Nonlinear control of reluctance synchronous machines

—Part I: Generic model and control—

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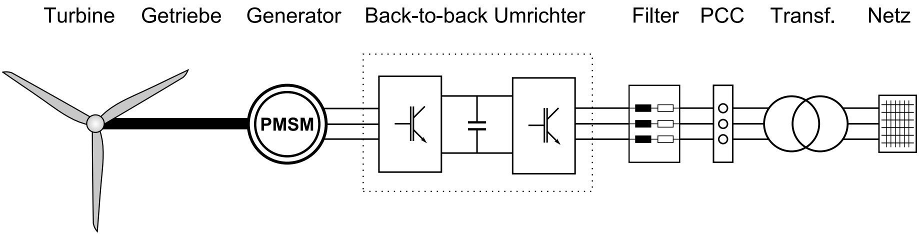
Introduction

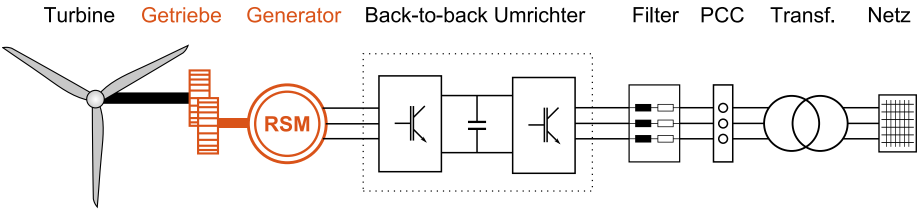
Simulation of large-scale wind turbine system (CRES book chapter)



- Current control (generator-side)
- Maximum torque per Ampere (MTPA)
- Speed control (Maximum power point tracking (MPPT))
- Current control (grid-side)
- DC-link voltage control
- Reactive power (feedforward) control

Introduction

Simulation of small-scale wind turbine system with RSM (Master's Thesis Julian Kullick [1])



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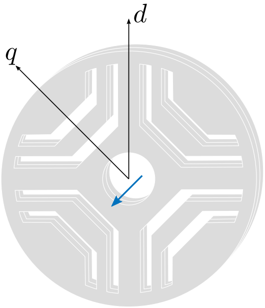
Outline

Outline

RSM: A viable alternative?



Pros:

- very cheap
- high efficiency possible
- high energy/power density (compact design)
- sensorless control feasible

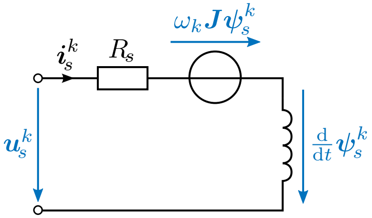
Cons:

- low power factor

Outline

System dynamics

Rotor-fixed reference frame $k = (d, q)$



Stator voltage equation:

$$\boxed{\boldsymbol{u}_s^k(t) = R_s \boldsymbol{i}_s^k(t) + \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_s^k(t) + \omega_k(t)\boldsymbol{J}\boldsymbol{\psi}_s^k(t), \quad [\mathrm{V}]^2 \qquad \boldsymbol{\psi}_s^k(0) = \boldsymbol{\psi}_{s,0}^k \quad [\mathrm{Wb}]^2} \tag{1}$$

where $\boldsymbol{J} := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

System dynamics

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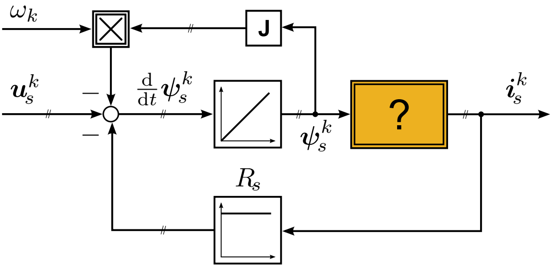

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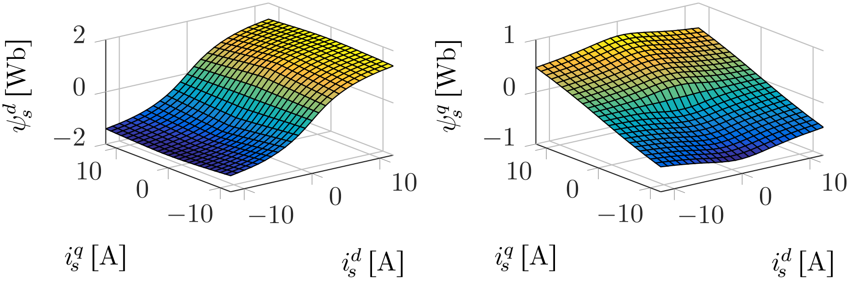
Flux linkage

