MPC with analytical solution and integral error feedback for LTI MIMO systems and its application to current control of grid-connected power converters with LCL-filter

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05.10.2015

PRECEDE 2015

Valparaiso, Chile

Model-predictive control: Problem statement

- **Optimality** with respect to performance index

$$J = \sum_{i=1}^{n} w_i J_i, \qquad n \in \mathbb{N},\ \forall i \in \{1, \dots, n\}: \quad J_i, w_i \geqslant 0. \tag{COST}$$

- but **not** necessarily "optimal" with respect to sub-cost J_i
  - How to choose weighting factors $w_1, \dots, w_n$?
    $\Longrightarrow$ "Increase w_i to "optimize" sub-cost J_i" does **not** hold in general [1]!
  - Does (COST) reflect the "human performance index" adequately?
    $\Longrightarrow$ Is the use of adaptive weights beneficial [1] ?
- How to achieve "good" control performance under **disturbances** or **uncertainties** (i.e. steady state accuracy and/or no overshoots) ?
  - Use of a PI controller (but not admissible) $\rightsquigarrow$ Integral error feedback
  - Use of long prediction horizons (not feasible) $\rightsquigarrow$ MPC with analytical solution

Contents

Model predictive control

Considered system class

LTI MIMO system (discrete) with **disturbances**

$$\left.\begin{array}{rcl} \boldsymbol{x}[k+1] & = & \boldsymbol{A}\boldsymbol{x}[k] + \boldsymbol{B}\boldsymbol{u}[k] + \boldsymbol{B}_{\mathrm{d}}\boldsymbol{d}[k], \quad \boldsymbol{x}[0] = \boldsymbol{x}_0 \in \mathbb{R}^n \\ \boldsymbol{y}[k] & = & \boldsymbol{C}\boldsymbol{x}[k], \end{array}\right\} \qquad (1)$$

Standard assumption [2, p. 48] on system (1):

(A_1) controllability, i.e. $\mathrm{rank}[\boldsymbol{B}, \boldsymbol{A}\boldsymbol{B}, \ldots, \boldsymbol{A}^{n-1}\boldsymbol{B}] = n$.

(A_2) full state feedback, i.e. $\boldsymbol{x}[k]$ is available for all k

(**or** observability, i.e. $\mathrm{rank}\begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{C}\boldsymbol{A} \\ \vdots \\ \boldsymbol{C}\boldsymbol{A}^{n-1} \end{bmatrix} = n$ (observer design).)

(A_3) model-based prediction, i.e. $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{B}_{\mathrm{d}}$ and $\boldsymbol{C}$ are known and $\boldsymbol{d}(\cdot)$ is measured. What if model is not accurate / disturbances are only roughly know?

Model predictive control

Problem & Solution

Problem: Find *optimal control action* (vector)

$$\boldsymbol{u}_\star^{H_\mathrm{p}}[k] := \left(\boldsymbol{u}_\star^{H_\mathrm{p}}[k+1]^\top, \ldots, \boldsymbol{u}_\star[k+n_\mathrm{p}]^\top\right)^\top \in \mathbb{R}^{mn_\mathrm{p}} \tag{2}$$

over $H_\mathrm{p}(k) := \{k+1, \ldots, k+n_\mathrm{p}+1\}$ which maximizes *performance index*

$$\left.\begin{array}{l} J_0\big(k,\boldsymbol{u},\boldsymbol{x},\boldsymbol{v}\big) = \boldsymbol{\gamma}\big(k,\boldsymbol{u},\boldsymbol{x}\big)^\top \boldsymbol{v} + \frac{1}{2}\boldsymbol{v}^\top \boldsymbol{\Lambda}\big(k,\boldsymbol{u},\boldsymbol{x}\big)\boldsymbol{v}, \\ \text{where } \forall (k,\boldsymbol{u},\boldsymbol{x}) \in \mathbb{N}_0 \times \mathbb{R} \times \mathbb{R}^n: \\ (\mathrm{C}_1)\ 0 < \boldsymbol{\Lambda}(k,\boldsymbol{u},\boldsymbol{x}) = \boldsymbol{\Lambda}(k,\boldsymbol{u},\boldsymbol{x})^\top \in \mathbb{R}^{n_\mathrm{p}\times n_\mathrm{p}} \text{ and} \\ (\mathrm{C}_2)\ \boldsymbol{\Lambda}(k,\boldsymbol{u},\boldsymbol{x})^{-1}\boldsymbol{\gamma}(k,\boldsymbol{u},\boldsymbol{x}) = \boldsymbol{0}_{n_\mathrm{p}} \Leftrightarrow \boldsymbol{\gamma}(k,\boldsymbol{u},\boldsymbol{x}) = \boldsymbol{0}_{n_\mathrm{p}} \end{array}\right\} \tag{3}$$

