A unified theory for optimal feedforward torque control of anisotropic synchronous machines

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MSE Research group “Control of Renewable Energy Systems (CRES)”

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Outline

1. MSE Group "Control of Renewable Energy Systems (CRES)"

2. A unified theory for optimal feedforward torque control of anisotropic synchronous machines [1–4]

Outline

1 MSE Group “Control of Renewable Energy Systems (CRES)”
- Team
- Projects
- Teaching
- Research

Group “Control of renewable energy systems (CRES)”

Team (7 PhD candidates,1 Post-Doc, 3 PhD collaborations)

H. Eldeeb, M.Sc.
(CRES, 09/2015)

C. Dirscherl, M.Sc.
(CRES, 01/2014)

C. Hackl, Dr.-Ing.
(CRES, 01/2014)

J. Kullick, M.Sc.
(CRES, 10/2015)

K. Schechner, M.Sc.
(CRES, 01/2014)

M. Landerer, M.Sc.
(CRES, 06/2016)

External PhD candidates

Z. Zhang, Dr.-Ing.
(CRES/EAL,07/2015)

S. Krüner, M.Sc.
(SINNPower,11/2015)

A. Birda, M.Sc.
(BMW, 04/2016)

. . . and collaborations with EAL (Prof. Kennel)

F. Bauer, M.Sc.
(EAL)

M. Abdelrahem,
M.Sc. (EAL)

A. Ayad,
M.Sc. (EAL)

Group “Control of renewable energy systems (CRES)”

Research projects (all related to the electrical system)

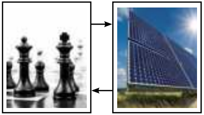
Large-scale WTS

Efficiency+Reliability

Small-scale WTS

Reluctance SM

Model predictive control for RES



Real-time applicability

Airborne Wind Energy

Fault-tolerant control

Geothermal energy

Fault-tolerant control

Wave energy (SinnPower)

Efficiency+Reliability

Electric vehicles (BMW)

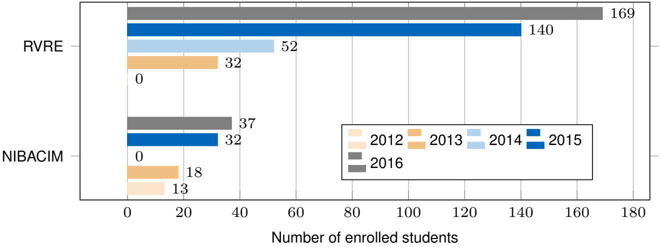
Efficiency+Reliability

Power systems

Four-wire system

Group “Control of renewable energy systems (CRES)”

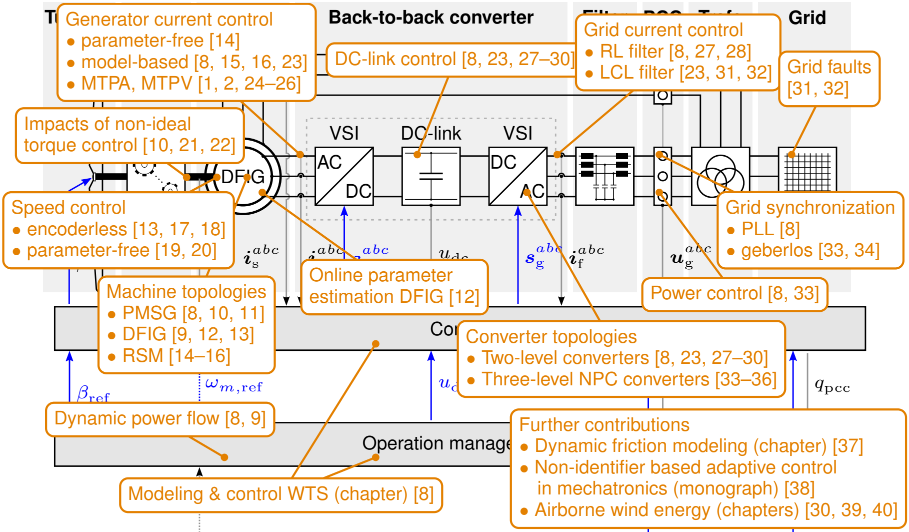
Teaching



- Lectures, tutorials, practical courses & student projects
  - “**R**egelung **v**on **r**egenerativen **E**nergiesystemen” (WS, 3SWS with C. Dirscherl & K. Schechner)
  - “**N**on-**i**dentifier **b**ased **a**daptive **c**ontrol in **m**echatronics” (SS, 4SWS)
  - “Projektstudium Antriebstechnik” [5–7] (EOF Lehrpreis 2013, WS1415): **13**
  - Supervised student projects: more than **50** (all CRES members, since 2014)
- Workshops
  - MSE Winter School “Holistic Modeling and Control of Energy Systems”: **47** (with F. Dörfler & A. Jokić)
  - H2020 ITN Workshop “Electrical Drives and Predictive Control”: **15** (with M. Diehl)

Group “Control of renewable energy systems (CRES)”

Research focus and publications: **Electrical** components of e.g. large-scale wind turbine systems