Nonlinear map: Dependency on stator currents



(a) d-*flux linkage* (b) q-*flux linkage*

- Magnetic saturation
- cross-coupling

$\Rightarrow$ **Nonlinear dynamical system!**

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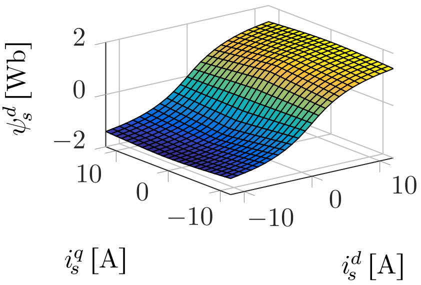
Flux linkage

Nonlinear map: Dependency on stator currents



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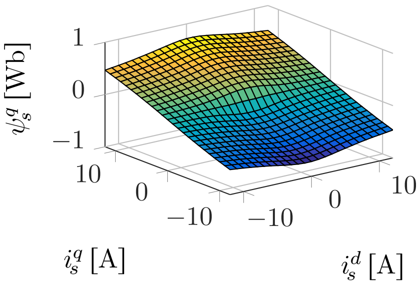

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- Magnetic saturation
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$\Rightarrow$ **Nonlinear dynamical system!**

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Outline

Derivation of the nonlinear current dynamics

- To be derived: Nonlinear current dynamics

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_s^k(t) = \ldots \tag{2}$$

- Given: Stator voltage equation

$$\boldsymbol{u}_s^k(t) = R_s\boldsymbol{i}_s^k(t) + \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_s^k(t) + \omega_k(t)\boldsymbol{J}\boldsymbol{\psi}_s^k(t), \qquad \boldsymbol{\psi}_s^k(0) = \boldsymbol{\psi}_{s,0}^k$$

Assumption 1

Flux linkage $\boldsymbol{\psi}_s^k\,[\mathrm{Wb}]^2$ *is a continuously differentiable function of the stator currents* $\boldsymbol{i}_s^k\,[\mathrm{A}]^2$*, i.e.*

$$\boldsymbol{\psi}_s^k: \quad \mathbb{R}^2 \to \mathbb{R}^2, \quad \boldsymbol{i}_s^k \mapsto \boldsymbol{\psi}_s^k(\boldsymbol{i}_s^k) \quad \wedge \quad \boldsymbol{\psi}_s^k(\cdot) \in \mathcal{C}^1(\mathbb{R}^2;\mathbb{R}^2) \tag{3}$$

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Inductance matrix

Definition 1

Inductance matrix $\boldsymbol{L}_s^k\,[\mathrm{H}]^{2\times 2}$ is defined by

$$\boldsymbol{L}_s^k(\boldsymbol{i}_s^k) = \begin{bmatrix} L_s^{dd}(\boldsymbol{i}_s^k) & L_s^{dq}(\boldsymbol{i}_s^k) \\ L_s^{qd}(\boldsymbol{i}_s^k) & L_s^{qq}(\boldsymbol{i}_s^k) \end{bmatrix} := \left.\begin{bmatrix} \frac{\partial \psi_s^d(\boldsymbol{i}_s^k)}{\partial i_s^d} & \frac{\partial \psi_s^d(\boldsymbol{i}_s^k)}{\partial i_s^q} \\ \frac{\partial \psi_s^q(\boldsymbol{i}_s^k)}{\partial i_s^d} & \frac{\partial \psi_s^q(\boldsymbol{i}_s^k)}{\partial i_s^q} \end{bmatrix}\right|_{\boldsymbol{i}_s^k} = \left.\frac{\partial \boldsymbol{\psi}_s^k(\boldsymbol{i}_s^k)}{\partial \boldsymbol{i}_s^k}\right|_{\boldsymbol{i}_s^k} \tag{4}$$

- positive definite (see [4]):

$$\forall \boldsymbol{i}_s^k \in \mathbb{R}^2:\; \boldsymbol{L}_s^k(\boldsymbol{i}_s^k) > 0 \;\Leftrightarrow\; L_s^{dd}(\boldsymbol{i}_s^k) > 0,\; L_s^{qq}(\boldsymbol{i}_s^k) > 0,\, \det(\boldsymbol{L}_s^k(\boldsymbol{i}_s^k)) > 0 \tag{5}$$

- symmetric (see [4]):

$$\forall \boldsymbol{i}_s^k \in \mathbb{R}^2:\; \boldsymbol{L}_s^k(\boldsymbol{i}_s^k) = \boldsymbol{L}_s^k(\boldsymbol{i}_s^k)^\top \;\Leftrightarrow\; L_s^{dq}(\boldsymbol{i}_s^k) = L_s^{qd}(\boldsymbol{i}_s^k) \tag{6}$$

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Current dynamics

- Time derivative of the flux linkage:

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_s^k(\boldsymbol{i}_s^k(t)) = \frac{\partial \boldsymbol{\psi}_s^k(\boldsymbol{i}_s^k(t))}{\partial \boldsymbol{i}_s^k(t)}\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_s^k(t) = \boldsymbol{L}_s^k(\boldsymbol{i}_s^k(t))\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_s^k(t) \tag{7}$$

- Inserting (7) into (1) gives **current dynamics**:

$$\boxed{\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_s^k(t) &= \boldsymbol{L}_s^k(\boldsymbol{i}_s^k(t))^{-1}\left[\boldsymbol{u}_s^k(t) - R_s\boldsymbol{i}_s^k(t) - \omega_k(t)\boldsymbol{J}\boldsymbol{\psi}_s^k(\boldsymbol{i}_s^k(t))\right]\\ \boldsymbol{i}_s^k(0) &= \boldsymbol{i}_{s,0}^k\ [\mathrm{A}]^2\end{aligned}} \tag{8}$$

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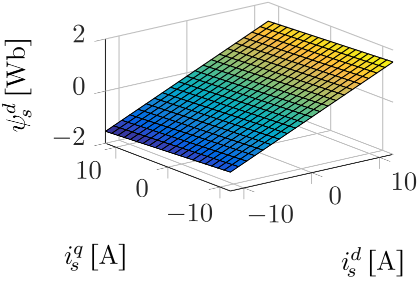
Current control (linear system)

Linear RSM: Flux linkage

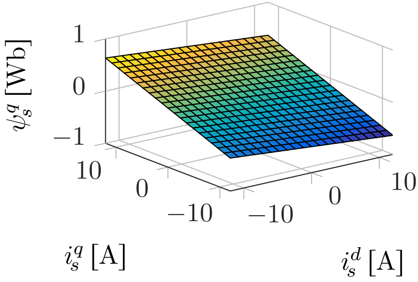
Assumption (temporary) 1

ψ_s^k is a linear function of the stator currents i_s^k. Then:

$$L_s^k(i_s^k) = L_s^k = \textit{const.} \qquad \textit{and} \qquad \psi_s^k(i_s^k) = L_s^k i_s^k \tag{9}$$



(a) d-flux linkage (linear)



(b) q-flux linkage (linear)

Current control (linear system)

Linear RSM: Deriviation of current dynamics

$$\frac{\mathrm{d}}{\mathrm{d}t} i_s^d(t) \overset{(8)}{=} \overbrace{\frac{L_s^{qq}}{L_s^{dd} L_s^{qq} - L_s^{dq} L_s^{qd}}}^{=:1/L_{s,\mathrm{aux}}^d} \Big[u_s^d(t) - R_s i_s^d(t) \underbrace{+ \omega_k(t) \psi_s^q(\boldsymbol{i}_s^k(t)) - \frac{L_s^{dq}}{L_s^{qq}} \Big(u_s^q(t) - R_s i_s^q(t) - \omega_k(t) \psi_s^d(\boldsymbol{i}_s^k(t)) \Big)}_{=:u_{s,\mathrm{dist}}^d(t)} \Big] \quad (10)$$

$$\frac{\mathrm{d}}{\mathrm{d}t} i_s^q(t) \overset{(8)}{=} \overbrace{\frac{L_s^{dd}}{L_s^{dd} L_s^{qq} - L_s^{dq} L_s^{qd}}}^{=:1/L_{s,\mathrm{aux}}^q} \Big[u_s^q(t) - R_s i_s^q(t) \underbrace{- \omega_k(t) \psi_s^q(\boldsymbol{i}_s^k(t)) - \frac{L_s^{qd}}{L_s^{dd}} \Big(u_s^d(t) - R_s i_s^d(t) + \omega_k(t) \psi_s^q(\boldsymbol{i}_s^k(t)) \Big)}_{=:u_{s,\mathrm{dist}}^q(t)} \Big] \quad (11)$$

Current control (linear system)

Linear RSM: Deriviation of current dynamics

$$
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& \underbrace{+ \omega_k(t)\psi_s^q(\boldsymbol{i}_s^k(t)) - \frac{L_s^{dq}}{L_s^{qq}} \left(u_s^q(t) - R_s i_s^q(t) - \omega_k(t)\psi_s^d(\boldsymbol{i}_s^k(t)) \right)}_{=:u^d_{s,\mathrm{dist}}(t)} \Big]
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\tag{10}
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Current control (linear system)

Linear RSM: Current dynamics

- Current dynamics:

$$\frac{\mathrm{d}}{\mathrm{d}t} i_s^d(t) = \frac{1}{L_{s,\mathrm{aux}}^d} \left[u_s^d(t) - R_s i_s^d(t) + u_{s,\mathrm{dist}}^d(t) \right]$$
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- ... with ideal compensation of the disturbances (see [2]):

$$\frac{\mathrm{d}}{\mathrm{d}t} i_s^d(t) = \frac{1}{L_{s,\mathrm{aux}}^d} \left[u_s^d(t) - R_s i_s^d(t) \right] \quad \circ\!\!-\!\!\bullet \quad \frac{i_s^d(s)}{u_s^d(s)} = \frac{\frac{1}{R_s}}{1 + s\frac{L_{s,\mathrm{aux}}^d}{R_s}} \tag{12}$$