Analytical solution: (*unique and globally optimal*)

$$\boldsymbol{u}_\star^{H_\mathrm{p}}[k] := \arg\min_{\boldsymbol{v}}\Big(J_0\big(k,\boldsymbol{u}[k],\boldsymbol{x}[k],\boldsymbol{v}\big)\Big) = -\boldsymbol{\Lambda}(k,\boldsymbol{u}[k],\boldsymbol{x}[k])^{-1}\boldsymbol{\gamma}(k,\boldsymbol{u}[k],\boldsymbol{x}[k]). \tag{4}$$

Model predictive control

Integral error feedback and system extension

Additional state for integration of error

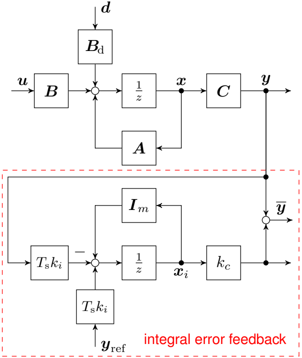
$$\boldsymbol{x}_i[k+1] = \boldsymbol{x}_i[k] + T_\mathrm{s}\, k_i \underbrace{\left(\widehat{\boldsymbol{y}}_\mathrm{ref}[k] - \boldsymbol{C}\boldsymbol{x}[k]\right)}_{=:\boldsymbol{e}[k]}, \quad k_i > 0, \tag{5}$$

Extended system

$$\overbrace{\begin{pmatrix} \boldsymbol{x}[k+1] \\ \boldsymbol{x}_i[k+1] \end{pmatrix}}^{:=\overline{\boldsymbol{x}}[k+1]} = \overbrace{\begin{bmatrix} \boldsymbol{A} & \boldsymbol{O}_{n\times m} \\ -T_\mathrm{s}\, k_i \boldsymbol{C} & \boldsymbol{I}_{m\times m} \end{bmatrix}}^{=:\overline{\boldsymbol{A}}\in\mathbb{R}^{(n+m)\times(n+m)}} \overline{\boldsymbol{x}}[k] + \overbrace{\begin{bmatrix} \boldsymbol{B} \\ \boldsymbol{O}_{m\times m} \end{bmatrix}}^{=:\overline{\boldsymbol{B}}\in\mathbb{R}^{(n+m)\times m}} \boldsymbol{u}[k]$$

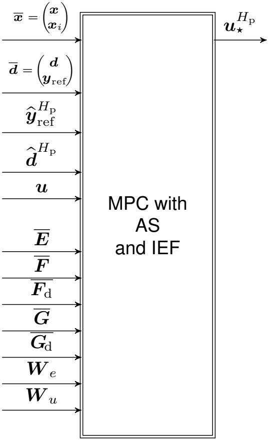
$$+ \underbrace{\begin{bmatrix} \boldsymbol{B}_\mathrm{d} & \boldsymbol{O}_{n\times m} \\ \boldsymbol{O}_{m\times l} & T_\mathrm{s}\, k_i \boldsymbol{I}_m \end{bmatrix}}_{=:\overline{\boldsymbol{B}}_\mathrm{d}\in\mathbb{R}^{(n+m)\times(l+m)}} \underbrace{\begin{pmatrix} \boldsymbol{d}[k] \\ \widehat{\boldsymbol{y}}_\mathrm{ref}[k] \end{pmatrix}}_{=:\overline{\boldsymbol{d}}[k]\in\mathbb{R}^{l+m}},$$

$$\overline{\boldsymbol{y}}[k] = \underbrace{\begin{bmatrix} \boldsymbol{C} & k_c \boldsymbol{I}_m \end{bmatrix}}_{=:\overline{\boldsymbol{C}}\in\mathbb{R}^{m\times(n+m)}} \overline{\boldsymbol{x}}[k], \qquad k_c > 0.$$