Outline

2 A unified theory for optimal feedforward torque control of anisotropic synchronous machines [1–4]

Motivation

Synchronous machines: Widely-used, compact and efficient actuators

www.bmw.de

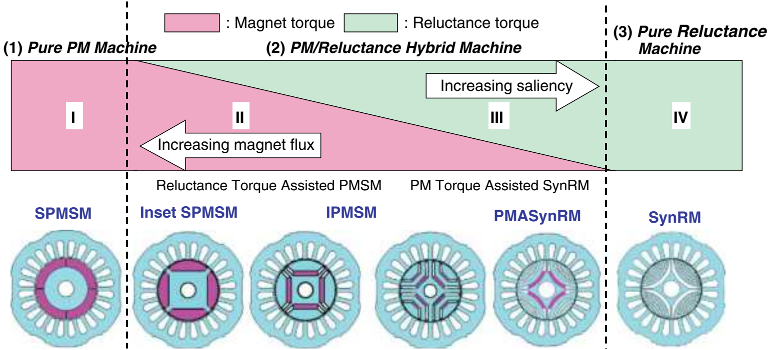
www.farmwood.co.uk

www.abb.com (from video)

- traction drives (e.g. electric vehicles or trains)
- air conditioning/fans
- pumps
- conventional power plants (e.g. hydro power plants)
- renewable energy systems (e.g. direct-drive wind turbine systems)

Motivation

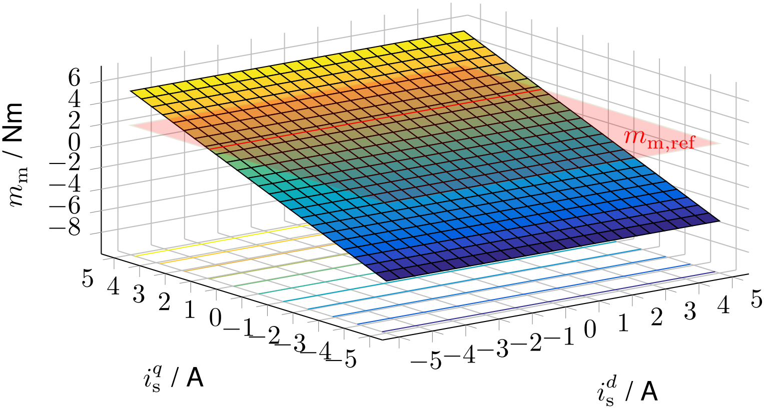
Examples of anisotropic synchronous machines [41] with "saliency ratio" $L_s^d / L_s^q \neq 1$



$$m_m(i_s^d, i_s^q) = \frac{3}{2} n_p \Big[\underbrace{\psi_{pm} i_s^q}_{\text{magnetic torque}} + \underbrace{(L_s^d - L_s^q) i_s^d i_s^q}_{\text{reluctance torque}} + \underbrace{L_m \big((i_s^q)^2 - (i_s^d)^2\big)}_{\text{cross-coupling torque}} \Big]$$

Motivation

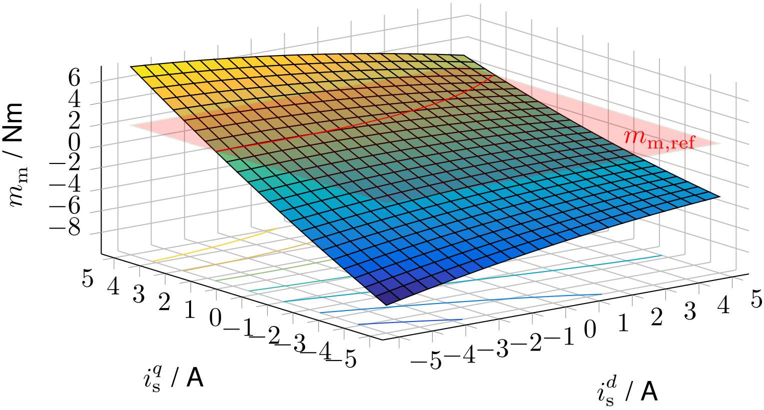
Optimal feedforward torque control problem: **Isotropic machine**



$$m_{\mathrm{m,ref}} \overset{!}{=} m_{\mathrm{m}}\left(i_{\mathrm{s}}^{d},\, i_{\mathrm{s}}^{q}\right) = \frac{3}{2}\, n_{\mathrm{p}}\, \psi_{\mathrm{pm}} i_{\mathrm{s}}^{q} \quad \left(\Longrightarrow\ i_{\mathrm{s,ref}}^{q} = \frac{2\, m_{\mathrm{m,ref}}}{3 n_{\mathrm{p}} \psi_{\mathrm{pm}}}\right)$$

