Funnel control for wind turbine systems

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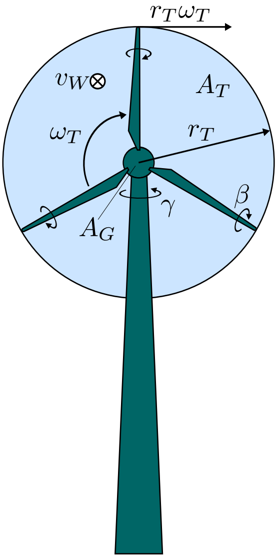
Outline

Introduction: Wind power and power coefficient



Wind power

$$p_W(t) = \frac{1}{2}\varrho \underbrace{\pi r_T^2}_{=A_T} v_W(t)^3, \quad v_W(\cdot) \in \mathcal{W}^{1,\infty}(\mathbb{R}_{\geqslant 0}; \mathbb{R}) \tag{1}$$

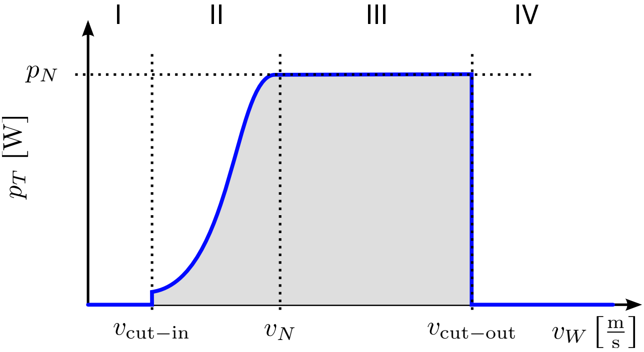
Tip speed ratio

$$\lambda := \lambda(\omega_G, v_W) := \frac{r_T \omega_G}{g_r v_W} \; [1] \quad \text{where} \quad \omega_T = \frac{\omega_G}{g_r}. \tag{2}$$

Power coefficient (approximation)

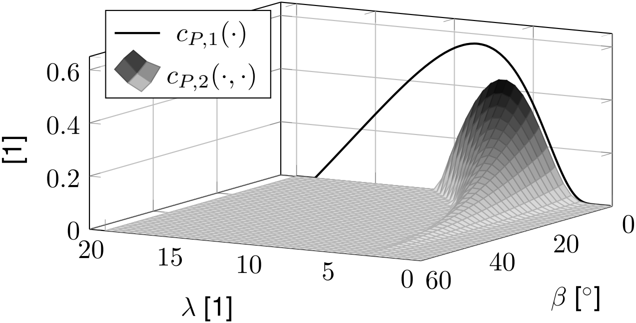
$$c_P : \overline{\mathcal{D}} \to \mathbb{R}_{\geqslant 0}, \qquad (\beta, \lambda) \mapsto c_P(\beta, \lambda) :=$$
$$c_1\left[c_2\, f(\beta, \lambda) - c_3\beta - c_4\beta^x - c_5\right]e^{-c_6\, f(\beta,\lambda)}$$
$$\text{with } \mathcal{D} := \{\, (\beta, \lambda) \in \mathbb{R}_{\geqslant 0} \times \mathbb{R}_{\geqslant 0} \mid c_P(\beta, \lambda) > 0 \,\} \tag{3}$$

Problem: Speed control of wind turbine systems



- Regime I: Standstill. No power output, i.e. $p_T = 0$.
- Regime II: Operation. Variable power output, i.e.. $0 < p_T < p_N \Longrightarrow$ Goal: Maximum power point tracking.
- Regime III: Nominal operation. Nominal power output, i.e. $p_T = p_N$.
- Regime IV: Standstill (emergency stop), i.e. $p_T = 0$.

Problem: Speed control of wind turbine systems

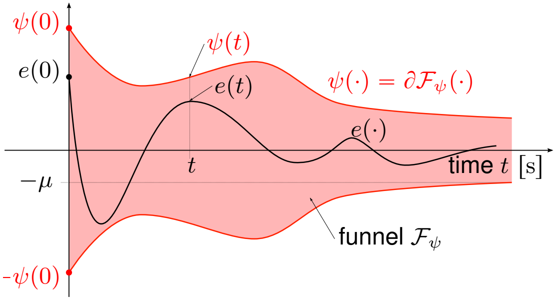


- Regime II: Maximum power point tracking, i.e. $\lambda \to \lambda^\star$, such that for given $\beta = \beta_0$,

$$c_P^\star(\beta_0) := \max_\lambda c_P(\lambda, \beta_0) < c_{P,\mathrm{Betz}} = \frac{16}{27} \approx 0{,}59.$$

- Tip speed ratio $\lambda := \lambda(\omega_G, v_W) \Longrightarrow \omega_G \to \omega_G^\star$ (speed control!)