Model predictive control

Implementation of MPC with analytical solution (AS) and integral error feedback (IEF)



Performance index (with tracking and control weights):

$$\boxed{J(k,\boldsymbol{u},\overline{\boldsymbol{x}},\boldsymbol{u}_\star^{H_{\mathrm{p}}}) = \underbrace{(\widehat{\boldsymbol{e}}^{H_{\mathrm{p}}})^\top \boldsymbol{W}_{\mathrm{e}}(k,\boldsymbol{u},\overline{\boldsymbol{x}})\,\widehat{\boldsymbol{e}}^{H_{\mathrm{p}}}}_{=:J_e(k,\boldsymbol{u},\overline{\boldsymbol{x}},\boldsymbol{u}_\star^{H_{\mathrm{p}}})} + \underbrace{(\boldsymbol{u}_\star^{H_{\mathrm{p}}})^\top \boldsymbol{W}_{\mathrm{u}}(k,\boldsymbol{u},\overline{\boldsymbol{x}})\,\boldsymbol{u}_\star^{H_{\mathrm{p}}}}_{=:J_u(\boldsymbol{u}_\star^{H_{\mathrm{p}}})}}$$

with

$$\widehat{\boldsymbol{e}}^{H_{\mathrm{p}}}[k] = \widehat{\boldsymbol{y}}_{\mathrm{ref}}^{H_{\mathrm{p}}}[k] - \big(\underbrace{\overline{\boldsymbol{E}}\,\overline{\boldsymbol{x}}[k] + \overline{\boldsymbol{F}}\boldsymbol{u}[k] + \overline{\boldsymbol{F}}_{\mathrm{d}}\,\overline{\boldsymbol{d}}[k] + \overline{\boldsymbol{G}}_{\mathrm{d}}\widehat{\boldsymbol{d}}^{H_{\mathrm{p}}}[k] + \overline{\boldsymbol{G}}\boldsymbol{u}_\star^{H_{\mathrm{p}}}[k]}_{=:\widehat{\boldsymbol{y}}[k]}\big)$$

where

$$\overline{\boldsymbol{E}} := \begin{bmatrix}\overline{\boldsymbol{C}}\overline{\boldsymbol{A}}\\ \overline{\boldsymbol{C}}\overline{\boldsymbol{A}}^2\\ \vdots\\ \overline{\boldsymbol{C}}\overline{\boldsymbol{A}}^{n_{\mathrm{p}}+1}\end{bmatrix},\ \overline{\boldsymbol{F}} := \begin{bmatrix}\overline{\boldsymbol{C}}\overline{\boldsymbol{B}}\\ \overline{\boldsymbol{C}}\overline{\boldsymbol{A}}\overline{\boldsymbol{B}}\\ \vdots\\ \overline{\boldsymbol{C}}\overline{\boldsymbol{A}}^{n_{\mathrm{p}}}\overline{\boldsymbol{B}}\end{bmatrix},\ \overline{\boldsymbol{F}}_{\mathrm{d}} := \begin{bmatrix}\overline{\boldsymbol{C}}\overline{\boldsymbol{B}}_{\mathrm{d}}\\ \overline{\boldsymbol{C}}\overline{\boldsymbol{A}}\overline{\boldsymbol{B}}_{\mathrm{d}}\\ \vdots\\ \overline{\boldsymbol{C}}\overline{\boldsymbol{A}}^{n_{\mathrm{p}}}\overline{\boldsymbol{B}}_{\mathrm{d}}\end{bmatrix},\ \overline{\boldsymbol{G}}(\overline{\boldsymbol{C}},\overline{\boldsymbol{A}},\overline{\boldsymbol{B}}),\ \overline{\boldsymbol{G}}_{\mathrm{d}}(\overline{\boldsymbol{C}},\overline{\boldsymbol{A}},\overline{\boldsymbol{B}}_{\mathrm{d}})$$