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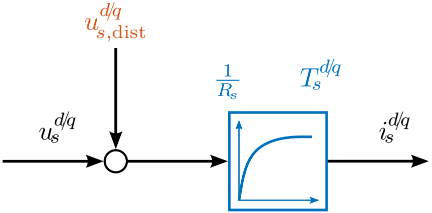
Current control (linear system)

Linear RSM: Current dynamics

$\Rightarrow$ **First-order lag system** with

- gain $\frac{1}{R_s}$ $\left[\frac{1}{\Omega}\right]$ and
- time constant $T_s^{d/q} = \frac{L_{s,\mathrm{aux}}^{d/q}}{R_s}$ [s]



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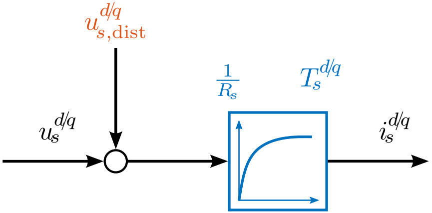

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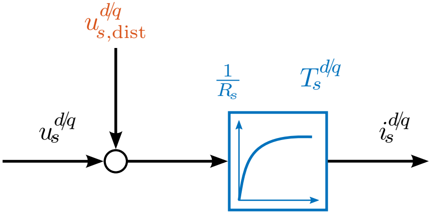
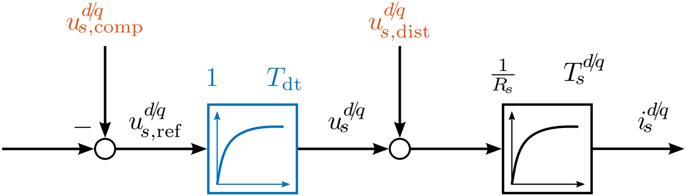


Current control (linear system)

Back-to-back converter (generator-side VSI)

- VSI generates stator voltages $\boldsymbol{u}_s^k$ according to reference $\boldsymbol{u}_{s,\mathrm{ref}}^k$ (PWM/SWM)
- VSI approximation als first-order lag system with $T_{\mathrm{dt}} = \frac{1}{f_{\mathrm{PWM}}}$ [s] (see [2])

$$F_{\mathrm{VSI}}(s) = \frac{u_s^{d/q}(s)}{u_{s,\mathrm{ref}}^{d/q}(s)} \approx \frac{1}{1 + sT_{\mathrm{dt}}} \tag{14}$$



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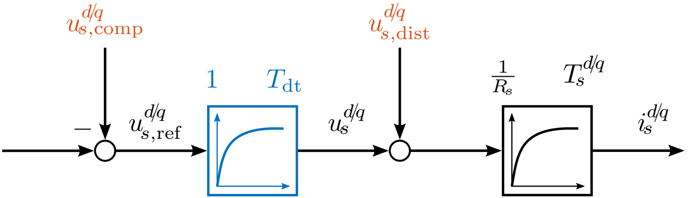

Current control (linear system)

Controller tuning: Magnitude Optimum

- Overall second-order lag system:

$$F_S(s) = \frac{i_s^{d/q}(s)}{u_{s,\mathrm{ref}}^{d/q}(s)} = \frac{\frac{1}{R_s}}{(1+sT_s^{d/q})(1+sT_{\mathrm{dt}})} \tag{15}$$

- PI **controller**

$$F_{R,i_s^{d/q}}(s) = V_{R,i_s^{d/q}} \frac{1+sT_{n,i_s^{d/q}}}{sT_{n,i_s^{d/q}}} \tag{16}$$

- Controller design according to *Magnitude (or Modulus) Optimum* (see [3]):

$$\boxed{\begin{array}{rl} \text{Controller gain:} & V_{R,i_s^{d/q}} = \dfrac{T_s^{d/q} R_s}{2T_{\mathrm{dt}}} \; [\Omega] \\ \text{Controller time constant:} & T_{n,i_s^{d/q}} = T_s^{d/q} \quad [\mathrm{s}] \end{array}}$$

Current control (linear system)

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$$F_S(s) = \frac{i_s^{d/q}(s)}{u_{s,\mathrm{ref}}^{d/q}(s)} = \frac{\frac{1}{R_s}}{(1 + sT_s^{d/q})(1 + sT_{\mathrm{dt}})} \tag{15}$$

- PI **controller**

$$F_{R,i_s^{d/q}}(s) = V_{R,i_s^{d/q}} \frac{1 + sT_{n,i_s^{d/q}}}{sT_{n,i_s^{d/q}}} \tag{16}$$

