Model predictive control with analytical solution and integral error feedback for current control of doubly-fed induction generators (DFIGs) with LC filter

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Outline

Model-predictive control: Problem statement

- **Optimality** with respect to performance index

$$J = \sum_{i=1}^{n} w_i J_i, \qquad n \in \mathbb{N},\ \forall i \in \{1, \ldots, n\}: \quad J_i, w_i \geqslant 0. \tag{COST}$$

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  - Use of long prediction horizons (not feasible) $\rightsquigarrow$ MPC with analytical solution

Contents

Outline

1. Model predictive control: Revisiting theory
   - Considered system class
   - Analytical solution
   - Integral error feedback and system extension
   - Implementation

Model predictive control

Considered system class

LTI MIMO system (discrete) with **disturbances**

$$\left.\begin{aligned} \boldsymbol{x}[k+1] &= \boldsymbol{A}\boldsymbol{x}[k] + \boldsymbol{B}\boldsymbol{u}[k] + \boldsymbol{B}_{\mathrm{d}}\,\boldsymbol{d}[k], \quad \boldsymbol{x}[0] = \boldsymbol{x}_0 \in \mathbb{R}^n \\ \boldsymbol{y}[k] &= \boldsymbol{C}\boldsymbol{x}[k], \end{aligned}\right\} \tag{1}$$

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Standard assumptions [2, p. 48] on system (1):

(A_1) controllability, i.e. $\mathrm{rank}[\boldsymbol{B}, \boldsymbol{A}\boldsymbol{B}, \ldots, \boldsymbol{A}^{n-1}\boldsymbol{B}] = n$.

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(A$_2$) full state feedback, i.e. $\boldsymbol{x}[k]$ is available for all k

(**or** observability, i.e. $\mathrm{rank}\begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{C}\boldsymbol{A} \\ \vdots \\ \boldsymbol{C}\boldsymbol{A}^{n-1} \end{bmatrix} = n$ (observer design).)

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(A_3) model-based prediction, i.e. $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{B}_{\mathrm{d}}$ and $\boldsymbol{C}$ are known and $\boldsymbol{d}(\cdot)$ is measured.

What if model is not accurate / disturbances are only roughly know?

Model predictive control

Problem & Solution

Problem: Find *optimal control action* (vector)

$$\boldsymbol{u}_{\star}^{H_{\mathrm{p}}}[k] := \left(\boldsymbol{u}_{\star}^{H_{\mathrm{p}}}[k+1]^{\top}, \ldots, \boldsymbol{u}_{\star}[k+n_{\mathrm{p}}]^{\top}\right)^{\top} \in \mathbb{R}^{m n_{\mathrm{p}}} \tag{2}$$

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over $H_\mathrm{p}(k) := \{k+1, \ldots, k+n_\mathrm{p}+1\}$ which maximizes *performance index*

$$\left.\begin{array}{l} J_0\big(k, \boldsymbol{u}, \boldsymbol{x}, \boldsymbol{v}\big) = \boldsymbol{\gamma}\big(k, \boldsymbol{u}, \boldsymbol{x}\big)^\top \boldsymbol{v} + \frac{1}{2}\boldsymbol{v}^\top \boldsymbol{\Lambda}\big(k, \boldsymbol{u}, \boldsymbol{x}\big)\boldsymbol{v}, \\ \text{where } \forall (k, \boldsymbol{u}, \boldsymbol{x}) \in \mathbb{N}_0 \times \mathbb{R} \times \mathbb{R}^n: \\ (\mathrm{C}_1)\ 0 < \boldsymbol{\Lambda}(k, \boldsymbol{u}, \boldsymbol{x}) = \boldsymbol{\Lambda}(k, \boldsymbol{u}, \boldsymbol{x})^\top \in \mathbb{R}^{n_\mathrm{p} \times n_\mathrm{p}} \ \text{ and} \\ (\mathrm{C}_2)\ \boldsymbol{\Lambda}(k, \boldsymbol{u}, \boldsymbol{x})^{-1}\boldsymbol{\gamma}(k, \boldsymbol{u}, \boldsymbol{x}) = \boldsymbol{0}_{n_\mathrm{p}} \Leftrightarrow \boldsymbol{\gamma}(k, \boldsymbol{u}, \boldsymbol{x}) = \boldsymbol{0}_{n_\mathrm{p}} \end{array}\right\} \tag{3}$$