Motivation

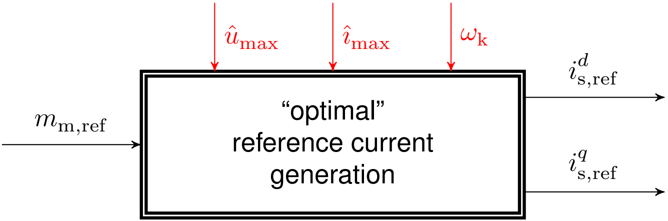
Optimal feedforward torque control problem: **Anisotropic machine**



$$m_{\mathrm{m,ref}} \stackrel{!}{=} m_{\mathrm{m}}\left(i_{\mathrm{s}}^{d},\, i_{\mathrm{s}}^{q}\right) = \tfrac{3}{2}\, n_{\mathrm{p}} \left[\psi_{\mathrm{pm}} i_{\mathrm{s}}^{q} + \left(L_{\mathrm{s}}^{d} - L_{\mathrm{s}}^{q}\right) i_{\mathrm{s}}^{d} i_{\mathrm{s}}^{q} + L_{\mathrm{m}}\left(\left(i_{\mathrm{s}}^{q}\right)^2 - \left(i_{\mathrm{s}}^{d}\right)^2\right)\right]$$

Motivation

Optimal feedforward torque control problem: **Goal**



$$\boldsymbol{i}^k_{s,ref}(m_{m,ref}, \hat{u}_{\max}, \hat{\imath}_{\max}, \omega_k) = \begin{pmatrix} i^d_{s,ref}(m_{m,ref}, \hat{u}_{\max}, \hat{\imath}_{\max}, \omega_k) \\ i^q_{s,ref}(m_{m,ref}, \hat{u}_{\max}, \hat{\imath}_{\max}, \omega_k) \end{pmatrix}$$

- ✓ Numerical solutions and/or look-up tables (but: limited storage, accuracy, real-time applicability)
- **?** Analytical solutions (some do exist but impose simplifying assumptions such as ~~$R_s = 0$ and/or $L_m = 0$~~)

Problem statement and proposed solution

Considered machines: Anisotropic synchronous machines (steady-state model)

$$\underbrace{\begin{pmatrix} u_s^d(\boldsymbol{i}_s^k,\omega_k) \\ u_s^q(\boldsymbol{i}_s^k,\omega_k) \end{pmatrix}}_{=:\boldsymbol{u}_s^k(\boldsymbol{i}_s^k,\omega_k)} = R_s \underbrace{\begin{pmatrix} i_s^d \\ i_s^q \end{pmatrix}}_{=:\boldsymbol{i}_s^k} + \omega_k \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{=:\boldsymbol{J}} \underbrace{\begin{pmatrix} \psi_s^d(\boldsymbol{i}_s^k) \\ \psi_s^q(\boldsymbol{i}_s^k) \end{pmatrix}}_{=:\boldsymbol{\psi}_s^k(\boldsymbol{i}_s^k)} + \cancel{\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_s^k(\boldsymbol{i}_s^k)}$$

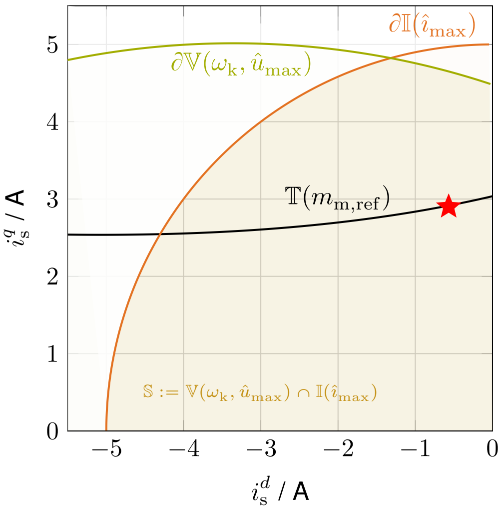
$$\begin{aligned} m_m(\boldsymbol{i}_s^k) &= \tfrac{3}{2} n_p (\boldsymbol{i}_s^k)^\top \boldsymbol{J} \boldsymbol{\psi}_s^k(\boldsymbol{i}_s^k) \\ &= \tfrac{3}{2} n_p \left[\psi_{pm} i_s^q + (L_s^d - L_s^q) i_s^d i_s^q + L_m\left((i_s^q)^2 - (i_s^d)^2\right)\right] \end{aligned}$$

Assumption (Affine flux linkage (at least locally))

$$\boldsymbol{\psi}_s^k(\boldsymbol{i}_s^k) = \underbrace{\begin{bmatrix} L_s^d & L_m \\ L_m & L_s^q \end{bmatrix}}_{=:\boldsymbol{L}_s^k \in \mathbb{R}^{2\times 2}} \boldsymbol{i}_s^k + \underbrace{\begin{pmatrix} \psi_{pm} \\ 0 \end{pmatrix}}_{=:\boldsymbol{\psi}_{pm}^k} \text{ with } \boldsymbol{L}_s^k = (\boldsymbol{L}_s^k)^\top > 0$$

