DC-link control: Understanding the nontrivial task

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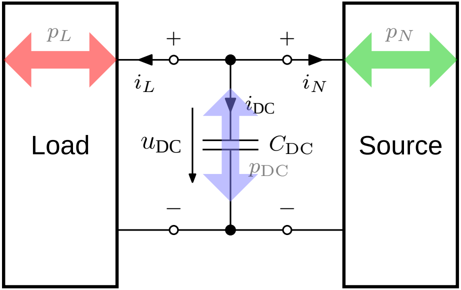
Outline

Motivation: DC-link control



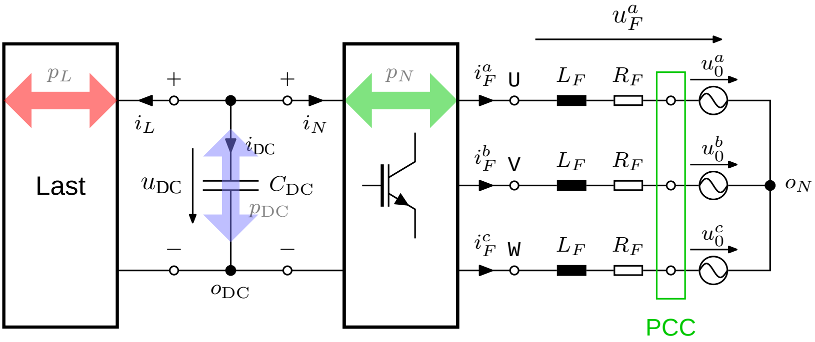
Power balance

$$p_{\mathrm{DC}} = u_{\mathrm{DC}} C_{\mathrm{DC}} \frac{\mathrm{d}}{\mathrm{d}t} u_{\mathrm{DC}} = -p_L - p_N \quad \Longrightarrow \quad \boxed{\frac{\mathrm{d}}{\mathrm{d}t} u_{\mathrm{DC}} = \frac{1}{u_{\mathrm{DC}} C_{\mathrm{DC}}} \big(-p_L - p_N \big)}$$

such that

$$\boxed{u_{\mathrm{DC}} = u_{\mathrm{DC,ref}} > 0}$$

Modeling: Grid side electrical network



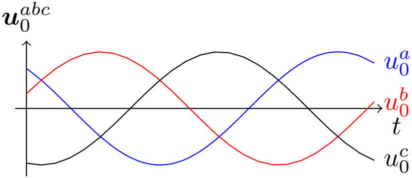
Electrical network

$$\boldsymbol{u}_F^{abc} = \left(u_F^a, \quad u_F^b, \quad u_F^c\right)^\top = R_F \boldsymbol{i}_F^{abc} + L_F \frac{\mathrm{d}}{\mathrm{d}t} \boldsymbol{i}_F^{abc} + \boldsymbol{u}_0^{abc}$$

Power (lossless converter)

$$p_N = i_N u_{DC} = u_F^a i_F^a + u_F^b i_F^b + u_F^c i_F^c = \left(\boldsymbol{u}_F^{abc}\right)^\top \boldsymbol{i}_F^{abc}$$

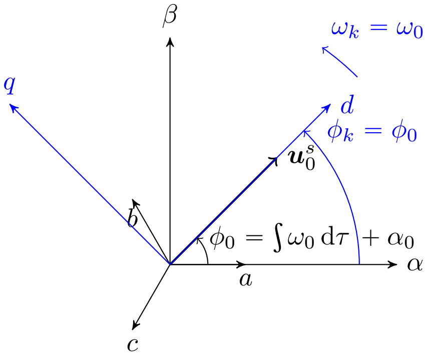
Modeling: Grid and voltage orientation



$$\boldsymbol{u}_0^{abc} = (u_0^a,\, u_0^b,\, u_0^c)^\top = \hat{u}_0 \begin{pmatrix} \cos(\omega_0 t + \alpha_0) \\ \cos(\omega_0 t - 2/3\pi + \alpha_0) \\ \cos(\omega_0 t - 4/3\pi + \alpha_0) \end{pmatrix}$$