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Analytical solution: (unique and globally optimal)

$$\boldsymbol{u}_\star^{H_\mathrm{p}}[k] := \arg\min_{\boldsymbol{v}}\Big(J_0\big(k, \boldsymbol{u}[k], \boldsymbol{x}[k], \boldsymbol{v}\big)\Big) = -\boldsymbol{\Lambda}(k, \boldsymbol{u}[k], \boldsymbol{x}[k])^{-1}\boldsymbol{\gamma}(k, \boldsymbol{u}[k], \boldsymbol{x}[k]). \tag{4}$$

Model predictive control

Integral error feedback and system extension

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Integral error feedback and system extension

Additional state for integration of error

$$\boldsymbol{x}_i[k+1] = \boldsymbol{x}_i[k] + T_\mathrm{s}\, k_i \underbrace{\left(\widehat{\boldsymbol{y}}_\mathrm{ref}[k] - \boldsymbol{C}\boldsymbol{x}[k]\right)}_{=:\boldsymbol{e}[k]}, \quad k_i > 0, \tag{5}$$

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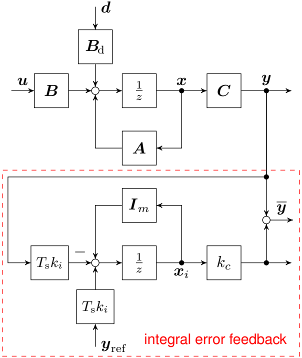
Extended system

$$\overbrace{\begin{pmatrix} \boldsymbol{x}[k+1] \\ \boldsymbol{x}_i[k+1] \end{pmatrix}}^{:=\overline{\boldsymbol{x}}[k+1]} = \overbrace{\begin{bmatrix} \boldsymbol{A} & \boldsymbol{O}_{n\times m} \\ -T_\mathrm{s}\, k_i \boldsymbol{C} & \boldsymbol{I}_{m\times m} \end{bmatrix}}^{=:\overline{\boldsymbol{A}}\in\mathbb{R}^{(n+m)\times(n+m)}} \overline{\boldsymbol{x}}[k] + \overbrace{\begin{bmatrix} \boldsymbol{B} \\ \boldsymbol{O}_{m\times m} \end{bmatrix}}^{=:\overline{\boldsymbol{B}}\in\mathbb{R}^{(n+m)\times m}} \boldsymbol{u}[k]$$
$$+ \underbrace{\begin{bmatrix} \boldsymbol{B}_\mathrm{d} & \boldsymbol{O}_{n\times m} \\ \boldsymbol{O}_{m\times l} & T_\mathrm{s}\, k_i \boldsymbol{I}_m \end{bmatrix}}_{=:\overline{\boldsymbol{B}}_\mathrm{d}\in\mathbb{R}^{(n+m)\times(l+m)}} \underbrace{\begin{pmatrix} \boldsymbol{d}[k] \\ \widehat{\boldsymbol{y}}_\mathrm{ref}[k] \end{pmatrix}}_{=:\overline{\boldsymbol{d}}[k]\in\mathbb{R}^{l+m}},$$

$$\overline{\boldsymbol{y}}[k] = \underbrace{\begin{bmatrix} \boldsymbol{C} & k_c \boldsymbol{I}_m \end{bmatrix}}_{=:\overline{\boldsymbol{C}}\in\mathbb{R}^{m\times(n+m)}} \overline{\boldsymbol{x}}[k], \qquad k_c > 0.$$



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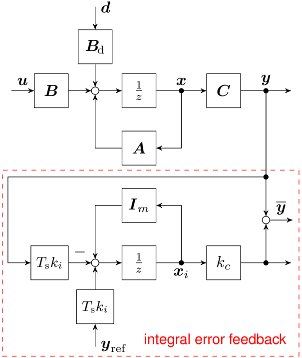
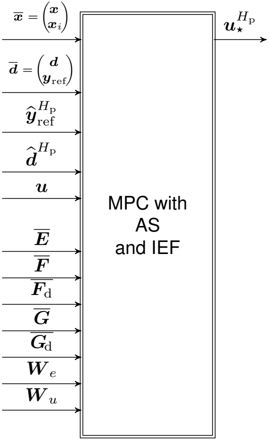


Open question: How to tune k_i & k_c?