Problem statement and proposed solution

Feedforward torque control: Optimization problem(s) with multiple constraints



$\max_{\boldsymbol{i}_{\mathrm{s}}^{k} \in \mathbb{S}} \; -f(\boldsymbol{i}_{\mathrm{s}}^{k})$ subject to

$$\left\{ \begin{array}{l} \|\boldsymbol{i}_{\mathrm{s}}^{k}\|^2 \leqslant \hat{\imath}_{\max}^2, \\ \text{(current circular area)} \\ \|\boldsymbol{u}_{\mathrm{s}}^{k}(\boldsymbol{i}_{\mathrm{s}}^{k}, \omega_{\mathrm{k}})\|^2 \leqslant \hat{u}_{\max}^2, \\ \text{(voltage elliptical area)} \\ |m_{\mathrm{m}}(\boldsymbol{i}_{\mathrm{s}}^{k})| \leqslant |m_{\mathrm{m,ref}}|, \text{ and} \\ \mathrm{sign}(m_{\mathrm{m,ref}}) = \\ \mathrm{sign}(m_{\mathrm{m}}(\boldsymbol{i}_{\mathrm{s}}^{k})). \end{array} \right.$$

e.g. $-f(\boldsymbol{i}_{\mathrm{s}}^{k}) = -\|\boldsymbol{i}_{\mathrm{s}}^{k}\|^2$ (minimize copper losses) $\Longrightarrow \boldsymbol{i}_{\mathrm{s,ref}}^{k}$ at ★

Problem statement and proposed solution

Proposed solution: Implicit formulation (quadrics), optimization and intersection points

Step 1: Derivation of quadrics $Q_A(\boldsymbol{i}_s^k) := (\boldsymbol{i}_s^k)^\top \boldsymbol{A}\, \boldsymbol{i}_s^k + 2\boldsymbol{a}^\top \boldsymbol{i}_s^k + \alpha$:

- Current circular area: $(i_s^d)^2 + (i_s^q)^2 \leqslant \hat{\imath}_{\max}^2 \iff \|\boldsymbol{i}_s^k\|^2 = (\boldsymbol{i}_s^k)^\top \boldsymbol{I}_2 \boldsymbol{i}_s^k \leqslant \hat{\imath}_{\max}^2$

$$\implies \quad \mathbb{I}(\hat{\imath}_{\max}) := \left\{\, \boldsymbol{i}_s^k \in \mathbb{R}^2 \;\middle|\; (\boldsymbol{i}_s^k)^\top \boldsymbol{I}_2 \boldsymbol{i}_s^k - \hat{\imath}_{\max}^2 \leqslant 0 \,\right\}$$

- Voltage elliptical area: $(u_s^d)^2 + (u_s^q)^2 \leqslant \hat{u}_{\max}^2 \iff \|\boldsymbol{u}_s^k(\boldsymbol{i}_s^k, \omega_k)\|^2 = \ldots \leqslant \hat{u}_{\max}^2$

$$\implies \quad \mathbb{V}(\omega_k, \hat{u}_{\max}) := \left\{\, \boldsymbol{i}_s^k \in \mathbb{R}^2 \;\middle|\; (\boldsymbol{i}_s^k)^\top \boldsymbol{V}(\omega_k)\, \boldsymbol{i}_s^k + 2\,\boldsymbol{v}(\omega_k)^\top \boldsymbol{i}_s^k + \nu(\omega_k, \hat{u}_{\max}) \leqslant 0 \right\}$$

- Reference torque hyperbola: $m_m(\boldsymbol{i}_s^k) = \frac{3}{2} n_p (\boldsymbol{i}_s^k)^\top \boldsymbol{J} \boldsymbol{\psi}_s^k(\boldsymbol{i}_s^k) \overset{!}{=} m_{m,\mathrm{ref}} \iff \ldots$

$$\implies \quad \mathbb{T}(m_{m,\mathrm{ref}}) := \left\{\, \boldsymbol{i}_s^k \in \mathbb{R}^2 \;\middle|\; (\boldsymbol{i}_s^k)^\top \boldsymbol{T} \boldsymbol{i}_s^k + 2\,\boldsymbol{t}^\top \boldsymbol{i}_s^k - m_{m,\mathrm{ref}} = 0 \,\right\}$$

- ...

Problem statement and proposed solution

Proposed solution: Implicit formulation (quadrics), optimization and intersection points (cont'd)

Step 2: Optimization problem with equality constraint

$$\max_{\boldsymbol{i}_s^k} -\Big(\underbrace{(\boldsymbol{i}_s^k)^\top \boldsymbol{A} \boldsymbol{i}_s^k + 2\boldsymbol{a}^\top \boldsymbol{i}_s^k + \alpha}_{=:Q_A(\boldsymbol{i}_s^k)} \Big) \text{ s.t. } \underbrace{(\boldsymbol{i}_s^k)^\top \boldsymbol{B} \boldsymbol{i}_s^k + 2\boldsymbol{b}^\top \boldsymbol{i}_s^k + \beta}_{=:Q_B(\boldsymbol{i}_s^k)} = 0$$

$\Longrightarrow$ Hyperbolas for MTPC, MTPV and MTPF (with $R_s \neq 0$ & $L_m \neq 0$!)