where

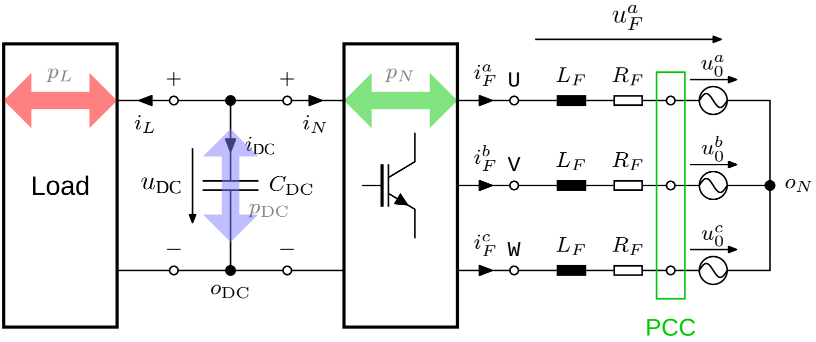
- $\hat{u}_0 = \sqrt{2} \cdot 230$ [V]
- $\omega_0 = 2\pi \cdot 50$ [rad/s]
- (here: $\alpha_0 = \pi/4$ [rad])



$$\|\boldsymbol{u}_0^s\| = \hat{u}_0 \quad \text{and} \quad \phi_0 = \operatorname{atan2}\begin{pmatrix} u_0^\alpha \\ \overline{u_0^\beta} \end{pmatrix}$$

$$\Longrightarrow \quad \boxed{u_0^d = \hat{u}_0 \quad \text{and} \quad u_0^q = 0}$$

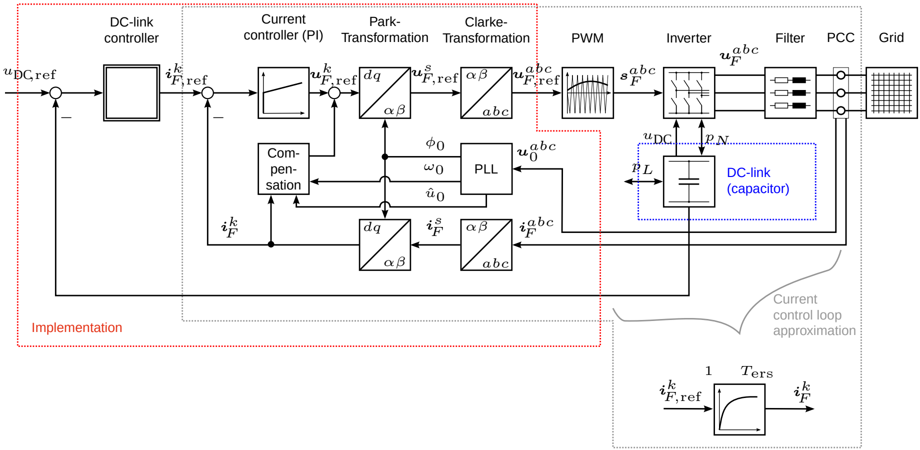
Modeling: Electrical network in voltage orientation



$$\boldsymbol{u}_F^k = R_F \boldsymbol{i}_F^k + L_F \frac{\mathrm{d}}{\mathrm{d}t} \boldsymbol{i}_F^k + \omega_0 L_F \begin{pmatrix} -i_F^q \\ i_F^d \end{pmatrix} + \begin{pmatrix} \hat{u}_0 \\ 0 \end{pmatrix}$$

$$p_N = \underbrace{\frac{3}{2} R_F \left\| \boldsymbol{i}_F^k \right\|^2}_{=: p_{V,R_F}} + \underbrace{\frac{3}{2} L_F \left(\frac{\mathrm{d}}{\mathrm{d}t} \boldsymbol{i}_F^k\right)^\top \boldsymbol{i}_F^k}_{=: p_{V,L_F}} + \underbrace{\frac{3}{2} \hat{u}_0 i_F^d}_{\substack{=: P_{\mathrm{PCC}} \\ \text{active power}}} \quad \& \quad \underbrace{Q_{\mathrm{PCC}} = -\frac{3}{2} \hat{u}_0 i_F^q}_{\text{reactive power}}$$