Model predictive control

Implementation of MPC with analytical solution (AS) and integral error feedback (IEF)



Performance index (with tracking and control weights):

$$\boxed{J(k,\boldsymbol{u},\overline{\boldsymbol{x}},\boldsymbol{u}_\star^{H_\mathrm{p}}) = \underbrace{(\widehat{\boldsymbol{e}}^{H_\mathrm{p}})^\top \boldsymbol{W}_\mathrm{e}(k,\boldsymbol{u},\overline{\boldsymbol{x}})\,\widehat{\boldsymbol{e}}^{H_\mathrm{p}}}_{=:J_e(k,\boldsymbol{u},\overline{\boldsymbol{x}},\boldsymbol{u}_\star^{H_\mathrm{p}})} + \underbrace{(\boldsymbol{u}_\star^{H_\mathrm{p}})^\top \boldsymbol{W}_\mathrm{u}(k,\boldsymbol{u},\overline{\boldsymbol{x}})\,\boldsymbol{u}_\star^{H_\mathrm{p}}}_{=:J_u(\boldsymbol{u}_\star^{H_\mathrm{p}})}}$$

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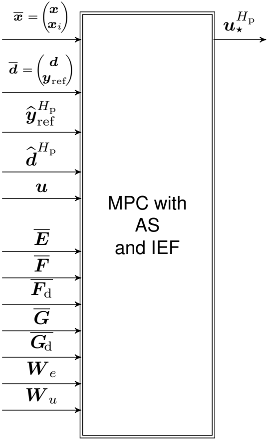

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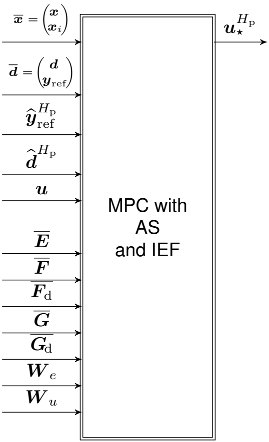

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with

$$\widehat{\boldsymbol{e}}^{H_{\mathrm{p}}}[k] = \widehat{\boldsymbol{y}}_{\mathrm{ref}}^{H_{\mathrm{p}}}[k] - \big(\underbrace{\overline{\boldsymbol{E}}\overline{\boldsymbol{x}}[k] + \overline{\boldsymbol{F}}\boldsymbol{u}[k] + \overline{\boldsymbol{F}}_{\mathrm{d}}\overline{\boldsymbol{d}}[k] + \overline{\boldsymbol{G}}_{\mathrm{d}}\widehat{\boldsymbol{d}}^{H_{\mathrm{p}}}[k] + \overline{\boldsymbol{G}}\boldsymbol{u}_\star^{H_{\mathrm{p}}}[k]}_{=:\widehat{\boldsymbol{y}}[k]}\big)$$