Step 3: Intersection of two quadrics (e.g. voltage ellipse and current circle)

$$\boldsymbol{i}_{s,\mathrm{ref}}^k := \arg\min_{\|\boldsymbol{i}_s^k\|} \left\{ \boldsymbol{i}_s^k \in \mathbb{R}^2 \,\middle|\, Q_A(\boldsymbol{i}_s^k) = 0 \ \wedge \ Q_B(\boldsymbol{i}_s^k) = 0 \right\}$$

$\Longrightarrow$ Optimal operation point ★ (reference current)

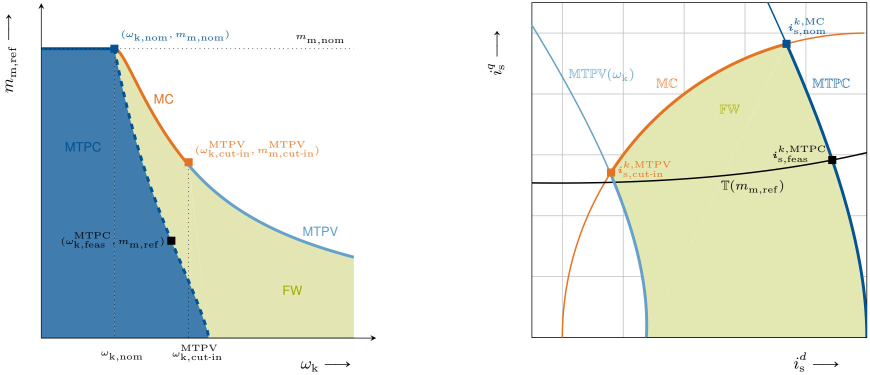
Both lead to subproblem of solving a fourth-order (quartic) polynomial

$$\chi(\lambda) := c_4\,\lambda^4 + c_3\,\lambda^3 + c_2\,\lambda^2 + c_1\,\lambda + c_0 \overset{!}{=} 0$$

$\Longrightarrow$ Analytical solutions exist (e.g. Euler's solution, see [42])

Operation management and operation strategies

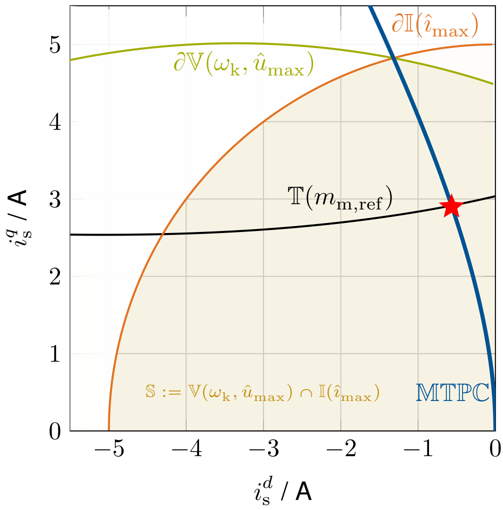
Operation management: Operation strategies and transition points



- Maximum Torque per Current (MTPC): $\omega_{\mathrm{k}} \leqslant \omega_{\mathrm{k,feas}}^{\mathrm{MTPC}}(m_{\mathrm{m,ref}})$ (and $m_{\mathrm{m,ref}} \leqslant m_{\mathrm{m,nom}}$)
- Field Weakening (FW): $\omega_{\mathrm{k,feas}}^{\mathrm{MTPC}}(m_{\mathrm{m,ref}}) < \omega_{\mathrm{k}}$ and $m_{\mathrm{m,ref}} < m_{\mathrm{m,feas}}^{\mathrm{FW}}(\omega_{\mathrm{k}})$
- Maximum Current (MC): $\omega_{\mathrm{k,nom}} < \omega_{\mathrm{k}} < \omega_{\mathrm{k,cut\text{-}in}}^{\mathrm{MTPV}}$ and $m_{\mathrm{m,ref}} \geqslant m_{\mathrm{m,feas}}^{\mathrm{FW}}(\omega_{\mathrm{k}})$
- Maximum Torque per Voltage (MTPV): $\omega_{\mathrm{k}} > \omega_{\mathrm{k,cut\text{-}in}}^{\mathrm{MTPV}}$ and $m_{\mathrm{m,ref}} \geqslant m_{\mathrm{m,cut\text{-}in}}^{\mathrm{MTPV}}(\omega_{\mathrm{k}})$

Operation management and operation strategies

Maximum Torque per Current (**MTPC**): Reference torque feasible



Optimization problem

$$\max_{\boldsymbol{i}_{\mathrm{s}}^{k} \in \mathbb{S}} -\|\boldsymbol{i}_{\mathrm{s}}^{k}\|^2 \quad \text{s.t.}$$

$$\underbrace{(\boldsymbol{i}_{\mathrm{s}}^{k})^\top \boldsymbol{T} \boldsymbol{i}_{\mathrm{s}}^{k} + 2\,\boldsymbol{t}^\top \boldsymbol{i}_{\mathrm{s}}^{k}}_{=m_{\mathrm{m}}(\boldsymbol{i}_{\mathrm{s}}^{k})} - m_{\mathrm{m,ref}} = 0$$