Funnel control: Objective and controller [6]



- Tracking with prescribed transient accuracy by design of $\psi(\cdot)$, i.e.

$$\forall\, t \geqslant 0: \qquad |e(t)| = |y_{\mathrm{ref}}(t) - y(t)| < \psi(t)$$

- Controller

$$u(t) = \mathrm{sign}(\gamma_0)\, \underbrace{\frac{1}{\psi(t) - |e(t)|}}_{=:k(t)}\, e(t)$$

Funnel control: Admissible system class $\mathcal{S}$ (simplified)

System

$$\left.\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}\big(u(t) + u_d(t)\big) \\ &\qquad\qquad + \boldsymbol{B}_{\mathfrak{T}}\Big(\boldsymbol{d}(t, \boldsymbol{x}(t)) + (\mathfrak{T}\boldsymbol{x})(t)\Big) \\ y(t) &= \boldsymbol{c}^\top \boldsymbol{x}(t), \end{aligned}\right\} \tag{4}$$

satisfying

(sp$_1$) the relative degree is one and the sign of the high-frequency gain is known, i.e.

$$\gamma_0 := \boldsymbol{c}^\top \boldsymbol{b} \neq 0 \quad \text{and} \quad \operatorname{sign}(\gamma_0) \text{ known;}$$

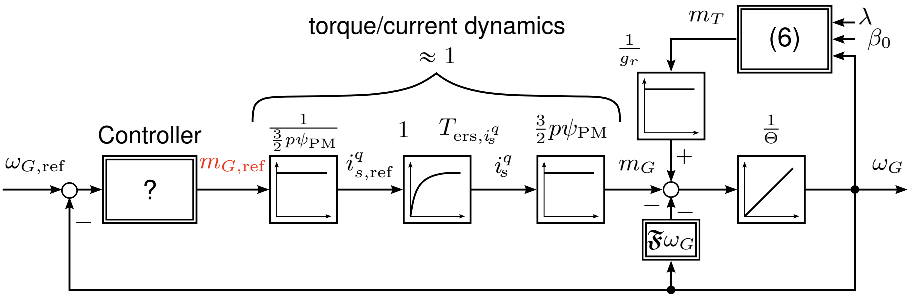
(sp$_2$) the unperturbed system is minimum-phase, i.e.

$$\forall\, s \in \mathbb{C} \text{ with } \Re(s) \geqslant 0: \quad \det \begin{bmatrix} s\boldsymbol{I}_n - \boldsymbol{A} & \boldsymbol{b} \\ \boldsymbol{c}^\top & 0 \end{bmatrix} \neq 0;$$

(sp$_3$) operator $\mathfrak{T}$ is of class[1] $\mathcal{T}$ (see Def. 1 in [6]) and globally bounded, i.e. $\mathfrak{T} \in \mathcal{T}$ and $M_{\mathfrak{T}} := \sup\{\|(\mathfrak{T}\boldsymbol{\xi})(t)\| \mid t \geqslant 0,\ \boldsymbol{\xi}(\cdot) \in \mathcal{C}([-h,\infty), \mathbb{R}^n)\} < \infty$;

(sp$_4$) disturbances are bounded, i.e. $u_d(\cdot) \in \mathcal{L}^\infty([-h,\infty); \mathbb{R})$ and $\boldsymbol{d}(\cdot,\cdot) \in \mathcal{L}^\infty([-h,\infty) \times \mathbb{R}^n; \mathbb{R}^m)$.

Speed control: Stiff model of wind turbine systems



$$\frac{\mathrm{d}}{\mathrm{d}t}\omega_G = \frac{1}{\Theta}\left(\frac{m_T(v_W,\beta_0,\lambda)}{g_r} - m_G - \mathfrak{F}\omega_G\right), \quad \omega_G(0) = \omega_G^0 \geqslant 0 \qquad (5)$$

where

$$m_T(v_W,\beta_0,\lambda) = \frac{c_P(\beta_0,\lambda)p_W(t)}{\omega_G/g_r} \overset{(2)}{=} \frac{\varrho\pi r_T^3 v_W(t)^2}{2g_r}\frac{c_P(\beta_0,\lambda)}{\lambda}. \qquad (6)$$

Speed control: Wind turbine system of class $\mathcal{S}$?