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$$\overline{\boldsymbol{E}} := \begin{bmatrix} \overline{\boldsymbol{C}\boldsymbol{A}} \\ \overline{\boldsymbol{C}\boldsymbol{A}^2} \\ \vdots \\ \overline{\boldsymbol{C}\boldsymbol{A}^{n_{\mathrm{p}}+1}} \end{bmatrix}, \overline{\boldsymbol{F}} := \begin{bmatrix} \overline{\boldsymbol{C}\boldsymbol{B}} \\ \overline{\boldsymbol{C}\boldsymbol{A}\boldsymbol{B}} \\ \vdots \\ \overline{\boldsymbol{C}\boldsymbol{A}^{n_{\mathrm{p}}}\boldsymbol{B}} \end{bmatrix}, \overline{\boldsymbol{F}}_{\mathrm{d}} := \begin{bmatrix} \overline{\boldsymbol{C}\boldsymbol{B}_{\mathrm{d}}} \\ \overline{\boldsymbol{C}\boldsymbol{A}\boldsymbol{B}_{\mathrm{d}}} \\ \vdots \\ \overline{\boldsymbol{C}\boldsymbol{A}^{n_{\mathrm{p}}}\boldsymbol{B}_{\mathrm{d}}} \end{bmatrix}, \overline{\boldsymbol{G}}(\overline{\boldsymbol{C}}, \overline{\boldsymbol{A}}, \overline{\boldsymbol{B}}), \overline{\boldsymbol{G}}_{\mathrm{d}}(\overline{\boldsymbol{C}}, \overline{\boldsymbol{A}}, \overline{\boldsymbol{B}}_{\mathrm{d}})$$

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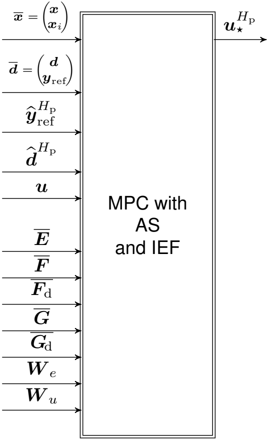

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Performance index (reformulated):

$$\boxed{J_0(k,\boldsymbol{u},\overline{\boldsymbol{x}},\boldsymbol{u}_\star^{H_{\mathrm{p}}}) = \boldsymbol{\gamma}(k,\boldsymbol{u},\overline{\boldsymbol{x}})^\top \boldsymbol{u}_\star^{H_{\mathrm{p}}} + \frac{1}{2}(\boldsymbol{u}_\star^{H_{\mathrm{p}}})^\top \boldsymbol{\Lambda}(k,\boldsymbol{u},\overline{\boldsymbol{x}})\boldsymbol{u}_\star^{H_{\mathrm{p}}}}$$

where

$$\boldsymbol{\gamma}(k,\boldsymbol{u},\boldsymbol{x})^\top = 2\Big(-\widehat{\boldsymbol{y}}_{\mathrm{ref}}^{H_{\mathrm{p}}}[k]^\top + \overline{\boldsymbol{x}}^\top\overline{\boldsymbol{E}}^\top + \boldsymbol{u}^\top\overline{\boldsymbol{F}}^\top + \overline{\boldsymbol{d}}[k]^\top\overline{\boldsymbol{F}}_{\mathrm{d}}^\top + \widehat{\boldsymbol{d}}^{H_{\mathrm{p}}}[k]^\top\overline{\boldsymbol{G}}_{\mathrm{d}}^\top\Big)\boldsymbol{W}_{\mathrm{e}}(k,\boldsymbol{u},\overline{\boldsymbol{x}})\overline{\boldsymbol{G}}.$$

and $\boldsymbol{\Lambda}(k,\boldsymbol{u}[k],\overline{\boldsymbol{x}}[k]) := 2\Big(\overline{\boldsymbol{G}}^\top\boldsymbol{W}_{\mathrm{e}}(k,\boldsymbol{u}[k],\overline{\boldsymbol{x}}[k])\overline{\boldsymbol{G}} + \boldsymbol{W}_{\mathrm{u}}(k,\boldsymbol{u}[k],\overline{\boldsymbol{x}}[k])\Big)$

$$\Longrightarrow \qquad \boxed{\boldsymbol{u}_\star^{H_{\mathrm{p}}}[k] = -\boldsymbol{\Lambda}(k,\boldsymbol{u}[k],\overline{\boldsymbol{x}}[k])^{-1}\boldsymbol{\gamma}(k,\boldsymbol{u}[k],\overline{\boldsymbol{x}}[k]).}$$

