Current PI-funnel control with anti-windup for reluctance synchronous machines

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Outline

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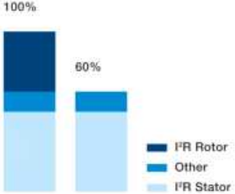
Motivation and problem formulation

Reluctance synchronous machine (RSM): A viable alternative?

Traditional IE2 induction motor

IE4 SynRM motor



Losses

`http://www.abb.com`

advantages [1–3]:

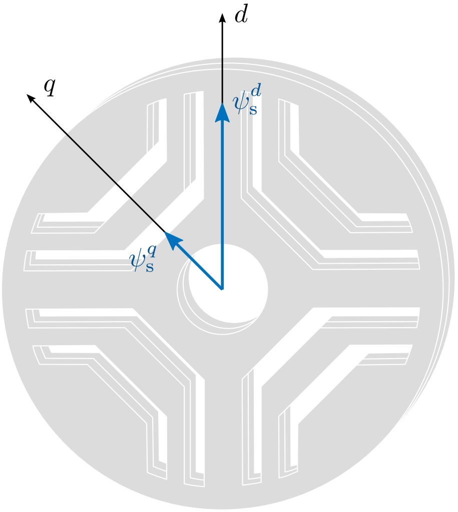
- (very) cheap
- high efficiency feasible (IE4)
- high power density and torque-per-volume ratio
- sensorless control feasible

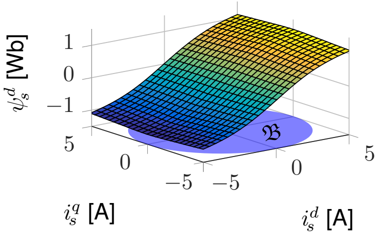
disadvantages:

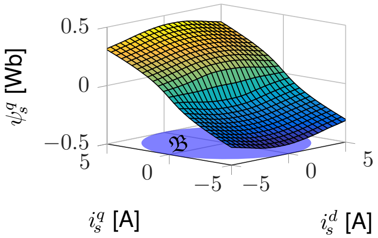
- power factor
- highly nonlinear
- inverter necessary
- (nonlinear control and good system knowledge necessary)

Motivation and problem formulation

Flux linkage in the RSM





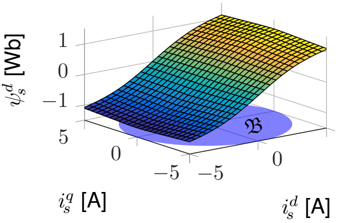


Motivation and problem formulation

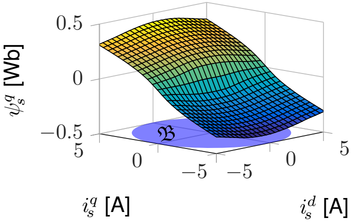
Control objective: Current reference tracking with prescribed transient accuracy

$$\boldsymbol{u}_{\mathrm{s}}^{k}(t) = R_{\mathrm{s}}\boldsymbol{i}_{\mathrm{s}}^{k}(t) + \tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_{\mathrm{s}}^{k}\big(\boldsymbol{i}_{\mathrm{s}}^{k}(t)\big) + \omega_k(t)\underbrace{\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}}_{=:\boldsymbol{J}}\boldsymbol{\psi}_{\mathrm{s}}^{k}\big(\boldsymbol{i}_{\mathrm{s}}^{k}(t)\big), \qquad (1)$$

$$\tfrac{\mathrm{d}}{\mathrm{d}t}\omega_k(t) = \frac{p}{\Theta}\Big[\underbrace{\frac{3}{2}p\,\boldsymbol{i}_{\mathrm{s}}^{k}(t)^\top\boldsymbol{J}\boldsymbol{\psi}_{\mathrm{s}}^{k}\big(\boldsymbol{i}_{\mathrm{s}}^{k}(t)\big)}_{\text{machine torque}} - \underbrace{m_l(t)}_{\text{loads}} - \underbrace{\big(\nu\omega_k(t) + (\mathfrak{F}\omega_k)(t)\big)}_{\text{friction}}\Big], \qquad (2)$$



(a) d-flux linkage



(b) q-flux linkage

→ Current tracking control without any system knowledge feasible?