Solution set

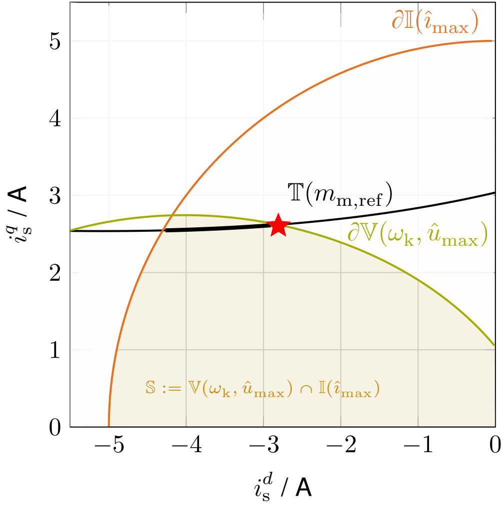
$$\mathbb{MTPC} := \Big\{ \boldsymbol{i}_{\mathrm{s}}^{k} \in \mathbb{R}^2 \;\Big|\; (\boldsymbol{i}_{\mathrm{s}}^{k})^\top \boldsymbol{M}_{\mathrm{C}}\, \boldsymbol{i}_{\mathrm{s}}^{k} + 2\,\boldsymbol{m}_{\mathrm{C}}^\top \boldsymbol{i}_{\mathrm{s}}^{k} = 0 \Big\}$$

Optimal current reference (★)

$$\boldsymbol{i}_{\mathrm{s,ref}}^{k} = \mathbb{MTPC} \cap \mathbb{T}(m_{\mathrm{m,ref}})$$

Operation management and operation strategies

Field Weakening (**FW**): Reference torque feasible



Optimization problem

$$\max_{\boldsymbol{i}_{\mathrm{s}}^k \in \mathbb{S}} -\|\boldsymbol{i}_{\mathrm{s}}^k\|^2 \quad \text{s.t.}$$

$$\underbrace{(\boldsymbol{i}_{\mathrm{s}}^k)^\top \boldsymbol{T} \boldsymbol{i}_{\mathrm{s}}^k + 2\,\boldsymbol{t}^\top \boldsymbol{i}_{\mathrm{s}}^k}_{=m_{\mathrm{m}}(\boldsymbol{i}_{\mathrm{s}}^k)} - m_{\mathrm{m,ref}} = 0$$

Feasible set

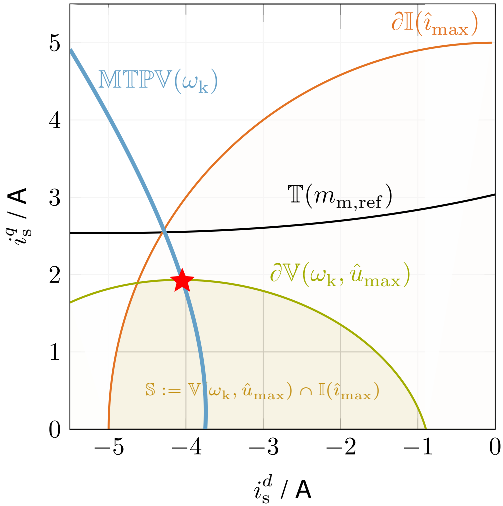
$$\mathbb{FW}(m_{\mathrm{m,ref}}, \omega_{\mathrm{k}}, \hat{u}_{\max}) := \mathbb{S}(\omega_{\mathrm{k}}, \hat{u}_{\max}, \hat{\imath}_{\max}) \cap \mathbb{T}(m_{\mathrm{m,ref}})$$

Optimal current reference (★)

$$\boldsymbol{i}_{\mathrm{s,ref}}^k = \partial\mathbb{V}(\omega_{\mathrm{k}}, \hat{u}_{\max}) \cap \mathbb{T}(m_{\mathrm{m,ref}})$$

Operation management and operation strategies

Maximum Torque per Voltage (**MTPV**): Reference torque **not** feasible



Optimization problem

$$\max_{\boldsymbol{i}_s^k \in \mathbb{S}} -\|\boldsymbol{u}_s^k(\boldsymbol{i}_s^k)\|^2 \quad \text{s.t.}$$

$$\underbrace{(\boldsymbol{i}_s^k)^\top \boldsymbol{T} \boldsymbol{i}_s^k + 2\,\boldsymbol{t}^\top \boldsymbol{i}_s^k}_{=m_m(\boldsymbol{i}_s^k)} - m_{m,\mathrm{ref}} = 0$$

Solution set

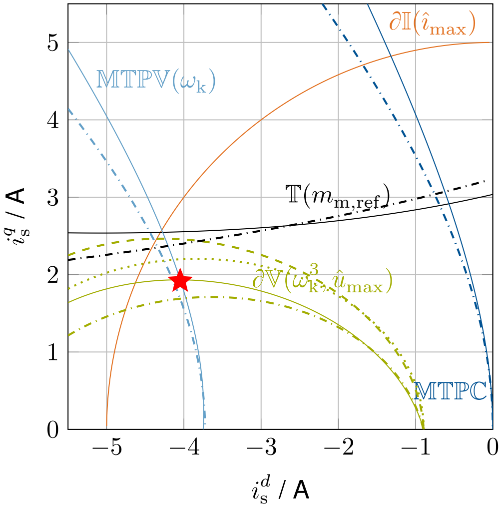
$$\mathbb{MTPV}(\omega_k) := \{\, \boldsymbol{i}_s^k \in \mathbb{R}^2 \mid (\boldsymbol{i}_s^k)^\top \boldsymbol{M}_V\, \boldsymbol{i}_s^k + 2\,\boldsymbol{m}_V^\top \boldsymbol{i}_s^k + \mu_V = 0 \}$$