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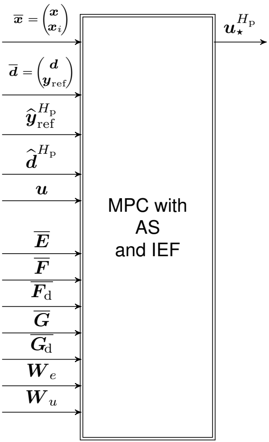


Performance index (with tracking and control weights):

$$\boxed{J(k,\boldsymbol{u},\overline{\boldsymbol{x}},\boldsymbol{u}_\star^{H_{\mathrm{p}}}) = \underbrace{(\widehat{\boldsymbol{e}}^{H_{\mathrm{p}}})^\top \boldsymbol{W}_{\mathrm{e}}(k,\boldsymbol{u},\overline{\boldsymbol{x}})\,\widehat{\boldsymbol{e}}^{H_{\mathrm{p}}}}_{=:J_e(k,\boldsymbol{u},\overline{\boldsymbol{x}},\boldsymbol{u}_\star^{H_{\mathrm{p}}})} + \underbrace{(\boldsymbol{u}_\star^{H_{\mathrm{p}}})^\top \boldsymbol{W}_{\mathrm{u}}(k,\boldsymbol{u},\overline{\boldsymbol{x}})\,\boldsymbol{u}_\star^{H_{\mathrm{p}}}}_{=:J_u(\boldsymbol{u}_\star^{H_{\mathrm{p}}})}}$$

with

$$\widehat{\boldsymbol{e}}^{H_{\mathrm{p}}}[k] = \widehat{\boldsymbol{y}}_{\mathrm{ref}}^{H_{\mathrm{p}}}[k] - \big(\underbrace{\overline{\boldsymbol{E}}\,\overline{\boldsymbol{x}}[k] + \overline{\boldsymbol{F}}\boldsymbol{u}[k] + \overline{\boldsymbol{F}}_{\mathrm{d}}\,\overline{\boldsymbol{d}}[k] + \overline{\boldsymbol{G}}_{\mathrm{d}}\widehat{\boldsymbol{d}}^{H_{\mathrm{p}}}[k] + \overline{\boldsymbol{G}}\boldsymbol{u}_\star^{H_{\mathrm{p}}}[k]}_{=:\widehat{\boldsymbol{y}}[k]}\big)$$

where

$$\overline{\boldsymbol{E}} := \begin{bmatrix} \overline{\boldsymbol{C}\boldsymbol{A}} \\ \overline{\boldsymbol{C}\boldsymbol{A}^2} \\ \vdots \\ \overline{\boldsymbol{C}\boldsymbol{A}^{n_{\mathrm{p}}+1}} \end{bmatrix},\ \overline{\boldsymbol{F}} := \begin{bmatrix} \overline{\boldsymbol{C}\boldsymbol{B}} \\ \overline{\boldsymbol{C}\boldsymbol{A}\boldsymbol{B}} \\ \vdots \\ \overline{\boldsymbol{C}\boldsymbol{A}^{n_{\mathrm{p}}}\boldsymbol{B}} \end{bmatrix},\ \overline{\boldsymbol{F}}_{\mathrm{d}} := \begin{bmatrix} \overline{\boldsymbol{C}\boldsymbol{B}_{\mathrm{d}}} \\ \overline{\boldsymbol{C}\boldsymbol{A}\boldsymbol{B}_{\mathrm{d}}} \\ \vdots \\ \overline{\boldsymbol{C}\boldsymbol{A}^{n_{\mathrm{p}}}\boldsymbol{B}_{\mathrm{d}}} \end{bmatrix},\ \overline{\boldsymbol{G}}(\overline{\boldsymbol{C}},\overline{\boldsymbol{A}},\overline{\boldsymbol{B}}),\ \overline{\boldsymbol{G}}_{\mathrm{d}}(\overline{\boldsymbol{C}},\overline{\boldsymbol{A}},\overline{\boldsymbol{B}}_{\mathrm{d}})$$