Outline

2 Structural system properties of the RSM

Structural system properties of the RSM

Derivation of the nonlinear current dynamics

- To be derived: Nonlinear current dynamics

$$\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_{\mathrm{s}}^{k}(t) = \ldots \tag{3}$$

- Given: Stator voltage equation

$$\boldsymbol{u}_{\mathrm{s}}^{k}(t) = R_{\mathrm{s}}\boldsymbol{i}_{\mathrm{s}}^{k}(t) + \tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_{\mathrm{s}}^{k}\big(\boldsymbol{i}_{\mathrm{s}}^{k}(t)\big) + \omega_k(t)\boldsymbol{J}\boldsymbol{\psi}_{\mathrm{s}}^{k}\big(\boldsymbol{i}_{\mathrm{s}}^{k}(t)\big), \qquad \boldsymbol{\psi}_{\mathrm{s}}^{k}\big(\boldsymbol{i}_{\mathrm{s}}^{k}(0)\big) = \boldsymbol{\psi}_{s,0}^{k}$$

Assumption 1

Flux linkage $\boldsymbol{\psi}_{\mathrm{s}}^{k}\ [\mathrm{Wb}]^2$ is a continuously differentiable function of the stator currents $\boldsymbol{i}_{\mathrm{s}}^{k}\ [\mathrm{A}]^2$, i.e.

$$\boldsymbol{\psi}_{\mathrm{s}}^{k}: \quad \mathbb{R}^2 \to \mathbb{R}^2, \quad \boldsymbol{i}_{\mathrm{s}}^{k} \mapsto \boldsymbol{\psi}_{\mathrm{s}}^{k}(\boldsymbol{i}_{\mathrm{s}}^{k}) \quad \wedge \quad \boldsymbol{\psi}_{\mathrm{s}}^{k}(\cdot) \in \mathcal{C}^1(\mathbb{R}^2;\mathbb{R}^2) \tag{4}$$

Structural system properties of the RSM

Inductance matrix

Definition 1

Inductance matrix $\boldsymbol{L}_{\mathrm{s}}^{k}\,[\mathrm{H}]^{2\times 2}$ is defined by

$$\boldsymbol{L}_{\mathrm{s}}^{k}(\boldsymbol{i}_{\mathrm{s}}^{k}) := \begin{bmatrix} L_{\mathrm{s}}^{dd}(\boldsymbol{i}_{\mathrm{s}}^{k}) & L_{\mathrm{s}}^{dq}(\boldsymbol{i}_{\mathrm{s}}^{k}) \\ L_{\mathrm{s}}^{qd}(\boldsymbol{i}_{\mathrm{s}}^{k}) & L_{\mathrm{s}}^{qq}(\boldsymbol{i}_{\mathrm{s}}^{k}) \end{bmatrix} := \left.\begin{bmatrix} \frac{\partial \psi_{\mathrm{s}}^{d}(\boldsymbol{i}_{\mathrm{s}}^{k})}{\partial i_{\mathrm{s}}^{d}} & \frac{\partial \psi_{\mathrm{s}}^{d}(\boldsymbol{i}_{\mathrm{s}}^{k})}{\partial i_{\mathrm{s}}^{q}} \\ \frac{\partial \psi_{\mathrm{s}}^{q}(\boldsymbol{i}_{\mathrm{s}}^{k})}{\partial i_{\mathrm{s}}^{d}} & \frac{\partial \psi_{\mathrm{s}}^{q}(\boldsymbol{i}_{\mathrm{s}}^{k})}{\partial i_{\mathrm{s}}^{q}} \end{bmatrix}\right|_{\boldsymbol{i}_{\mathrm{s}}^{k}} = \left.\frac{\partial \boldsymbol{\psi}_{\mathrm{s}}^{k}(\boldsymbol{i}_{\mathrm{s}}^{k})}{\partial \boldsymbol{i}_{\mathrm{s}}^{k}}\right|_{\boldsymbol{i}_{\mathrm{s}}^{k}} \tag{5}$$

- symmetric [4]:

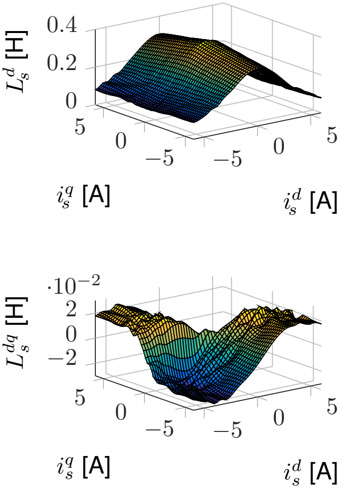
$$\forall \boldsymbol{i}_{\mathrm{s}}^{k} \in \mathbb{R}^{2}:\; \boldsymbol{L}_{\mathrm{s}}^{k}(\boldsymbol{i}_{\mathrm{s}}^{k}) = \boldsymbol{L}_{\mathrm{s}}^{k}(\boldsymbol{i}_{\mathrm{s}}^{k})^{\top} \;\Leftrightarrow\; L_{\mathrm{s}}^{dq}(\boldsymbol{i}_{\mathrm{s}}^{k}) = L_{\mathrm{s}}^{qd}(\boldsymbol{i}_{\mathrm{s}}^{k}) \tag{6}$$

