cdot 10^{-6}$ s and $\mu_a = 7{,}23 \cdot 10^{-6}$ s; and
- Standard deviations $\sigma_n = 1{,}61 \cdot 10^{-6}$ s and $\sigma_a = 0{,}73 \cdot 10^{-6}$ s.

Implementation results

Experiments: MTPC at a IPMSM with non-negligible mutual inductance and nonlinear flux linkage



Conclusion

Summary and future work

To take home

- **unified** theory for MTPC, FW, MTPV/MTPF or MC based on
  - **quadrics** (implicit formulation),
  - **optimization problems** with **equality constraint**, and
  - **intersection** of two quadrics
- **analytical** solution for reference currents (& transition points)
- $R_{\mathrm{s}} \neq 0$ and $L_{\mathrm{m}} \neq 0$ (both can be considered **simultaneously**)
- **fast(er)** and **more accurate** computation
- **applicable** in real-world (nonlinear RSM and IPMSM)

Future work

- convergence (mathematical proof or consideration of nonlinear flux linkages)
- consideration of iron losses (including e.g. $R_{\mathrm{fe}}(\omega_{\mathrm{k}})$; article in preparation)
- impact of parameter uncertainty (e.g. $R_{\mathrm{s}}^{a} \neq R_{\mathrm{s}}^{b} \neq R_{\mathrm{s}}^{c}$; article in preparation)

References I

[1] R.W.D Nickalls. The quartic equation: Invariants and Euler's solution revealed. *Mathematical Gazette*, 93(526):66–75, 2009.