Performance index (reformulated):

$$\boxed{J_0(k,\boldsymbol{u},\overline{\boldsymbol{x}},\boldsymbol{u}_\star^{H_{\mathrm{p}}}) = \boldsymbol{\gamma}(k,\boldsymbol{u},\overline{\boldsymbol{x}})^\top \boldsymbol{u}_\star^{H_{\mathrm{p}}} + \frac{1}{2}(\boldsymbol{u}_\star^{H_{\mathrm{p}}})^\top \boldsymbol{\Lambda}(k,\boldsymbol{u},\overline{\boldsymbol{x}})\boldsymbol{u}_\star^{H_{\mathrm{p}}}}$$

where

$$\boldsymbol{\gamma}(k,\boldsymbol{u},\boldsymbol{x})^\top = 2\Big(-\widehat{\boldsymbol{y}}_{\mathrm{ref}}^{H_{\mathrm{p}}}[k]^\top + \overline{\boldsymbol{x}}^\top\overline{\boldsymbol{E}}^\top + \boldsymbol{u}^\top\overline{\boldsymbol{F}}^\top + \overline{\boldsymbol{d}}[k]^\top\overline{\boldsymbol{F}}_{\mathrm{d}}^\top + \widehat{\boldsymbol{d}}^{H_{\mathrm{p}}}[k]^\top\overline{\boldsymbol{G}}_{\mathrm{d}}^\top\Big)\boldsymbol{W}_{\mathrm{e}}(k,\boldsymbol{u},\overline{\boldsymbol{x}})\overline{\boldsymbol{G}}.$$

and $\boldsymbol{\Lambda}(k,\boldsymbol{u}[k],\overline{\boldsymbol{x}}[k]) := 2\Big(\overline{\boldsymbol{G}}^\top\boldsymbol{W}_{\mathrm{e}}(k,\boldsymbol{u}[k],\overline{\boldsymbol{x}}[k])\overline{\boldsymbol{G}} + \boldsymbol{W}_{\mathrm{u}}(k,\boldsymbol{u}[k],\overline{\boldsymbol{x}}[k])\Big)$

$$\Longrightarrow \quad \boxed{\boldsymbol{u}_\star^{H_{\mathrm{p}}}[k] = -\boldsymbol{\Lambda}(k,\boldsymbol{u}[k],\overline{\boldsymbol{x}}[k])^{-1}\boldsymbol{\gamma}(k,\boldsymbol{u}[k],\overline{\boldsymbol{x}}[k]).}$$

$\Longrightarrow$ (Almost) arbitrary lengths $n_{\mathrm{p}} \gg 1$ of prediction horizon feasible!

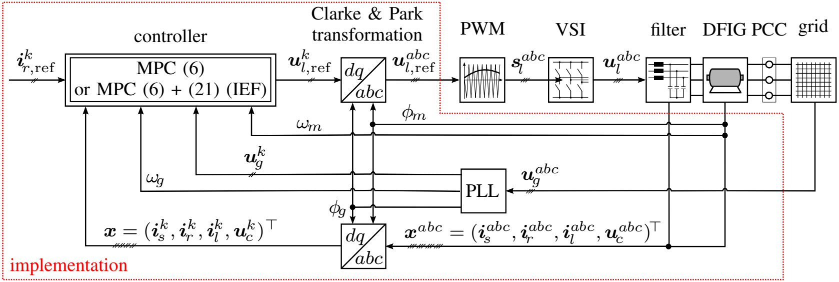
Outline

2 Application

- Rotor current control of doubly-fed induction generators with LC-filter
- Simulation results

Application: Block diagram

Rotor current control of doubly-fed induction generators with LC-filter



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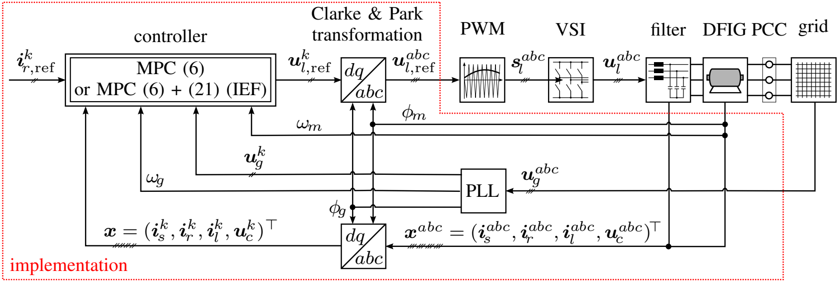