- Controller design according to *Magnitude (or Modulus) Optimum* (see [3]):

Controller gain: $V_{R,i_s^{d/q}} = \frac{T_s^{d/q} R_s}{2T_{\mathrm{dt}}}$ $[\Omega]$

Controller time constant: $T_{n,i_s^{d/q}} = T_s^{d/q}$ $[\mathrm{s}]$

Current control (linear system)

Controller tuning: Magnitude Optimum

- Overall second-order lag system:

$$F_S(s) = \frac{i_s^{d/q}(s)}{u_{s,\mathrm{ref}}^{d/q}(s)} = \frac{\frac{1}{R_s}}{(1+sT_s^{d/q})(1+sT_{\mathrm{dt}})} \tag{15}$$

- PI **controller**

$$F_{R,i_s^{d/q}}(s) = V_{R,i_s^{d/q}} \frac{1+sT_{n,i_s^{d/q}}}{sT_{n,i_s^{d/q}}} \tag{16}$$

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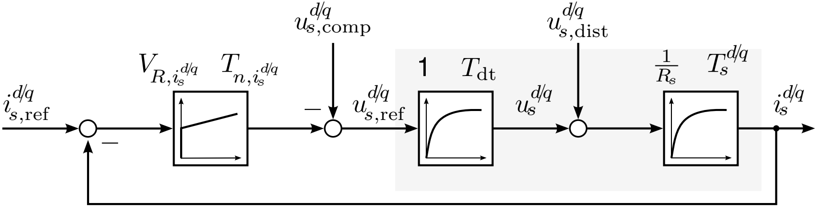
Controller time constant:	$T_{n,i_s^{d/q}} = T_s^{d/q}$ $[\mathrm{s}]$

Current control (linear system)

Closed-loop system: PI controller, VSI (approximation) & linear RSM



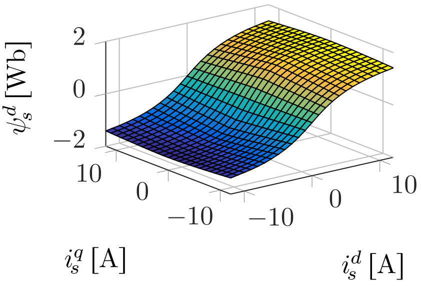
Current control (**non**linear system)

Nonlinear (real) RSM: Flux map

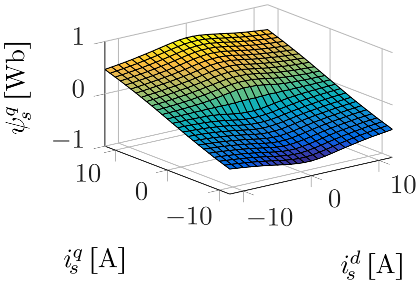
Assumption 2

ψ_s^k is a nonlinear function of the stator currents $\boldsymbol{i}_s^k$. Then:

$$\boldsymbol{L}_s^k(\boldsymbol{i}_s^k) \neq \textit{const.} \qquad \textit{and} \qquad \boldsymbol{\psi}_s^k(\boldsymbol{i}_s^k) \tag{17}$$



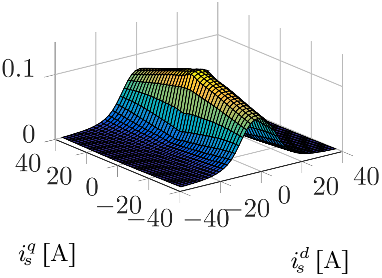
(a) d-flux linkage



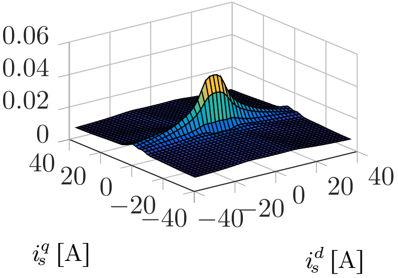
(b) q-flux linkage

Current control (**non**linear system)

Nonlinear (real) RSM: Inductances



(a) L_s^{dd} [H]

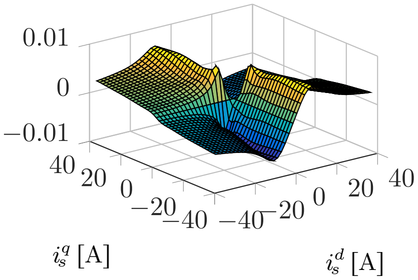


(b) L_s^{qq} [H]

Figure: *Self inductances*

Current control (**non**linear system)

Nonlinear (real) RSM: Inductances



(a) L_s^{dq} [H] (b) L_s^{qd} [H]

Figure: *Cross-coupling inductances*

Current control (**non**linear system)