$\frac{\mathrm{d}}{\mathrm{d}t}\,\omega_G = \frac{1}{\Theta}\left(\frac{m_T(v_W,\beta_0,\lambda)}{g_r} - m_G - \mathfrak{F}\omega_G\right)$ is of class $\mathcal{S}$, since

(sp$_1$) $\gamma_0 = -\frac{1}{\Theta} \quad \Longrightarrow \quad \mathrm{sign}(\gamma_0) = -1$ ✓

(sp$_2$) $\forall\, s \in \mathbb{C}\colon \;\; \det\begin{bmatrix} s & -\frac{1}{\Theta} \\ 1 & 0 \end{bmatrix} = \frac{1}{\Theta} \neq 0$ ✓

(sp$_3$) $\mathfrak{F} \in \mathcal{T}$ ✓ (see [1, Sec. 5.1.2])

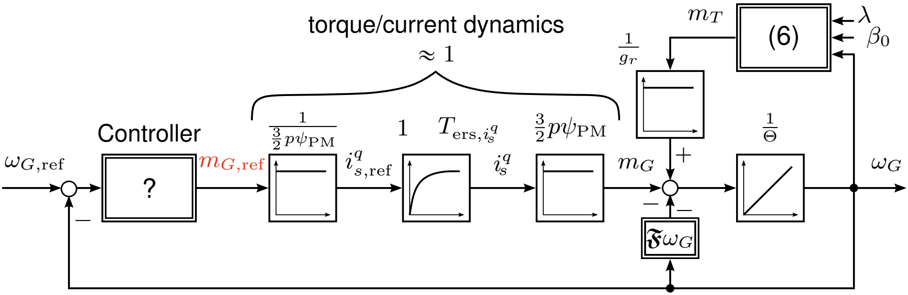
(sp$_4$) "disturbance = turbine torque" bounded ✓ by (see paper, Sec. III.D)

$$\forall\, \beta_0, c_0 \geqslant 0\; \forall\, v_W(\cdot) \in \mathcal{W}^{1,\infty}(\mathbb{R}_{\geqslant 0};\mathbb{R})\; \forall\, \lambda \in [c_0\beta_0, \infty):$$

$$0 \leqslant m_T(v_W(\cdot), \beta_0, \lambda) \leqslant \frac{\varrho\pi r_T^3 \|v_W\|_\infty}{2g_r}\,\frac{c_P^\star}{c_0\beta_0}. \qquad (7)$$

$\Longrightarrow$ Application of funnel control for speed control is admissible!

Speed control: Funnel controller



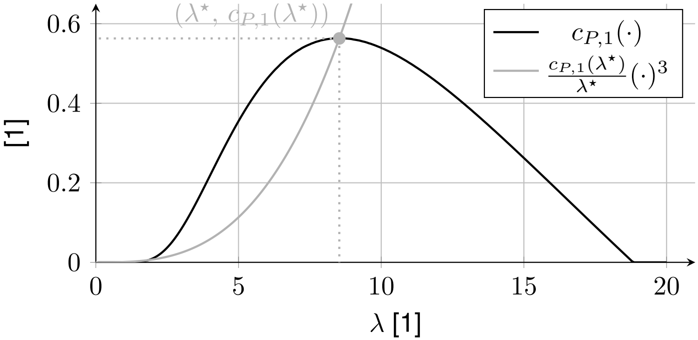
Controller with $\omega_{G,\mathrm{ref}}(t) := \frac{g_r \lambda^\star}{r_T} v_W(t) =: \omega_G^\star(t)$:

$$u(t) = m_{G,\mathrm{ref}}(t) = \underbrace{-}_{\mathrm{sign}\,\gamma_0} \underbrace{\frac{\varsigma(t)}{\psi(t) - |e(t)|}}_{=:k(t)} \underbrace{\left(\omega_{G,\mathrm{ref}}(t) - \omega_G(t)\right)}_{=:e(t)} \tag{8}$$

Properties:

$$\forall\, t \geqslant 0: \qquad |e(t)| = |\omega_G^\star(t) - \omega_G(t)| < \psi(t).$$

Speed control: Classical controller [8]