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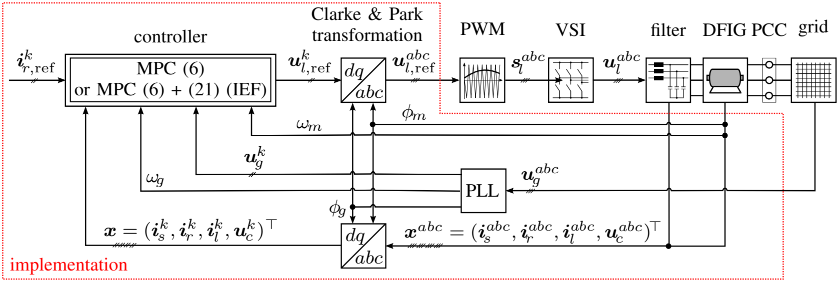

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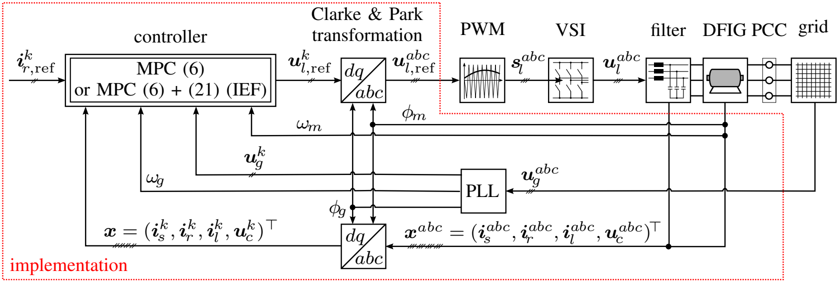

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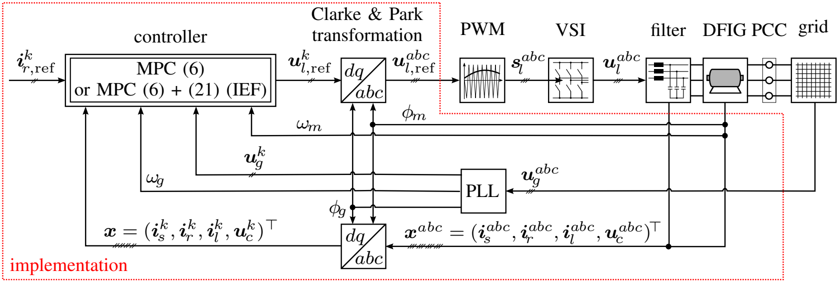

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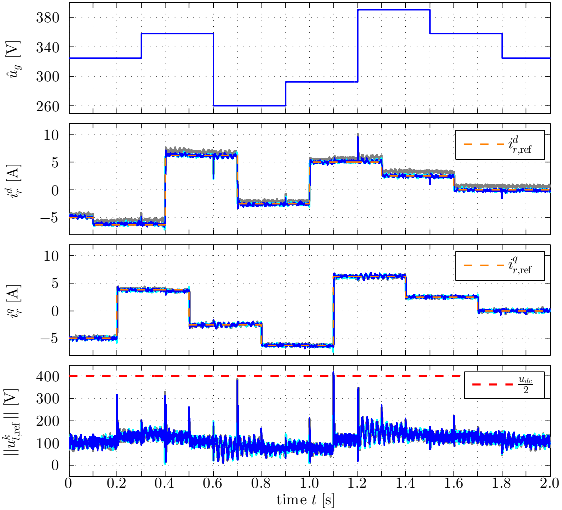
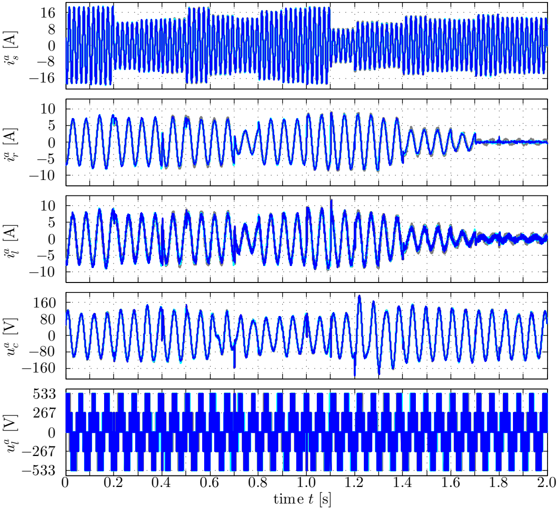
(d,q)-signals: MPC, MPC with IEF & MPC with IEF and $y_{\text{ref}}^{H_p}$ $(n_p = 24)$