- positive definite [4]:

$$\forall \boldsymbol{i}_{\mathrm{s}}^{k} \in \mathbb{R}^{2}:\; \boldsymbol{L}_{\mathrm{s}}^{k}(\boldsymbol{i}_{\mathrm{s}}^{k}) > 0 \;\Leftrightarrow\; L_{\mathrm{s}}^{dd}(\boldsymbol{i}_{\mathrm{s}}^{k}) > 0,\; L_{\mathrm{s}}^{qq}(\boldsymbol{i}_{\mathrm{s}}^{k}) > 0,\; \det(\boldsymbol{L}_{\mathrm{s}}^{k}(\boldsymbol{i}_{\mathrm{s}}^{k})) > 0 \tag{7}$$

Structural system properties of the RSM

Nonlinear inductances $L_{\mathrm{s}}^{d} := \frac{\partial \psi_{\mathrm{s}}^{d}}{\partial i_{\mathrm{s}}^{d}}$, $L_{\mathrm{s}}^{q} := \frac{\partial \psi_{\mathrm{s}}^{q}}{\partial i_{\mathrm{s}}^{q}}$, $L_{\mathrm{s}}^{dq} := \frac{\partial \psi_{\mathrm{s}}^{d}}{\partial i_{\mathrm{s}}^{q}} = \frac{\partial \psi_{\mathrm{s}}^{q}}{\partial i_{\mathrm{s}}^{d}} =: L_{\mathrm{s}}^{qd}$



Structural system properties of the RSM

Generic model with nonlinear current dynamics

Time derivatives of the flux linkage are given by:

$$\tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_{\mathrm{s}}^{k}(\boldsymbol{i}_{\mathrm{s}}^{k}(t)) = \frac{\partial \boldsymbol{\psi}_{\mathrm{s}}^{k}(\boldsymbol{i}_{\mathrm{s}}^{k}(t))}{\partial \boldsymbol{i}_{\mathrm{s}}^{k}(t)} \tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_{\mathrm{s}}^{k}(t) \overset{(5)}{=} \boldsymbol{L}_{\mathrm{s}}^{k}(\boldsymbol{i}_{\mathrm{s}}^{k}(t)) \tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_{\mathrm{s}}^{k}(t) \tag{8}$$

Inserting (8) into (1) yields **nonlinear current dynamics**:

$$\boxed{\begin{aligned} \tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_{\mathrm{s}}^{k}(t) &= \boldsymbol{L}_{\mathrm{s}}^{k}\big(\boldsymbol{i}_{\mathrm{s}}^{k}(t)\big)^{-1}\Big[\boldsymbol{u}_{\mathrm{s}}^{k}(t) - R_{\mathrm{s}}\boldsymbol{i}_{\mathrm{s}}^{k}(t) - \omega_k(t)\boldsymbol{J}\boldsymbol{\psi}_{\mathrm{s}}^{k}\big(\boldsymbol{i}_{\mathrm{s}}^{k}(t)\big)\Big] \\ \tfrac{\mathrm{d}}{\mathrm{d}t}\omega_k(t) &= \tfrac{p}{\Theta}\Big[\underbrace{\tfrac{3}{2}p\,\boldsymbol{i}_{\mathrm{s}}^{k}(t)^{\top}\boldsymbol{J}\boldsymbol{\psi}_{\mathrm{s}}^{k}\big(\boldsymbol{i}_{\mathrm{s}}^{k}(t)\big)}_{\text{machine torque}} - \underbrace{m_l(t)}_{\text{loads}} - \underbrace{\big(\nu\omega_k(t) + (\mathfrak{F}\omega_k)(t)\big)}_{\text{friction}}\Big] \end{aligned}} \tag{9}$$

with system properties (see Remark IV.1):

