Path-following funnel control for rigid-link revolute-joint robotic systems

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Outline

Position control of industrial robots

Kuka KR 150-2 (Serie 2000)
(http://www.kuka-robotics.com)

Applications

- (spot-)welding
- painting/enameling
- mounting/assembling
- laser-beam cutting
- ...

Goals:

- precise position control (of end effector), i.e. for given $\lambda > 0$:

$$\boxed{\forall\, t \geqslant t_0: \qquad \|\boldsymbol{e}_E(t)\| = \left\|\boldsymbol{y}_{\mathrm{ref,E}}(t) - \boldsymbol{y}_E(t)\right\| \leqslant \lambda.}$$

- controller: simple and easy to tune

Rigid robotic manipulator with n revolute joints

Robot model (in joint space)

$$\left.\begin{aligned} \boldsymbol{M}(\boldsymbol{y}(t))\ddot{\boldsymbol{y}}(t) + \boldsymbol{C}(\boldsymbol{y}(t),\dot{\boldsymbol{y}}(t))\dot{\boldsymbol{y}}(t) + (\mathfrak{F}\dot{\boldsymbol{y}})(t) + \boldsymbol{g}(\boldsymbol{y}(t)) + \boldsymbol{d}(t) = \boldsymbol{u}(t),\\ (\boldsymbol{y}(0),\, \dot{\boldsymbol{y}}(0))^\top = (\boldsymbol{y}_0,\, \boldsymbol{y}_1)^\top \in \mathbb{R}^{2n} \end{aligned}\right\} \tag{RO}$$

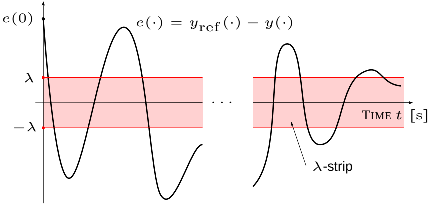
- standard assumptions
  - $\exists\, \overline{c}_{\boldsymbol{M}}, \underline{c}_{\boldsymbol{M}} > 0\ \forall\, \boldsymbol{y} \in \mathbb{R}^n: \quad 0 < \underline{c}_{\boldsymbol{M}} \boldsymbol{I}_n \leqslant \boldsymbol{M}(\boldsymbol{y}) = \boldsymbol{M}(\boldsymbol{y})^\top \leqslant \overline{c}_{\boldsymbol{M}} \boldsymbol{I}_n$
  - $\exists\, c_{\boldsymbol{C}} > 0\ \forall\, \boldsymbol{y}, \boldsymbol{v} \in \mathbb{R}^n: \quad \|\boldsymbol{C}(\boldsymbol{y},\boldsymbol{v})\boldsymbol{v}\| \leqslant c_{\boldsymbol{C}} \|\boldsymbol{v}\|^2$
  - $\mathfrak{F}\colon \mathcal{C}(\mathbb{R}_{\geqslant 0}; \mathbb{R}) \to \mathcal{L}^\infty_{\mathrm{loc}}(\mathbb{R}_{\geqslant 0}; \mathbb{R})$ (causal friction operator)
  - $\exists\, c_{\boldsymbol{g}} > 0\ \forall\, \boldsymbol{y} \in \mathbb{R}^n: \quad \|\boldsymbol{g}(\boldsymbol{y})\| \leqslant c_{\boldsymbol{g}}$
  - $\boldsymbol{d}(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geqslant 0}; \mathbb{R}^n)$ (disturbance)
  - $\boldsymbol{u}(\cdot) \in \mathcal{C}(\mathbb{R}_{\geqslant 0}; \mathbb{R}^n)$ (control input)
  - feedback signals: $\boldsymbol{y}(\cdot)$ *and* $\dot{\boldsymbol{y}}(\cdot)$
- $\Longrightarrow$ 'structural properties'
  - (strict) relative degree $r = 2$
  - positive (definite) high-frequency gain
  - no internal dynamics (minimum-phase)
  - derivative feedback admissible

High-gain adaptive control

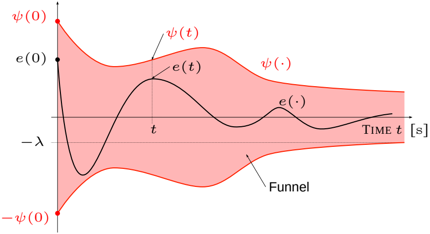
Promising advantages

- ‘structural system knowledge’ sufficient (robustness)
  - known relative degree $r \geqslant 1$
  - positive high-frequency gain (or known sign)
  - minimum-phase
- high feedback gains reduce ‘stick-slip’
- reference tracking with
  - ‘prescribed asymptotic accuracy’ (see e.g. [1, 2]) and
  - ‘prescribed transient accuracy’ (see e.g. [3–5])