Application: Simulation results

Phase (a) signals: MPC, MPC with IEF & MPC with IEF and $\boldsymbol{y}_{\mathrm{ref}}^{H_{\mathrm{p}}}$ $(n_{\mathrm{p}} = 24)$



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- Apply results to nonlinear systems (with online linearization) ?

References

[1] C. M. Hackl, F. Larcher, A. Dötlinger, and R. M. Kennel, "Is multiple-objective model-predictive control 'optimal'?," in *Proceedings of the 2013 IEEE International Symposium on Predictive Control of Electrical Drives and Power Electronics (PRECEDE 2013)*, (Munich, Germany), pp. 1–8, 2013.

[2] B. D. Anderson and J. B. Moore, *Optimal Control: Linear Quadratic Methods*. Precentice Hall Information and System Sciences Series, Englewood Cliffs, New Jersey: Prentice Hall International Inc., 1990.

[3] C. M. Hackl, *Contributions to High-gain Adaptive Control in Mechatronics*. PhD thesis, Lehrstuhl für Elektrische Antriebssysteme und Leistungselektronik, Technische Universität München (TUM), Germany, 2012.

Simulation results

Implementation data

descr.	*symbols & values*
DFIG	$L_\mathrm{s} = 78\,\mathrm{mH}$, $L_\mathrm{r} = 80\,\mathrm{mH}$, $L_\mathrm{m} = 72\,\mathrm{mH}$, $n_\mathrm{pp} = 2$, $R_\mathrm{s} = 0{,}5\,\Omega$, $R_\mathrm{r} = 0{,}41\,\Omega$, $\omega_\mathrm{m} = 100\,\frac{\mathrm{rad}}{\mathrm{s}}$,
grid	$\omega_\mathrm{g} = 2\pi 50\,\frac{\mathrm{rad}}{\mathrm{s}}$, $\hat{u}_\mathrm{g} = \sqrt{2} \cdot 230\,\mathrm{V}$,
filter	$L_\mathrm{l} = 15\,\mathrm{mH}$, $C_\mathrm{c} = 50\,\mu\mathrm{F}$, $R_\mathrm{l} = 0{,}5\,\Omega$, $R_\mathrm{c} = 0{,}25\,\Omega$
VSI	$f_\mathrm{pwm} = 8\,\mathrm{kHz}$, $u_\mathrm{dc} = 800\,\mathrm{V}$
MPC	$\boldsymbol{W}_\mathrm{u} = 0{,}5\,\frac{1}{\mathrm{A}^2} \cdot \boldsymbol{I}_{48}$, $\boldsymbol{W}_\mathrm{e} = 1\,000\,\frac{1}{\mathrm{V}^2} \cdot \boldsymbol{I}_{50}$, $k_i = 30$, $k_c = -0.5$ (IEF), $n_\mathrm{p} = 24$,
sampling time	$T_\mathrm{s} = 1{,}25 \cdot 10^{-4}\,\mathrm{s}$.

Table: *System, implementation and controller data.*