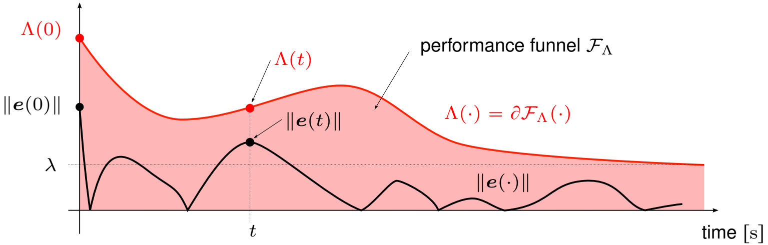
- 2-input 2-output system with vector relative degree **one**!
- BIBO stable internal dynamics, i.e. $\boldsymbol{i}_{\mathrm{s}}^{k}(\cdot) \in \mathcal{L}^{\infty} \wedge m_{\mathrm{l}}(\cdot) \in \mathcal{L}^{\infty} \Rightarrow \omega_k(\cdot) \in \mathcal{L}^{\infty}$
- input saturation, i.e. $\|\boldsymbol{u}_{\mathrm{s}}^{k}(t)\| \leqslant \hat{u} := \frac{u_{\mathrm{dc}}}{2}$ for all $t \geqslant 0$

Outline

3 Current PI-funnel control with anti-windup for RSMs

Current PI-funnel control with anti-windup for RSMs

Funnel control: Tracking with prescribed transient accuracy



Controller (applicable if feasibility condition $\hat{u} \geqslant \hat{u}_{\mathrm{feas}}$ holds, see Theorem IV.2):

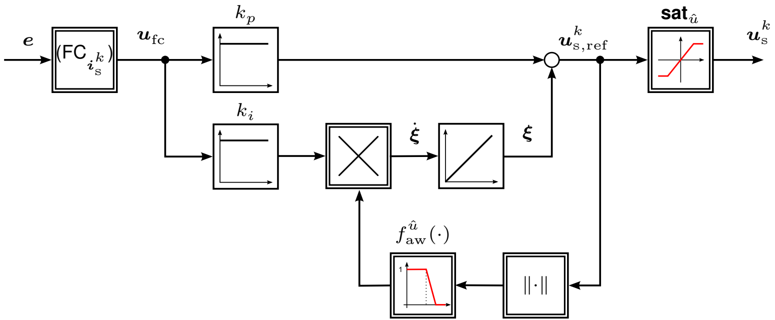
$$\boxed{\boldsymbol{u}_{\mathrm{fc}}(t) = k(t)\boldsymbol{e}(t)} \quad \Longrightarrow \quad \boxed{\|\boldsymbol{e}(t)\| < \Lambda(t)} \qquad (\mathrm{FC}_{\boldsymbol{i}_{\mathrm{s}}^k})$$

where

- $\boldsymbol{e}(t) = (e_s^d(t),\, e_s^q(t))^\top = \boldsymbol{i}_{\mathrm{s,ref}}^k(t) - \boldsymbol{i}_{\mathrm{s}}^k(t)$
- $k(t) = \dfrac{1}{\Lambda(t) - \|\boldsymbol{e}(t)\|} \geqslant \dfrac{1}{\Lambda(t)}$ and $\|\boldsymbol{e}(t)\| = \sqrt{e_s^d(t)^2 + e_s^q(t)^2}$

Current PI-funnel control with anti-windup for RSMs

PI-funnel control with anti-windup



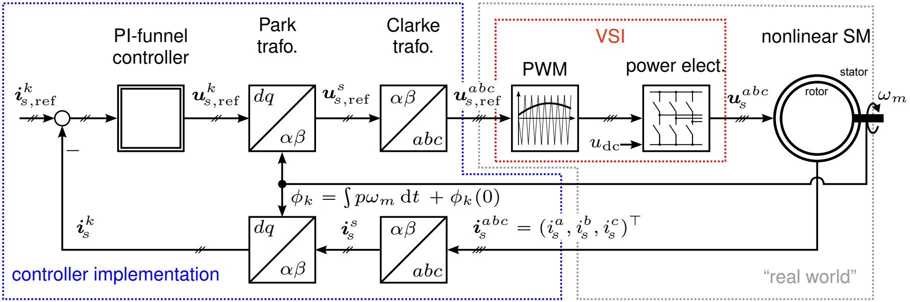
$$\left.\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\xi}(t) &= k_i\, f_{\mathrm{aw}}^{\widehat{u}}\Big(\boldsymbol{\xi}(t) + k_p\,\boldsymbol{u}_{\mathrm{fc}}(t)\Big)\,\boldsymbol{u}_{\mathrm{fc}}(t), & \boldsymbol{\xi}(0) &= \boldsymbol{\xi}^0 \in \mathbb{R}^n \\ \boldsymbol{u}_{\mathrm{s,ref}}^k(t) &= \boldsymbol{\xi}(t) + k_p\,\boldsymbol{u}_{\mathrm{fc}}(t), & \text{where }\; k_p &> 0,\, k_i \geqslant 0. \end{aligned}\right\} \quad (\mathrm{PI}^{\mathrm{aw}})$$