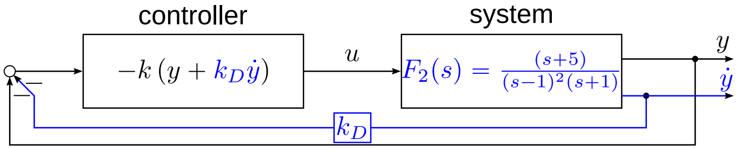
Adaptive λ-tracking control



Funnel control



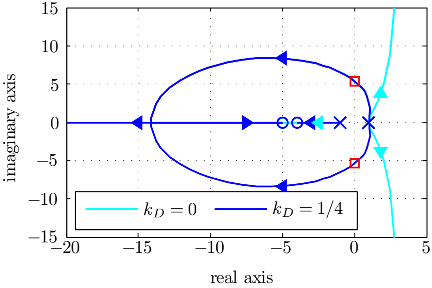
Relative degree two, high gains & derivative feedback



'Structural properties' of $F_2(s)$

- relative degree (pole excess): $r = 2$
- positive high-frequency gain ($\lim_{s\to\infty} s^2 F_2(s) = 1$)
- minimum-phase (Hurwitz numerator)

Root locus ($\times$ pole, $\circ$ zero)

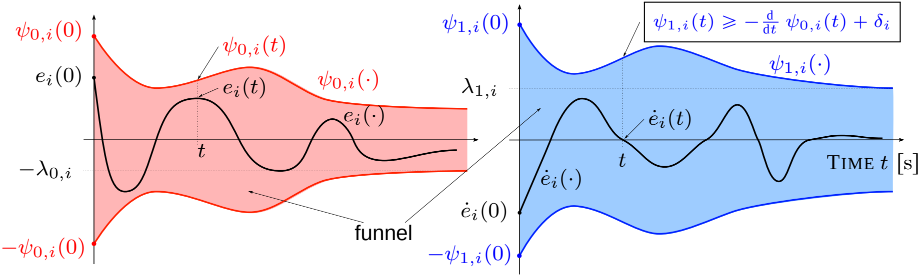


$\exists\, k^\star > 0$, such that closed-loop is stable for $k_D > 0$ and all $k > k^\star$

Control objective

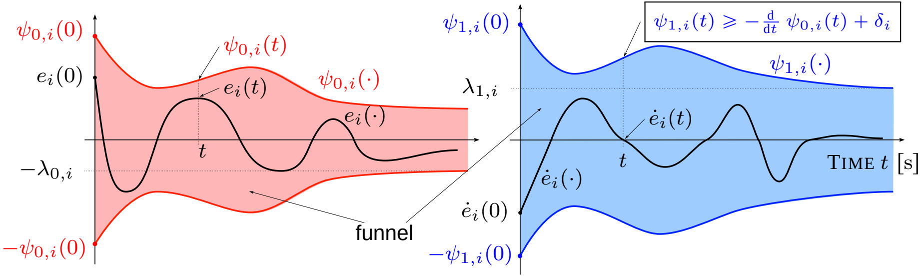
- reference tracking with '**prescribed transient accuracy**' for **each joint** $i \in \{1, \ldots, n\}$:

$$\boxed{\forall\, t \geqslant 0: \quad |e_i(t)| = |y_{\mathrm{ref,i}}(t) - y_i(t)| < \psi_{0,i}(t) \quad \text{and} \quad |\dot{e}_i(t)| < \psi_{1,i}(t)}$$



- joint reference $y_{\mathrm{ref,i}}(\cdot) \in \mathcal{C}^1(\mathbb{R}_{\geqslant 0}; \mathbb{R})$, $y_{\mathrm{ref,i}}(\cdot), \dot{y}_{\mathrm{ref,i}}(\cdot), \ddot{y}_{\mathrm{ref,i}}(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geqslant 0}; \mathbb{R})$
- joint funnel boundary $(\psi_{0,i}(\cdot),\ \psi_{1,i}(\cdot)) \in \mathcal{C}^1(\mathbb{R}_{\geqslant 0}; [\lambda, \infty)^2)$, $\lambda > 0$