Overview: DC-link control



Linearization at equilibrium $(u_{\mathrm{DC}}^\star, i_F^{d,\star}, i_F^{q,\star})$

$$\frac{\mathrm{d}}{\mathrm{d}t} u_{\mathrm{DC}} = \frac{1}{C_{\mathrm{DC}} u_{\mathrm{DC}}} \left[-p_L - \frac{3}{2} \left(R_F \left\| \boldsymbol{i}_F^k \right\|^2 + L_F \left(\frac{\mathrm{d}}{\mathrm{d}t} \boldsymbol{i}_F^k \right)^\top \boldsymbol{i}_F^k + \hat{u}_0 i_F^d \right) \right]$$

$$\Longrightarrow \frac{\mathrm{d}}{\mathrm{d}t} \tilde{u}_{\mathrm{DC}} = -\frac{1}{C_{\mathrm{DC}} u_{\mathrm{DC}}^\star} \Bigg[\tilde{p}_L + \frac{3}{2} \left(\left(\hat{u}_0 + 2R_F i_F^{d,\star} \right) \tilde{i}_F^d + L_F i_F^{d,\star} \frac{\mathrm{d}}{\mathrm{d}t} \tilde{i}_F^d \right) + \frac{3}{2} \left(2R_F i_F^{q,\star} \tilde{i}_F^q + L_F i_F^{d,\star} \frac{\mathrm{d}}{\mathrm{d}t} \tilde{i}_F^q \right) \Bigg]$$

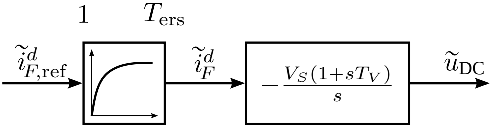
$$\circ\!\!-\!\!\bullet\ s\tilde{u}_{\mathrm{DC}}(s) = -\frac{1}{C_{\mathrm{DC}} u_{\mathrm{DC}}^\star} \Bigg[\tilde{p}_L(s) + \frac{3}{2} \left(\left(\hat{u}_0 + 2R_F i_F^{d,\star} \right) + sL_F i_F^{d,\star} \right) \tilde{i}_F^d(s) + \frac{3}{2} \left(2R_F + sL_F \right) i_F^{q,\star} \tilde{i}_F^q(s) \Bigg]$$

Transfer function:

$$\Longrightarrow \boxed{\frac{\tilde{u}_{\mathrm{DC}}(s)}{\tilde{i}_F^d(s)} = -V_S \frac{1 + T_V s}{s}} \quad \text{where } T_V := \frac{L_F i_F^{d,\star}}{\hat{u}_0 + 2R_F i_F^{d,\star}}, \quad V_S := \frac{3}{2} \frac{\hat{u}_0 + 2R_F i_F^{d,\star}}{C_{\mathrm{DC}} u_{\mathrm{DC}}^\star}$$

Structurally varying system

Current control loop | Linearized system F_S



(S$_1$) $i_F^{d,\star} = 0 \implies T_V = 0 \implies \dfrac{u_{\text{DC}}(s)}{i_{F,\text{ref}}^{d}(s)} = \dfrac{V_S}{s(1+sT_{\text{ers}})}$ (IT$_1$)