Performance index (reformulated):

$$\boxed{J_0(k,\boldsymbol{u},\overline{\boldsymbol{x}},\boldsymbol{u}_\star^{H_{\mathrm{p}}}) = \boldsymbol{\gamma}(k,\boldsymbol{u},\overline{\boldsymbol{x}})^\top \boldsymbol{u}_\star^{H_{\mathrm{p}}} + \frac{1}{2}(\boldsymbol{u}_\star^{H_{\mathrm{p}}})^\top \boldsymbol{\Lambda}(k,\boldsymbol{u},\overline{\boldsymbol{x}})\boldsymbol{u}_\star^{H_{\mathrm{p}}}}$$

where

$$\boldsymbol{\gamma}(k,\boldsymbol{u},\boldsymbol{x})^\top = 2\Big(-\widehat{\boldsymbol{y}}_{\mathrm{ref}}^{H_{\mathrm{p}}}[k]^\top + \overline{\boldsymbol{x}}^\top\overline{\boldsymbol{E}}^\top + \boldsymbol{u}^\top\overline{\boldsymbol{F}}^\top + \overline{\boldsymbol{d}}[k]^\top\overline{\boldsymbol{F}}_{\mathrm{d}}^\top + \widehat{\boldsymbol{d}}^{H_{\mathrm{p}}}[k]^\top\overline{\boldsymbol{G}}_{\mathrm{d}}^\top\Big)\boldsymbol{W}_{\mathrm{e}}(k,\boldsymbol{u},\overline{\boldsymbol{x}})\overline{\boldsymbol{G}}.$$

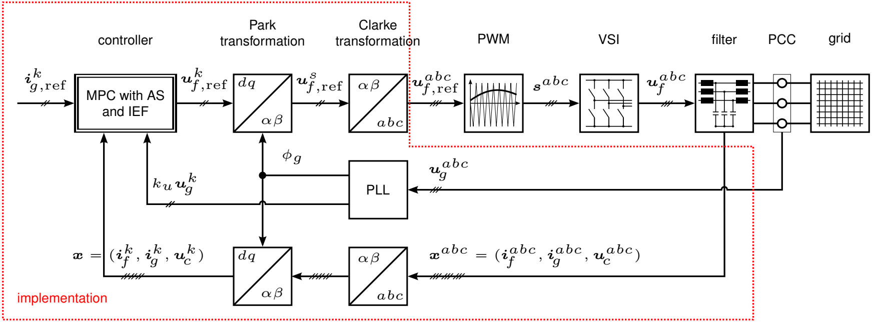
and $\boldsymbol{\Lambda}(k,\boldsymbol{u}[k],\overline{\boldsymbol{x}}[k]) := 2\Big(\overline{\boldsymbol{G}}^\top\boldsymbol{W}_{\mathrm{e}}(k,\boldsymbol{u}[k],\overline{\boldsymbol{x}}[k])\overline{\boldsymbol{G}} + \boldsymbol{W}_{\mathrm{u}}(k,\boldsymbol{u}[k],\overline{\boldsymbol{x}}[k])\Big)$

$$\Longrightarrow \quad \boxed{\boldsymbol{u}_\star^{H_{\mathrm{p}}}[k] = \boldsymbol{\Lambda}(k,\boldsymbol{u}[k],\overline{\boldsymbol{x}}[k])^{-1}\boldsymbol{\gamma}(k,\boldsymbol{u}[k],\overline{\boldsymbol{x}}[k]).}$$

$\Longrightarrow$ (Almost) arbitrary lengths $n_{\mathrm{p}} \gg 1$ of prediction horizon feasible!

Application

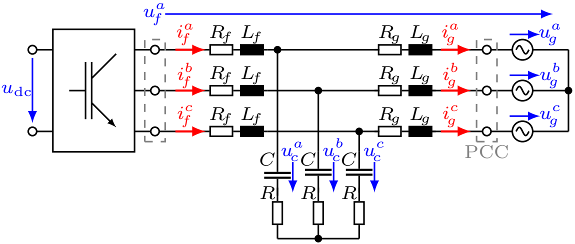
Current control of grid-connected power converter with LCL filter



Assumption: Full state feedback available (otherwise observer design needed [3]).