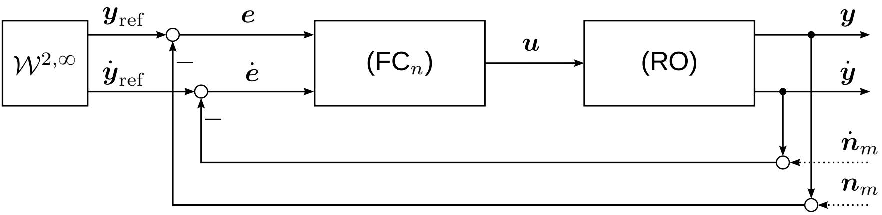
MIMO funnel controller (extension of [6, 7])



$$\boxed{\boldsymbol{u}(t) = \boldsymbol{M}(\boldsymbol{y}(t))\Big(\underbrace{\boldsymbol{K}_0(t)^2\boldsymbol{e}(t)}_{\textbf{P}} + \underbrace{\boldsymbol{K}_0(t)\boldsymbol{K}_1(t)\dot{\boldsymbol{e}}(t)}_{\textbf{D}}\Big)} \qquad (\mathrm{FC}_n)$$

- $\boldsymbol{e}(t) = \big(e_1(t), \quad \ldots, \quad e_n(t)\big)^\top = \big(y_{\mathrm{ref},1}(t) - y_1(t), \quad \ldots, \quad y_{\mathrm{ref,n}}(t) - y_n(t)\big)^\top$
- $\boldsymbol{K}_0(t) = \mathrm{diag}\left\{k_{0,1}(t), \ldots, k_{0,n}(t)\right\}$ & $\boldsymbol{K}_1(t) = \mathrm{diag}\left\{k_{1,1}(t), \ldots, k_{1,n}(t)\right\}$
- $\forall\, i \in \{1, \ldots, n\}: \quad k_{0,i}(t) = \dfrac{1}{\psi_{0,i}(t) - |e_i(t)|}$ and $k_{1,i}(t) = \dfrac{1}{\psi_{1,i}(t) - |\dot{e}_i(t)|}$

Properties of closed-loop system [8, Theorem 3.1]

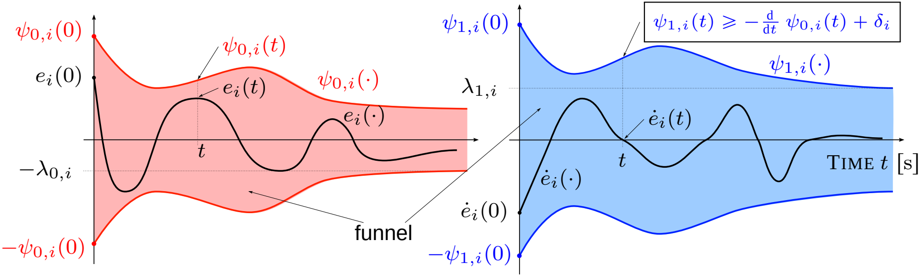


- (RO):$\boldsymbol{M}(\boldsymbol{y}(t))\ddot{\boldsymbol{y}}(t) + \boldsymbol{C}(\boldsymbol{y}(t), \dot{\boldsymbol{y}}(t))\dot{\boldsymbol{y}}(t) + (\mathfrak{F}\dot{\boldsymbol{y}})(t) + \boldsymbol{g}(\boldsymbol{y}(t)) + \boldsymbol{d}(t) = \boldsymbol{u}(t)$
- (FC$_n$): $\boldsymbol{u}(t) = \boldsymbol{M}(\boldsymbol{y}(t))\left(\boldsymbol{K}_0(t)^2\boldsymbol{e}(t) + \boldsymbol{K}_0(t)\boldsymbol{K}_1(t)\dot{\boldsymbol{e}}(t)\right)$
- For **known** $\boldsymbol{M}(\boldsymbol{y})$:
  - $\boldsymbol{n}_m(\cdot) \in \mathcal{W}^{2,\infty}(\mathbb{R}_{\geqslant 0}; \mathbb{R}^n)$ (measurement errors/noise admissible)
  - $\forall i \in \{1, \ldots, n\}\ \exists \varepsilon_i > 0\ \forall\, t \geqslant 0\colon\ \psi_{0,i}(t) - |e_i(t)| \geqslant \varepsilon_i$ and $\psi_{1,i}(t) - |\dot{e}_i(t)| \geqslant \varepsilon_i$
  - $\boldsymbol{K}_0(\cdot), \boldsymbol{K}_1(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geqslant 0}; \mathbb{R}^n_{>0})$
  - $\boldsymbol{u}(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geqslant 0}; \mathbb{R}^n)$