Optimal current reference (★)

$$\boldsymbol{i}_{s,\mathrm{ref}}^k = \mathbb{MTPV}(\omega_k) \cap \partial\mathbb{V}(\omega_k, \hat{u}_{\max})$$

Operation management and operation strategies

Illustration of the effect of neglecting R_s and/or L_m



$R_\mathrm{s} = 0$ (and $L_\mathrm{m} \neq 0$)

$L_\mathrm{m} = 0$ (and $R_\mathrm{s} \neq 0$)

$R_\mathrm{s} = 0$ and $L_\mathrm{m} = 0$

$\Longrightarrow$ **Reduced efficiency**! (and wrong torque and/or possibly wrong operation strategies)

Analytical solution

Example: Analytical solution for intersection points of two quadrics

Problem

$$\mathbb{X} := \left\{ \boldsymbol{x} \in \mathbb{R}^2 \;\middle|\; \underbrace{\boldsymbol{x}^\top \boldsymbol{A}\boldsymbol{x} + 2\boldsymbol{a}^\top \boldsymbol{x} + \alpha}_{=:Q_A(\boldsymbol{x})} = 0 \;\wedge\; \underbrace{\boldsymbol{x}^\top \boldsymbol{B}\boldsymbol{x} + 2\boldsymbol{b}^\top \boldsymbol{x} + \beta}_{=:Q_B(\boldsymbol{x})} = 0 \right\}$$

Solution: Relate both (e.g., if $\alpha \neq 0$ and $\beta \neq 0$)

$$Q_D(\boldsymbol{x}) := \frac{Q_A(\boldsymbol{x})}{\alpha} - \frac{Q_B(\boldsymbol{x})}{\beta} = \boldsymbol{x}^\top \underbrace{\left(\frac{\boldsymbol{A}}{\alpha} - \frac{\boldsymbol{B}}{\beta}\right)}_{=:\boldsymbol{D}} \boldsymbol{x} + 2 \underbrace{\left(\frac{\boldsymbol{a}}{\alpha} - \frac{\boldsymbol{b}}{\beta}\right)^\top}_{=:\boldsymbol{d}^\top} \boldsymbol{x} + \underbrace{\left(\frac{\alpha}{\alpha} - \frac{\beta}{\beta}\right)}_{=0} = 0$$

and note that

$$Q_D(\boldsymbol{x}) := \boldsymbol{x}^\top \big(\underbrace{\boldsymbol{D}\boldsymbol{x} + 2\boldsymbol{d}}_{\overset{!}{=}\lambda \boldsymbol{J}\boldsymbol{x}} \big) \overset{!}{=} 0 \quad \Longleftrightarrow \quad \boldsymbol{x}^\star(\lambda) = -2\big(\boldsymbol{D} - \lambda \boldsymbol{J}\big)^{-1} \boldsymbol{d}$$

Analytical solution

Example: Analytical solution for intersection points of two quadrics (cont'd)

then insert $\boldsymbol{x}^\star(\lambda)$ in $Q_A(\boldsymbol{x})$ (or $Q_B(\boldsymbol{x})$), i.e.

$$Q_A(\boldsymbol{x}^\star(\lambda)) =$$

$$4\boldsymbol{d}^\top\big(\boldsymbol{D}-\lambda\boldsymbol{J}\big)^{-\top}\boldsymbol{A}\big(\boldsymbol{D}-\lambda\boldsymbol{J}\big)^{-1}\boldsymbol{d} - 4\boldsymbol{a}^\top\big(\boldsymbol{D}-\lambda\boldsymbol{J}\big)^{-1}\boldsymbol{d} + \alpha = 0 \Big| \cdot \big[\underbrace{\det\big(\boldsymbol{D}-\lambda\boldsymbol{J}\big)}_{\propto\lambda^2}\big]^2$$

$$\Longrightarrow \boxed{\chi(\lambda) := c_4\,\lambda^4 + c_3\,\lambda^3 + c_2\,\lambda^2 + c_1\,\lambda + c_0 \overset{!}{=} 0}$$

$\Longrightarrow$ Analytical (real) solutions $\lambda^\star \in \{\lambda_1, \ldots, \lambda_4\}$ can be computed [42]:

$$\Longrightarrow \boxed{\boldsymbol{x}^\star(\lambda^\star) = -2\big(\boldsymbol{D} - \lambda^\star\boldsymbol{J}\big)^{-1}\,\boldsymbol{d}}$$