$\Longrightarrow$ $\boldsymbol{\xi}(\cdot)$ acts as bounded input disturbance (see Lemma III.1)

$\Longrightarrow$ PI-like extension for steady state accuracy

Current PI-funnel control with anti-windup for RSMs

Control-loop and implementation

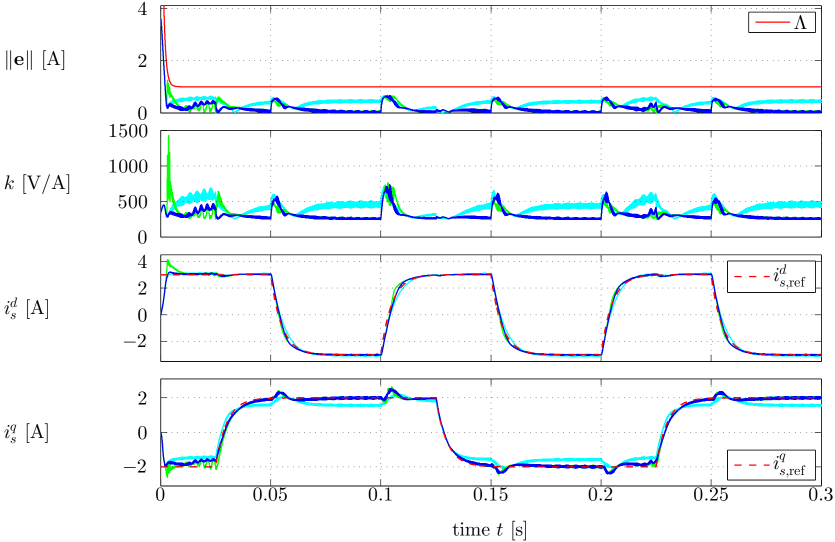


Realistic modeling:

- three-phase machine
- pulse width modulation (PWM) and switching behavior of voltage source inverter (VSI)
- Park-/Clarke transformation

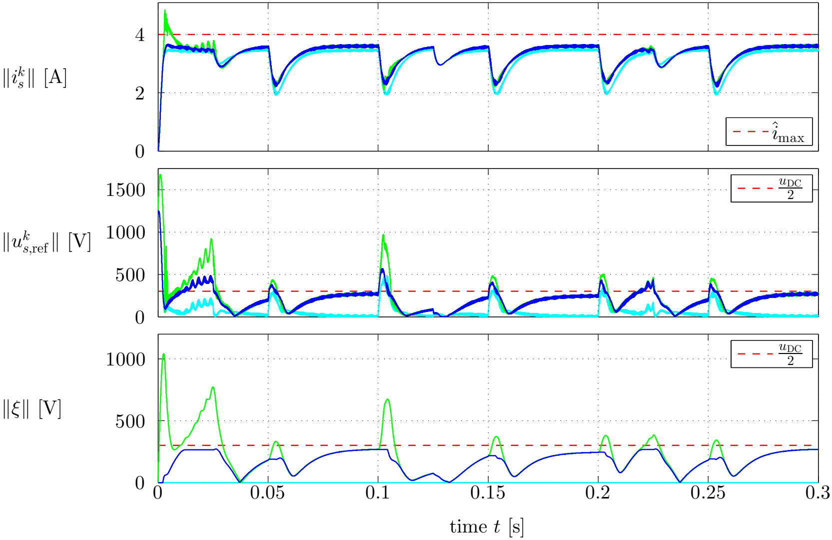
Current PI-funnel control with anti-windup for RSMs

Simulation results (1.1 kW RSM): FC, FC+PI and FC+PI$^{\mathrm{aw}}$



Current PI-funnel control with anti-windup for RSMs

Simulation results (1.1 kW RSM): FC, FC+PI and FC+PI^{aw}



Outline

Conclusion

To take home:

Current PI-funnel control with anti-windup for synchronous machines ...

- is simple
- is robust (*no* system knowledge is required)
- achieves steady state accuracy

Next step: Real-time implementation and measurements ...

References

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[4] M. T. Ivrlač, "Lecture notes on Circuit Theory and Communications," lecture notes (version 2.1), Institute for Circuit Theory and Signal Processing, Technische Universität München, 2013.