Control objective

- **path-following** with '**prescribed transient accuracy**' for **each joint** $i \in \{1, \ldots, n\}$:

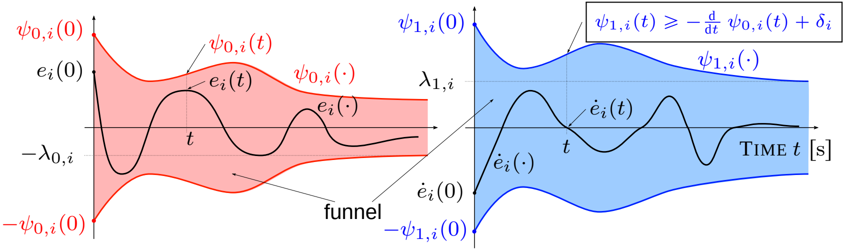
$$\boxed{\forall\, t \geqslant 0: \quad |e_i(t)| = |p_i(\theta(t)) - y_i(t)| < \psi_{0,i}(t) \quad \text{and} \quad |\dot{e}_i(t)| < \psi_{1,i}(t)}$$



- path: $\boldsymbol{p}(\cdot) \in \mathcal{C}^2(\mathbb{R};\mathbb{R}^n)$ and $\boldsymbol{p}(\cdot),\ \frac{\mathrm{d}}{\mathrm{d}\theta}\,\boldsymbol{p}(\cdot),\ \frac{\mathrm{d}^2}{\mathrm{d}\theta^2}\,\boldsymbol{p}(\cdot) \in \mathcal{L}^\infty(\mathbb{R};\mathbb{R}^n)$
- timing dynamics with virtual input v:

$$\boxed{\frac{\mathrm{d}^2}{\mathrm{d}t^2}\,\theta(t) = v(t), \qquad (\theta(0),\ \dot{\theta}(0))^\top = (\theta_0,\ \theta_1)^\top \in \mathbb{R}^2} \qquad \text{(TD)}$$

MIMO path-following funnel controller



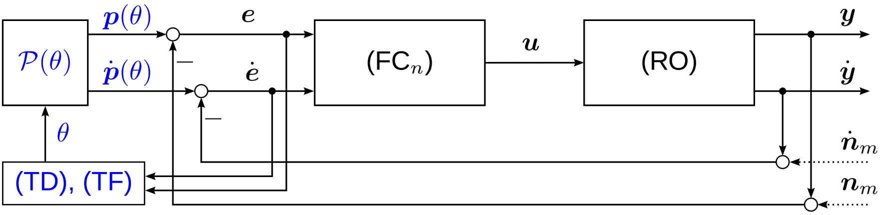
- funnel controller

$$\boxed{\boldsymbol{u}(t) = \boldsymbol{M}(\boldsymbol{y}(t))\Big(\underbrace{\boldsymbol{K}_0(t)^2\boldsymbol{e}(t)}_{\textbf{P}} + \underbrace{\boldsymbol{K}_0(t)\boldsymbol{K}_1(t)\dot{\boldsymbol{e}}(t)}_{\textbf{D}}\Big)} \qquad (\text{FC}_n)$$

- timing feedback (virtual input v)

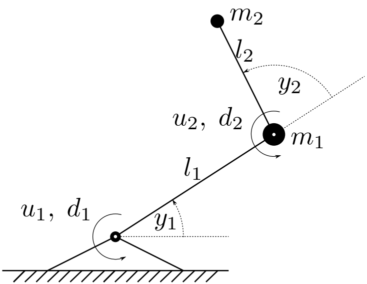
$$\boxed{v(t) = -\boldsymbol{\kappa}\big(\boldsymbol{e}(t), \dot{\boldsymbol{e}}(t)\big)^\top \begin{pmatrix}\theta(t)\\ \dot{\theta}(t)\end{pmatrix}, \; \boldsymbol{\kappa}(\cdot,\cdot) = \begin{pmatrix}\kappa_1(\cdot,\cdot)\\ \kappa_2(\cdot,\cdot)\end{pmatrix} \in \mathcal{C}^1(\mathbb{R}^n \times \mathbb{R}^n; \mathbb{R}^2)} \qquad \text{(TF)}$$