Application

Model of grid-connected power converter with LCL filter

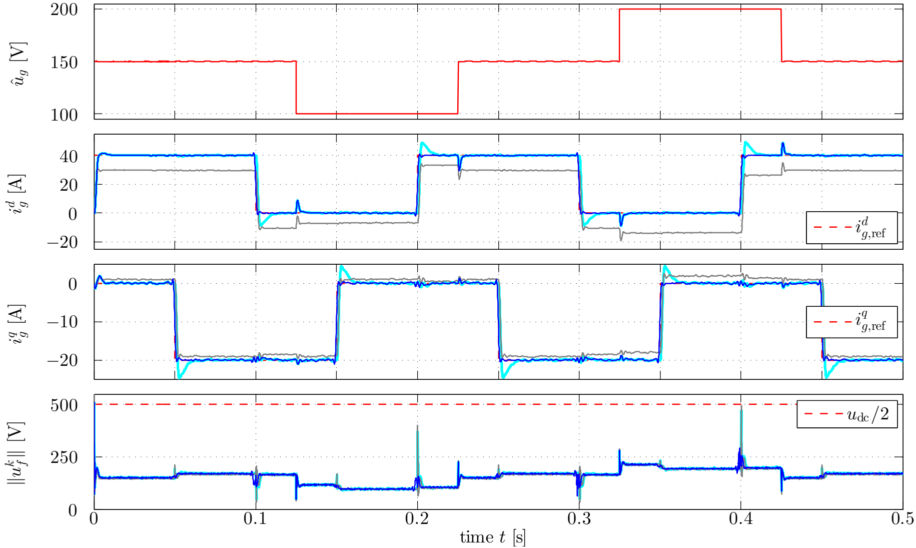


$$\left.\begin{aligned} \boldsymbol{x}[k+1] &= \overbrace{(\boldsymbol{I}_6 + T_{\mathrm{s}}\boldsymbol{A}_{\mathrm{c}})}^{=:\boldsymbol{A}}\,\boldsymbol{x}[k] + \overbrace{T_{\mathrm{s}}\boldsymbol{B}_{\mathrm{c}}}^{=:\boldsymbol{B}}\,\boldsymbol{u}[k] + \overbrace{T_{\mathrm{s}}\boldsymbol{B}_{\mathrm{c,d}}}^{=:\boldsymbol{B}_{\mathrm{d}}}\,\boldsymbol{d}[k] \\ \boldsymbol{y}[k] &= \underbrace{\boldsymbol{C}_{\mathrm{c}}}_{=:\boldsymbol{C}}\,\boldsymbol{x}[k], \end{aligned}\right\} \tag{6}$$

- system is controllable [3]
- full state feedback available (system is observable [3])
- grid voltage is measured ($\boldsymbol{d} = k_u \boldsymbol{u}_g^k$, non-ideal measurement assumed)

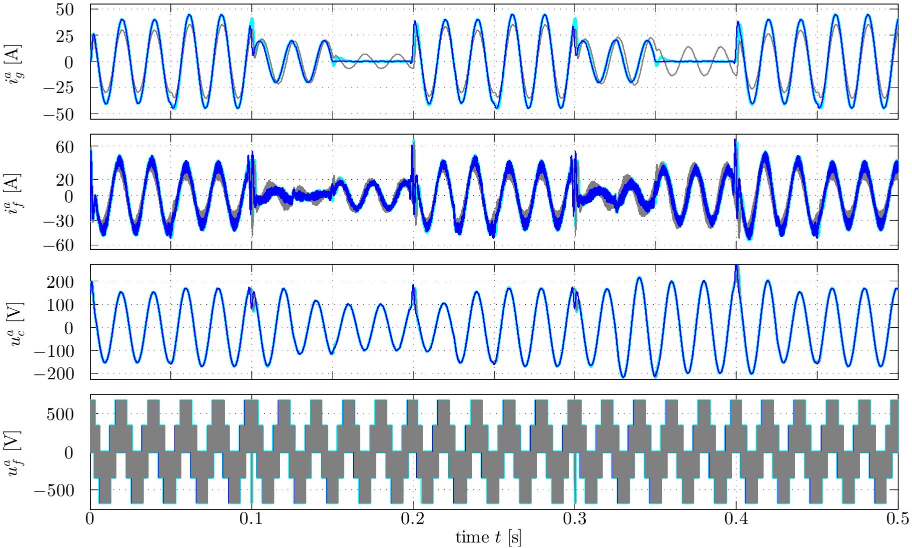
Simulation results ($k_u = 0.85$)