Nonlinear (real) RSM: State-dependent parameters

- Auxiliary inductances $L^{d}_{s,\mathrm{aux}}$ and $L^{q}_{s,\mathrm{aux}}$ become **state-dependent**

$$L^{d/q}_{s,\mathrm{aux}} = \frac{L_s^{qq/dd}}{L_s^{dd} L_s^{qq} - L_s^{dq} L_s^{qd}} \quad \Rightarrow \quad L^{d/q}_{s,\mathrm{aux}}(\boldsymbol{i}_s^k) = \frac{L_s^{qq/dd}(\boldsymbol{i}_s^k)}{L_s^{dd}(\boldsymbol{i}_s^k) L_s^{qq}(\boldsymbol{i}_s^k) - L_s^{dq}(\boldsymbol{i}_s^k) L_s^{qd}(\boldsymbol{i}_s^k)}$$

- Time constant $T_s^{d/q}$ becomes **state-dependent**

$$T_s^{d/q} = \frac{L^{d/q}_{s,\mathrm{aux}}}{R_s} \quad \Rightarrow \quad T_s^{d/q}(\boldsymbol{i}_s^k) := \frac{L^{d/q}_{s,\mathrm{aux}}(\boldsymbol{i}_s^k)}{R_s}$$

$\Rightarrow$ **Constant** controller design incorrect
(bad performance & or even instability)

Current control (**non**linear system)

Nonlinear (real) RSM: State-dependent parameters

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- Time constant $T^{d/q}_{s}$ becomes **state-dependent**

$$T^{d/q}_{s} = \frac{L^{d/q}_{s,\mathrm{aux}}}{R_{s}} \quad \Rightarrow \quad T^{d/q}_{s}(\boldsymbol{i}^{k}_{s}) := \frac{L^{d/q}_{s,\mathrm{aux}}(\boldsymbol{i}^{k}_{s})}{R_{s}}$$

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- Time constant $T_s^{d/q}$ becomes **state-dependent**

$$T_s^{d/q} = \frac{L^{d/q}_{s,\mathrm{aux}}}{R_s} \quad \Rightarrow \quad T_s^{d/q}(\boldsymbol{i}_s^k) := \frac{L^{d/q}_{s,\mathrm{aux}}(\boldsymbol{i}_s^k)}{R_s}$$

$\Rightarrow$ **Constant** controller design incorrect
(bad performance & or even instability)

Current control (**non**linear system)

Nonlinear (real) RSM: Online adjustment of controller parameters

Assumption 3

Sampling time $0 < T_{\mathrm{ctrl}} \ll 1\,[\mathrm{s}]$ *is sufficiently small, such that*

$$\forall \tau \in [t - T_{\mathrm{ctrl}}, t]: \quad \boldsymbol{L}_s^k(\boldsymbol{i}_s^k(\tau)) := \boldsymbol{L}_s^k(t - T_{\mathrm{ctrl}}) = \mathrm{const.} \tag{18}$$

$\Rightarrow$ **Update of the controller parameters in each sampling step:**

Controller gain: $V_{R,i_s^{d/q}}(\boldsymbol{i}_s^k) = \dfrac{T_s^{d/q}(\boldsymbol{i}_s^k) R_s}{2T_{\mathrm{dt}}} = \dfrac{L_{s,\mathrm{aux}}^{d/q}(\boldsymbol{i}_s^k)}{2T_{\mathrm{dt}}}\ [\Omega]$

Controller time constant: $T_{n,i_s^{d/q}}(\boldsymbol{i}_s^k) = T_s^{d/q}(\boldsymbol{i}_s^k) = \dfrac{L_{s,\mathrm{aux}}^{d/q}(\boldsymbol{i}_s^k)}{R_s}\ [\mathrm{s}]$

Simulation results (9.6 kW RSM)

Nonlinear (real) RSM: Nonlinear PI current control with online adjustment of parameters

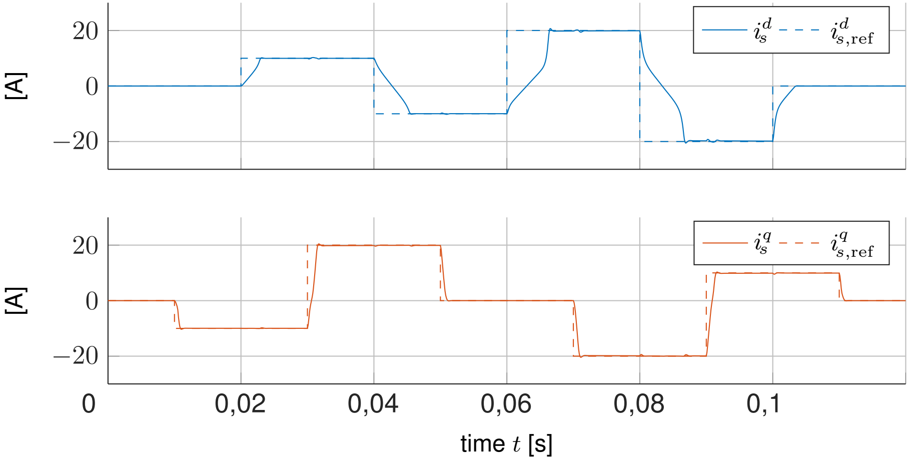


Figure: *Nonlinear PI current control with inverter approximation as first-order lag system*