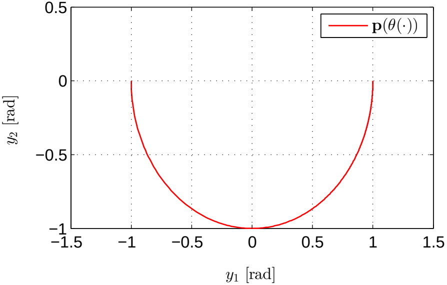
Properties of closed-loop system [Theorem 2]



- (RO):$\boldsymbol{M}(\boldsymbol{y}(t))\ddot{\boldsymbol{y}}(t) + \boldsymbol{C}(\boldsymbol{y}(t), \dot{\boldsymbol{y}}(t))\dot{\boldsymbol{y}}(t) + (\mathfrak{F}\dot{\boldsymbol{y}})(t) + \boldsymbol{g}(\boldsymbol{y}(t)) + \boldsymbol{d}(t) = \boldsymbol{u}(t)$
- (FC$_n$): $\boldsymbol{u}(t) = \boldsymbol{M}(\boldsymbol{y}(t))\left(\boldsymbol{K}_0(t)^2\boldsymbol{e}(t) + \boldsymbol{K}_0(t)\boldsymbol{K}_1(t)\dot{\boldsymbol{e}}(t)\right)$
- (TD), (TF): $\ddot{\theta}(t) = -\boldsymbol{\kappa}\big(\boldsymbol{e}(t),\, \dot{\boldsymbol{e}}(t)\big)^\top \begin{pmatrix}\theta(t)\\ \dot{\theta}(t)\end{pmatrix},\ \boldsymbol{\kappa}(\cdot,\cdot) \in \mathcal{C}^1(\mathbb{R}^n \times \mathbb{R}^n; \mathbb{R}^2)$
- For known $\boldsymbol{M}(\boldsymbol{y})$ and $\exists\, 0 < \underline{\kappa} \leqslant \overline{\kappa}\colon\ \underline{\kappa} \leqslant \kappa_1(\boldsymbol{\alpha}, \boldsymbol{\beta}) \leqslant \kappa_2(\boldsymbol{\alpha}, \boldsymbol{\beta}) \leqslant \overline{\kappa}$ and $1 + \underline{\kappa} \leqslant \kappa_2(\boldsymbol{\alpha}, \boldsymbol{\beta})$ for all $\boldsymbol{\alpha}, \boldsymbol{\beta} \in \mathbb{R}^n$ then
  - $\boldsymbol{n}_m(\cdot) \in \mathcal{W}^{2,\infty}(\mathbb{R}_{\geqslant 0}; \mathbb{R}^n)$ (measurement errors admissible)
  - $\forall i \in \{1, \ldots, n\}\ \exists \varepsilon_i > 0\ \forall\, t \geqslant 0\colon\ \psi_{0,i}(t) - |e_i(t)| \geqslant \varepsilon_i$ and $\psi_{1,i}(t) - |\dot{e}_i(t)| \geqslant \varepsilon_i$
  - $\boldsymbol{K}_0(\cdot), \boldsymbol{K}_1(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geqslant 0}; \mathbb{R}^n_{>0})$ and $\boldsymbol{u}(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geqslant 0}; \mathbb{R}^n)$
  - $\lim_{t\to\infty} \theta(t) = 0$ (timing dynamics are asymptotically stable [9, Thm. 2.15])