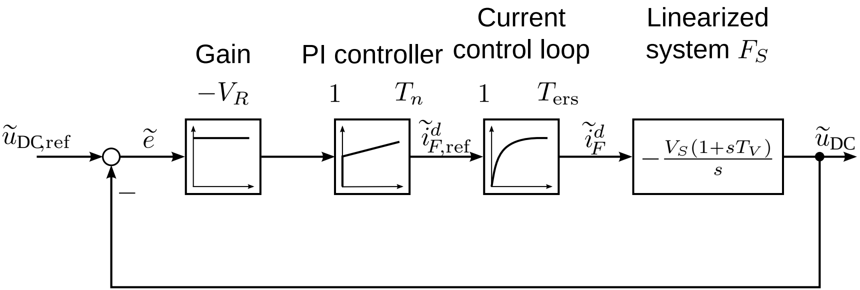
(S$_2$) $i_F^{d,\star} > 0 \implies T_V > 0 \implies \dfrac{u_{\text{DC}}(s)}{i_{F,\text{ref}}^{d}(s)} = \dfrac{V_S(1+sT_V)}{s(1+sT_{\text{ers}})}$

(IT$_1$ + PD, minimum-phase)

(S$_3$) $i_F^{d,\star} < 0 \implies T_V < 0 \implies \dfrac{u_{\text{DC}}(s)}{i_{F,\text{ref}}^{d}(s)} = \dfrac{V_S(1-s|T_V|)}{s(1+sT_{\text{ers}})}$

(IT$_1$ + PD, **non** minimum-phase)

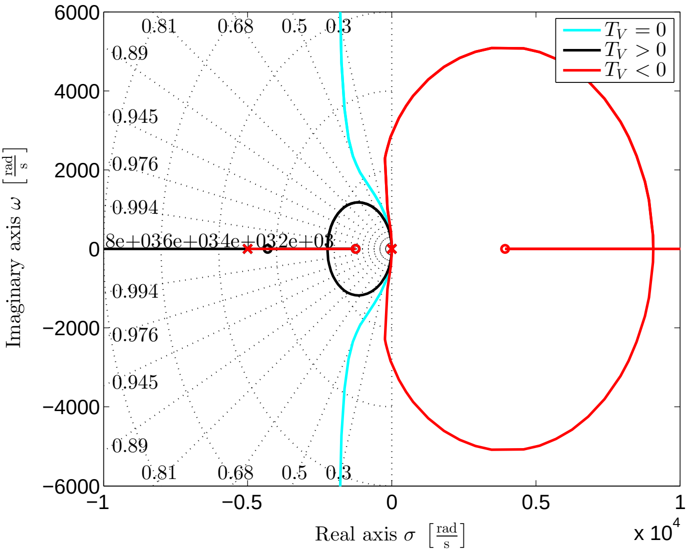
Root locus of linearized control system



Open loop

$$\frac{\widetilde{u}_{\mathrm{DC}}(s)}{\widetilde{e}(s)} = \frac{V_R V_S(1+sT_n)(1+sT_V)}{s^2 T_n(1+sT_{\mathrm{ers}})}, \quad \begin{cases} T_{\mathrm{ers}}, T_n > 0,\ V_S > 0 \\ T_V \in \mathbb{R} \ \text{ (depends on equilibrium!)} \\ V_R \to \infty \ \text{ gain} \end{cases}$$

Root loci for different equilibria



Stability analysis (Hurwitz criterion)

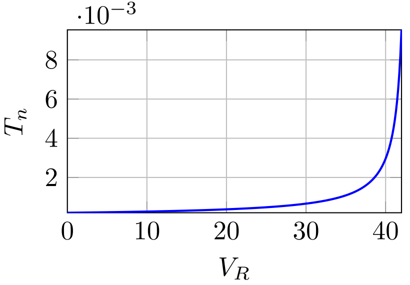
Closed-loop system

$$F_{RK}(s) = \frac{u_{\mathrm{DC}}(s)}{u_{\mathrm{DC,ref}}(s)} = \frac{V_R V_S (1 + sT_{\mathrm{ers}})(1 + sT_V)}{s^3 \underbrace{T_n T_{\mathrm{ers}}}_{=:a_3} + s^2 \underbrace{T_n(1 + T_V V_S V_R)}_{=:a_2} + s \underbrace{(T_V + T_n) V_S V_R}_{=:a_1} + \underbrace{V_R V_S}_{=:a_0}}$$