(d, q)-signals: MPC, MPC with IEF & MPC with IEF and $y_{\text{ref}}^{H_p}$ ($n_p = 25$)



Simulation results ($k_u = 0.85$)

Phase a signals: MPC, MPC with IEF & MPC with IEF and $\boldsymbol{y}_{\mathrm{ref}}^{H_{\mathrm{p}}}$ ($n_{\mathrm{p}} = 25$)



Summary

To take home and open questions

- MPC with integral error feedback
  - **fits theory** (extended system with additional state(s) is still a LTI MIMO system) and
  - **is beneficial** (compensates for disturbances and uncertainties)!
- Long prediction horizons
  - **are beneficial** (faster tracking performance; e.g. without overshoots) and
  - **are feasible with analytical solution** (computational load does not increase exponentially)!

- Exploitation of non-constant weights $\boldsymbol{W}_{\mathrm{e}}(k, \boldsymbol{u}, \overline{\boldsymbol{x}})$ and $\boldsymbol{W}_{\mathrm{u}}(k, \boldsymbol{u}, \overline{\boldsymbol{x}})$ feasible? E.g. to incorporate soft-constraints for
  - saturated control actions [1], i.e. $w_u \propto \frac{1}{u_{\max} - \|\boldsymbol{u}_s^k[k]\|}$ or
  - prescribed transient accuracy, i.e. $w_e \propto \frac{1}{\psi[k] - \|\boldsymbol{e}[k]\|}$ [1, 4] ?
- Apply results to nonlinear systems (with online linearization) ?

References

[1] C. M. Hackl, F. Larcher, A. Dötlinger, and R. M. Kennel, "Is multiple-objective model-predictive control "optimal"?," in *Proceedings of the 2013 IEEE International Symposium on Sensorless Control for Electrical Drives and Predictive Control of Electrical Drives and Power Electronics*, pp. 1–8, 2013.

[2] B. D. Anderson and J. B. Moore, *Optimal Control: Linear Quadratic Methods*. Precentice Hall Information and System Sciences Series, Englewood Cliffs, New Jersey: Prentice Hall International Inc., 1990.

[3] C. Dirscherl, J. Fessler, C. M. Hackl, and H. Ipach, "State-feedback controller and observer design for grid-connected voltage source power converters with LCL-filter," in *Proceedings of the 2015 IEEE International Conference on Control Applications*, p. (to be published), 2015.

[4] C. M. Hackl, *Contributions to High-gain Adaptive Control in Mechatronics*. PhD thesis, Lehrstuhl für Elektrische Antriebssysteme und Leistungselektronik, Technische Universität München (TUM), Germany, 2012.

Simulation results

Implementation data

description	*symbols & values*
sampling time	$T_\mathrm{s} = 1{,}25 \cdot 10^{-4}\,\mathrm{s}$
LCL-filter	$L_f = 6{,}25 \cdot 10^{-3}\,\mathrm{H}$, $L_g = 26{,}4 \cdot 10^{-4}\,\mathrm{H}$, $C = 4{,}95 \cdot 10^{-4}\,\mathrm{F}$, $R = R_f = R_g = 0\,\Omega$
VSI	$f_\mathrm{pwm} = 8\,\mathrm{kHz}$, $u_\mathrm{dc} = 1\,000\,\mathrm{V}$
PLL	$k_u = 0.85$ (15 % grid amplitude error)
MPC with AS	$n_\mathrm{p} = 25$, $\boldsymbol{W}_\mathrm{u} = 5000 \cdot \boldsymbol{I}_{54}\ 1/\mathrm{A}$, $\boldsymbol{W}_\mathrm{e} = 50 \cdot \boldsymbol{I}_{50}\ 1/\mathrm{V}$. $k_i = 50$, $k_c = 2$ (integral error feedback)

Table: System, implementation and controller data.