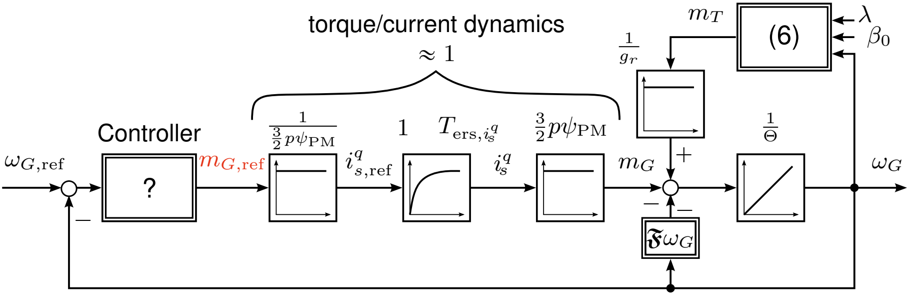
Controller:

$$m_{G,\mathrm{ref}}(t) = k_P^\star \omega_G(t)^2 \text{ with } k_P^\star := \frac{\varrho \pi r_T^5}{2 g_r^3} \frac{c_P(\lambda^\star, \beta_0)}{(\lambda^\star)^3} \;\wedge\; \omega_{G,\mathrm{ref}} = 0. \qquad (9)$$

Properties for all $0 < \lambda_{\min} \leqslant \lambda(0)$, $v_W(t) = v_0 > 0$, $\beta = \beta_0 \geqslant 0 \wedge \mathfrak{F}\omega_G = 0$ [7]:

- locally asymptotically stable
- $\lambda^\star$ attractive, i.e. $\lim_{t\to\infty} \lambda(t) = \lambda^\star$ ($\Leftrightarrow \lim_{t\to\infty} \omega_G(t) = \omega_G^\star$).

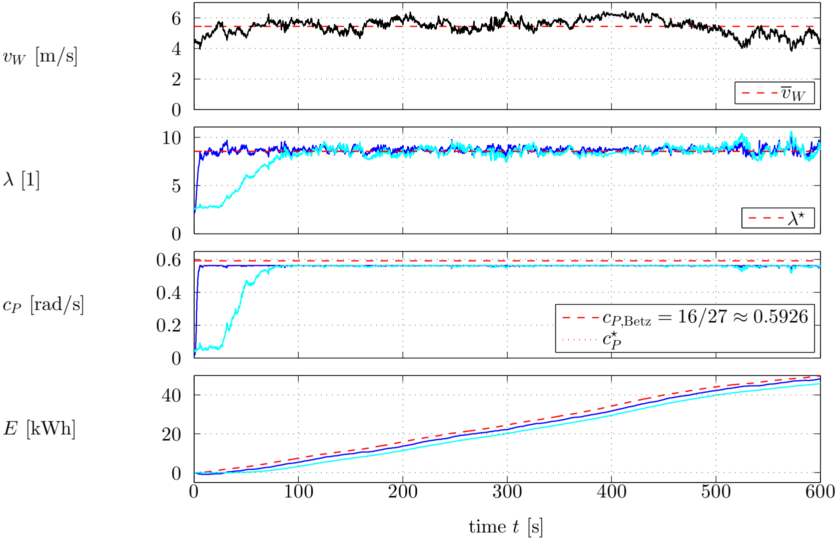
Speed control: Simulation and comparison



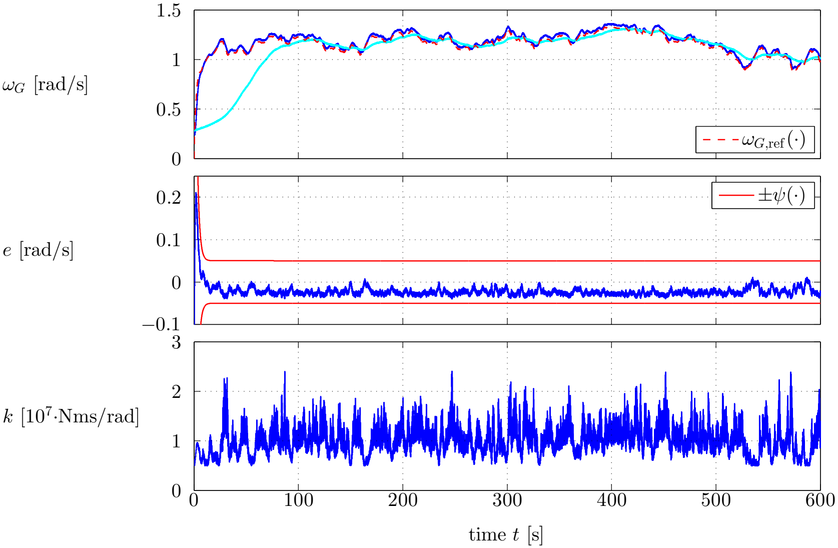
Implementation of funnel controller (FC) and classical controller (CC):