Simulation results (9.6 kW RSM)

Nonlinear (real) RSM: Nonlinear PI current control with online adjustment of parameters

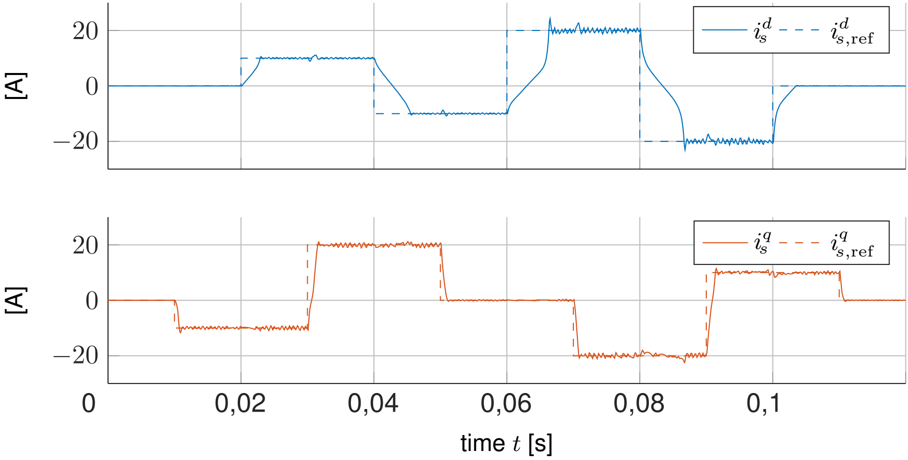
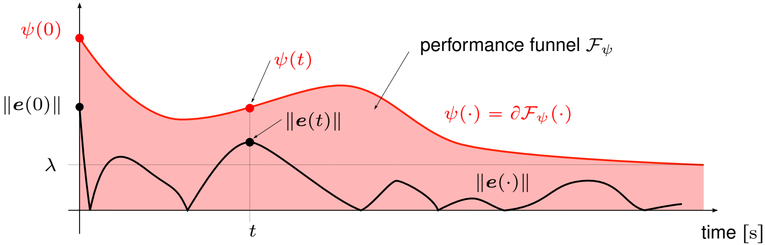


Figure: *Nonlinear PI control with inverter and PWM (more realistic scenario)*

Outline

Principle idea of funnel control

Nonlinear (real) RSM: Prescribed transient current accuracy



Funnel controller:

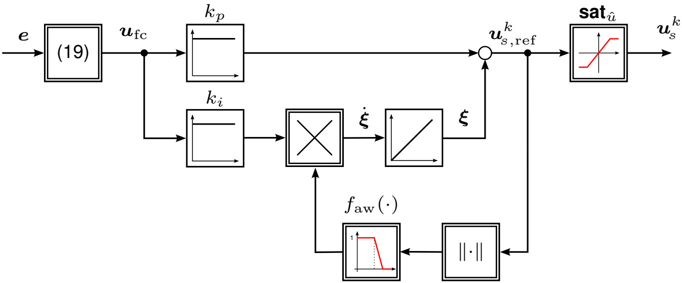
$$\boxed{\boldsymbol{u}_{\text{fc}}(t) = k(t)\boldsymbol{e}(t)} \tag{19}$$

where

- $\boldsymbol{e}(t) = (e_s^d(t),\, e_s^q(t))^\top = \boldsymbol{i}_{s,\text{ref}}^k(t) - \boldsymbol{i}_s^k(t)$
- $k(t) = \dfrac{1}{\psi(t) - \|\boldsymbol{e}(t)\|} \geqslant \dfrac{1}{\psi(t)}$ with $\|\boldsymbol{e}(t)\| = \sqrt{e_s^d(t)^2 + e_s^q(t)^2}$