Implementation results

Laboratory setup

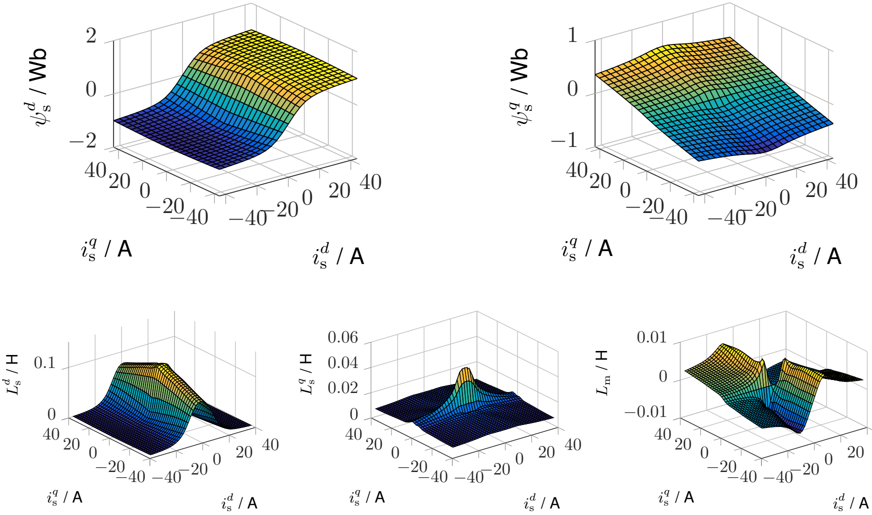
Reluctance SM (RSM) and Permanent-magnet SM

Host PC (rapid prototyping)

Real-time system and VSIs

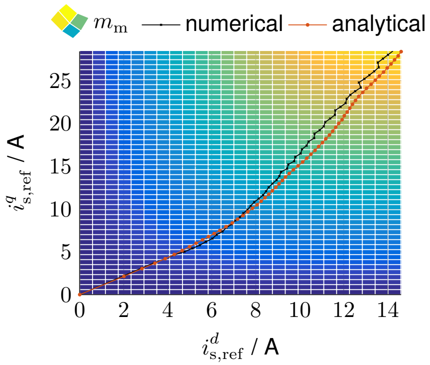
Implementation results

Nonlinear flux linkages and inductances of RSM

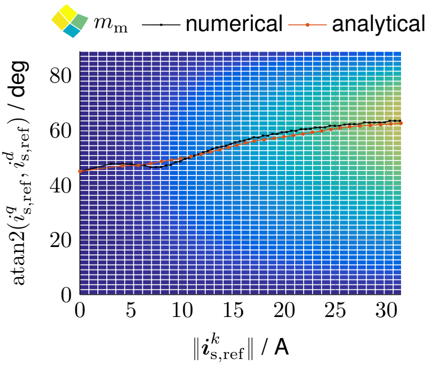


Implementation results

Comparison of numerical and analytical solutions for MTPC



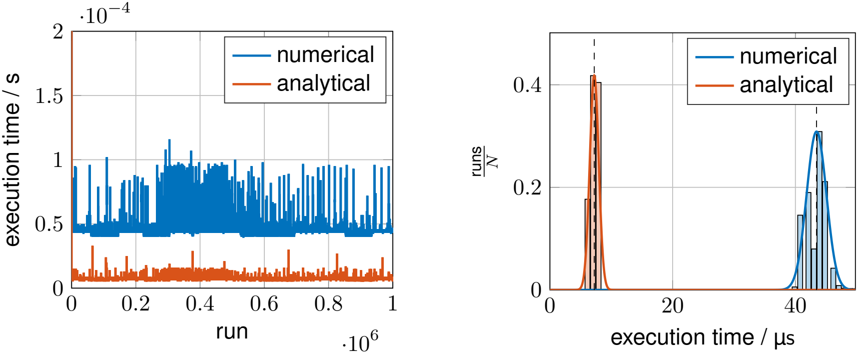
Cartesian coordinates



polar coordinates

Implementation results

Comparison of the computational load (Euler's solution for fourth-order polynomials)



- Execution times for $N = 10^6$ runs (downsampled by factor 100);
- Average execution times $\mu_n = 43{,}4 \cdot 10^{-6}$ s and $\mu_a = 7{,}23 \cdot 10^{-6}$ s; and
- Standard deviations $\sigma_n = 1{,}61 \cdot 10^{-6}$ s and $\sigma_a = 0{,}73 \cdot 10^{-6}$ s.

Conclusion

Summary and future work

To take home

- **unified** theory for MTPC, FW, MTPV/MTPF or MC based on
  - **quadrics** (implicit formulation),
  - **optimization problems** with **equality constraint**, and
  - **intersection** of two quadrics
- **analytical** solution for reference currents (& transition points)
- $R_\mathrm{s} \neq 0$ and $L_\mathrm{m} \neq 0$ (both can be considered **simultaneously**)
- **fast(er)** and **more accurate** computation
- **applicable** in real-world (nonlinear RSM)

Future work

- convergence (mathematical proof or consideration of nonlinear flux linkages)
- overall implementation, validation and test
- consideration of iron losses (including e.g. $R_\mathrm{fe}(\omega_\mathrm{k})$)
- impact of parameter uncertainty (e.g. $R_\mathrm{s}^a \neq R_\mathrm{s}^b \neq R_\mathrm{s}^c$)

References I

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