Example: Planar robot and path

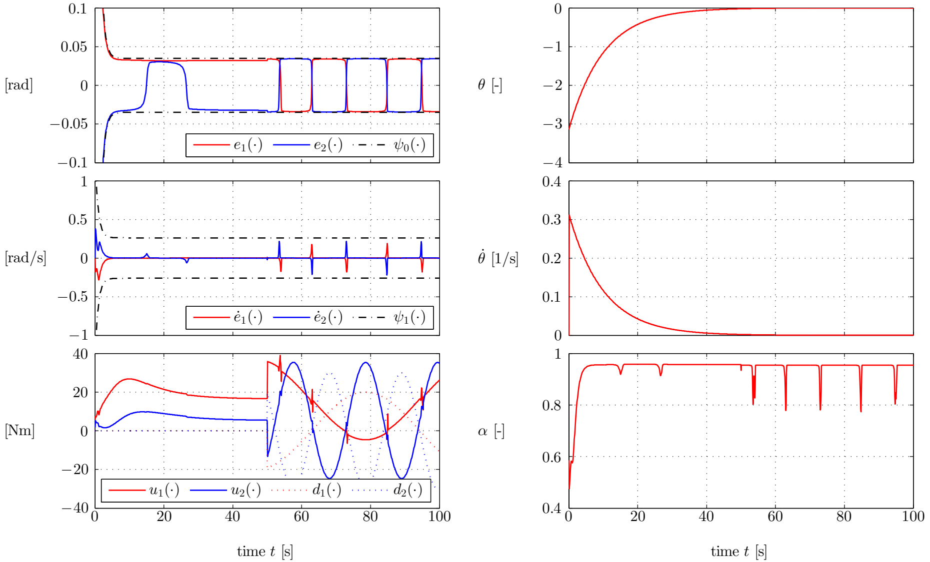




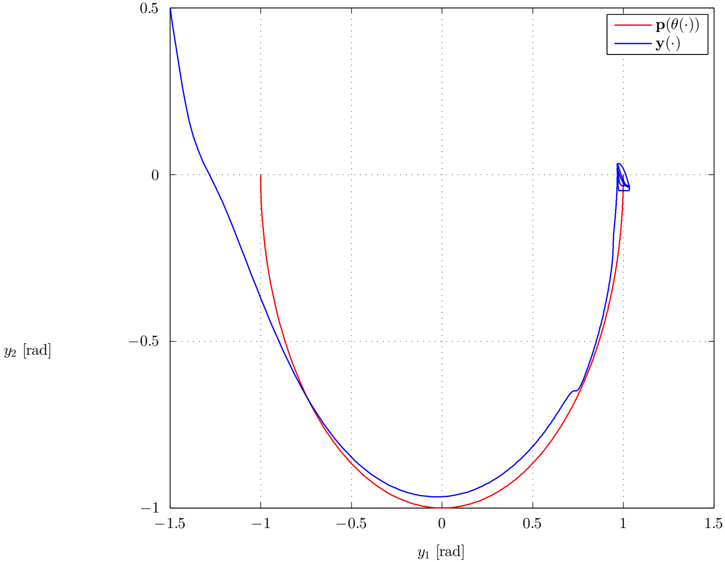
- $\boldsymbol{u}(t) = \boldsymbol{M}(\boldsymbol{y}(t))\left(\underbrace{\begin{bmatrix} k_{0,1}(t)^2 & 0 \\ 0 & k_{0,2}(t)^2 \end{bmatrix}\boldsymbol{e}(t)}_{\mathbf{P}} + \underbrace{\begin{bmatrix} k_{0,1}(t)k_{1,1}(t) & 0 \\ 0 & k_{0,2}(t)k_{1,2}(t) \end{bmatrix}\dot{\boldsymbol{e}}(t)}_{\mathbf{D}}\right)$
- $v(t) = -\alpha(\boldsymbol{e}(t),\, \dot{\boldsymbol{e}}(t))\boldsymbol{\kappa}^\top \begin{pmatrix} \theta(t) \\ \dot\theta(t) \end{pmatrix}$ with $\boldsymbol{\kappa} = (10,\, 100)^\top$ and

$$0 < \alpha_{\min} \leqslant \alpha(\boldsymbol{e},\, \dot{\boldsymbol{e}}) = \frac{\alpha_{\max} + \alpha_{\min}\big(\|\boldsymbol{e}\| + \|\dot{\boldsymbol{e}}\|\big)}{\|\boldsymbol{e}\| + \|\dot{\boldsymbol{e}}\| + 1} \leqslant \alpha_{\max}$$

Simulation results: Errors, control & timing dynamics



Simulation results: Path and trajectory



Summary and future work

To take home:

- position funnel control: simple but time-varying PD-controller
- measurement noise admissible
- **no** compensation of
  - friction
  - disturbances (unknown but bounded)
  - Coriolis and/or gravity effects
- however: inertia matrix must be **known**
- path-following funnel control with prescribed path accuracy
- timing dynamics with virtual input (one degree of freedom more)

Open questions

- derivation of feasibility condition (actuator saturation)?
- utilization of timing dynamics to assure feasible control actions?
- parametrization of funnel boundary by timing variables?
- path-following in finite time?

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