PI with anti-windup

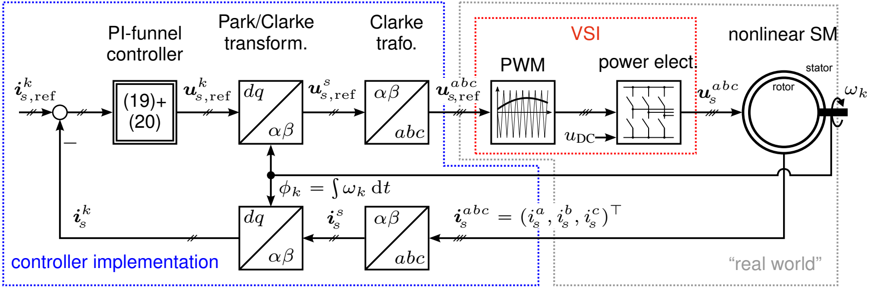
Nonlinear (real) RSM: PI controller with anti-windup implementation



$$\left.\begin{array}{l} \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\xi}(t) = f_{\mathrm{aw}}\Big(\boldsymbol{\xi}(t) + k_p\,\boldsymbol{u}_{\mathrm{fc}}(t)\Big)\,k_i\,\boldsymbol{u}_{\mathrm{fc}}(t), \\ \boldsymbol{u}_{s,\mathrm{ref}}^k(t) = \boldsymbol{\xi}(t) + k_p\,\boldsymbol{u}_{\mathrm{fc}}(t), \qquad \boldsymbol{\xi}(0) = \boldsymbol{\xi}^0 \in \mathbb{R}^n \\ \quad \text{where } \; k_p > 0,\, k_i \geqslant 0. \end{array}\right\} \tag{20}$$

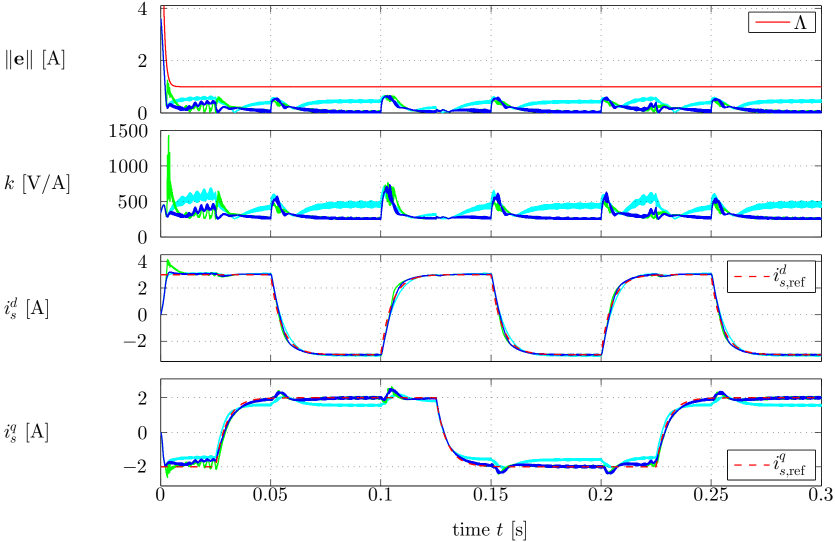
Closed-loop system for simulation

Nonlinear (real) RSM: Simulation environment



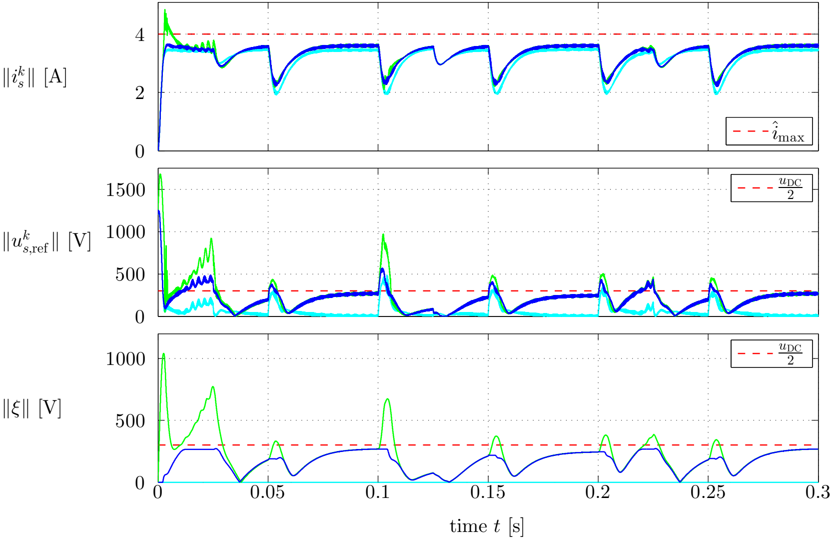
Simulation results (1.1 kW RSM)

Nonlinear (real) RSM: PI-funnel control — FC, — FC+PI and — FC+PI$^{\mathrm{aw}}$



Simulation results (1.1 kW RSM)

Nonlinear (real) RSM: PI-funnel control — FC, — FC+PI and — FC+PI$^{\mathrm{aw}}$



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[2] C. Dirscherl, C. M. Hackl und K. Schechner. "Regelung von Windkraftanlagen: Eine Einführung". In: D. "Schröder: Elektrische Antriebe - Regelung von Antriebssystemen" (Kap. 24). 4. Aufl. Berlin: Springer-Verlag GmbH, 2015.

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