- FC: $m_{G,\mathrm{ref}}(t) = -\frac{\varsigma(t)}{\psi(t)-|e(t)|}\big(\omega_{G,\mathrm{ref}}(t) - \omega_G(t)\big)$ with $\omega_{G,\mathrm{ref}}(t) = \frac{g_r\lambda^\star}{r_T} v_W(t)$
- CC: $m_{G,\mathrm{ref}}(t) = k_P^\star \omega_G(t)^2$ with $k_P^\star := \frac{\varrho\pi {r_T}^5}{2{g_r}^3}\frac{c_P(\lambda^\star,\beta_0)}{(\lambda^\star)^3} \;\wedge\; \omega_{G,\mathrm{ref}}(t) = 0.$

Speed control: Simul. results (# 1) —— FC, —— CC



Speed control: Simul. results (# 2) —— FC, —— CC



Conclusion and outlook

To take home:

- funnel control applicable for speed control of wind turbine systems
- tracking with prescribed transient accuracy (on generator side)
- improved performance and power output compared to classical controller
- more robust to torque/parameter deviations than classical controller
- wind measurement required (drawback!)

Open questions:

- ... impact of actuator saturation?
  → theoretical results exist [5].
- ... damping of oscillations due to elasticity in shaft (non-stiff mechanics)?
  → use of disturbance observer possible [2]
- ... use of internal models beneficial (wind predictions)?
  → theoretical and experimental results for electrical drives exist [3, 4].

References

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[8] L. Y. Pao and K. E. Johnson. Control of wind turbines: Approaches, challenges, and recent developments. *IEEE Control Systems Magazine*, 31(2):44–62, 2011.

Implementation data

description	symbols & values [unit]
wind turbine (5) (without pitch control)	$\Theta = 9.9 \cdot 10^6 \left[\mathrm{kgm}^2\right]$, $g_r = 1\ [1]$, $r_T = 40\ [\mathrm{m}]$, $\varrho = 1.293 \left[\mathrm{kg/m}^3\right]$ $c_P(\lambda) = c_{P,1}(\lambda)$ as in (3) & see next table.
friction	$\mathfrak{F}\omega_G = \mathfrak{F}^\star\omega_G + \nu_F\omega_G\ [\mathrm{Nm}]$ with $\mathfrak{F}^\star$ as in [1, Sec. 1.4.5] (bounded), $\nu_F = 0.5\ [\mathrm{Nms/rad}]$ (viscous coeff.)
wind initial value	$v_W(\cdot)$ from FINO project (see simulation results part I) $\omega_G^0 = 0.3\omega_G^\star \Rightarrow e(0) = 0.7\omega_G^\star$
funnel controller as in (8)	$\psi(\cdot) = (\Lambda - \mu)\exp\left(-t/T_E\right) + \mu$ with $\Lambda = 1\ [\mathrm{rad/s}]$, $\mu = 0.05\ [\mathrm{rad/s}]$, $T_E = 2\ [\mathrm{s}]$, $\varsigma(t) = 0.5 \cdot 10^7\psi(t)\ [\mathrm{rad/s}]^{11}$
controller (9)	$k_P^\star = 1.89 \cdot 10^5 \left[\mathrm{Nms}^2/(\mathrm{rad})^2\right]$

Gain scaling $\varsigma(\cdot)$ is chosen, such that $k(t) = \varsigma(t)/(\psi(t) - \lvert e(t)\rvert) \geqslant 1$ for all $t \geqslant 0$.

Power coefficient approximations

	$c_{P,1}$ (w/o pitch control)	$c_{P,2}$
c_1	0	0.73
c_2	46.6	151
c_3	0	0.58
c_4	0	0.002
c_5	2.0	13.2
c_6	15.6	18.4
$f(\beta,\lambda)$	$\frac{1}{\lambda} - 0.01$	$\frac{1}{\lambda - 0{,}02\beta} - \frac{0.003}{\beta^3+1}$
x	–	2.14