Two conditions (sufficient)

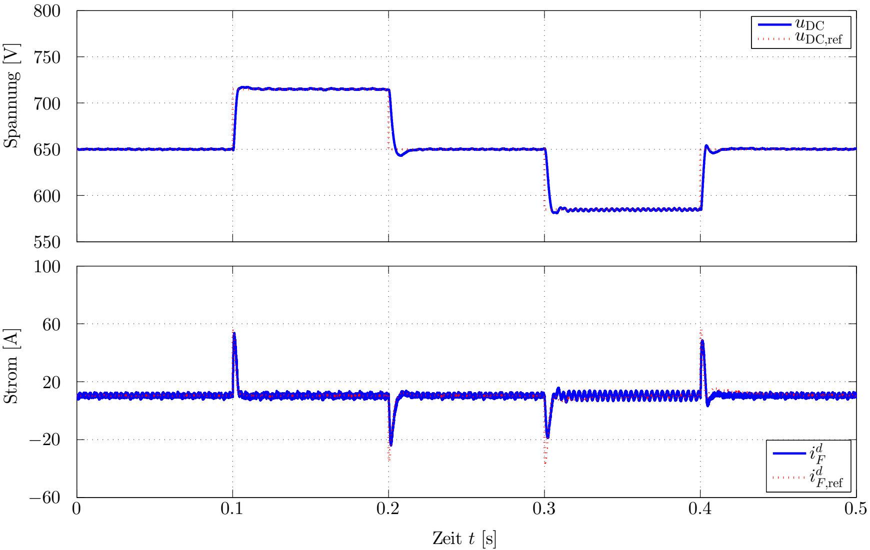
(B$_1$) $0 < V_R < \frac{1}{|T_V| V_S}$

(B$_2$) $T_n > \frac{T_{\mathrm{ers}}}{1 - V_R V_S |T_V|} + |T_V| > 0$

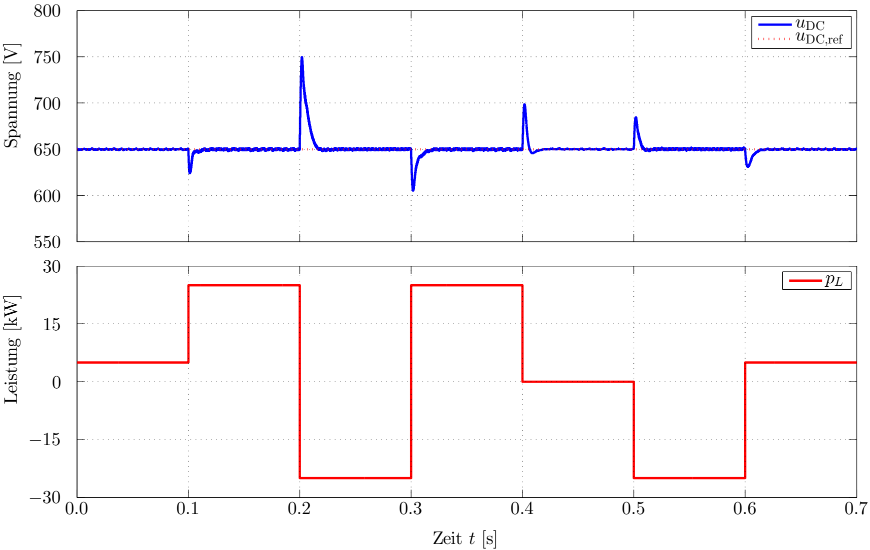


$\Longrightarrow$ **local stability**, since $a_0, a_1, a_2, a_3 > 0$ and $D_2 = a_2 a_1 - a_3 a_0 > 0$.

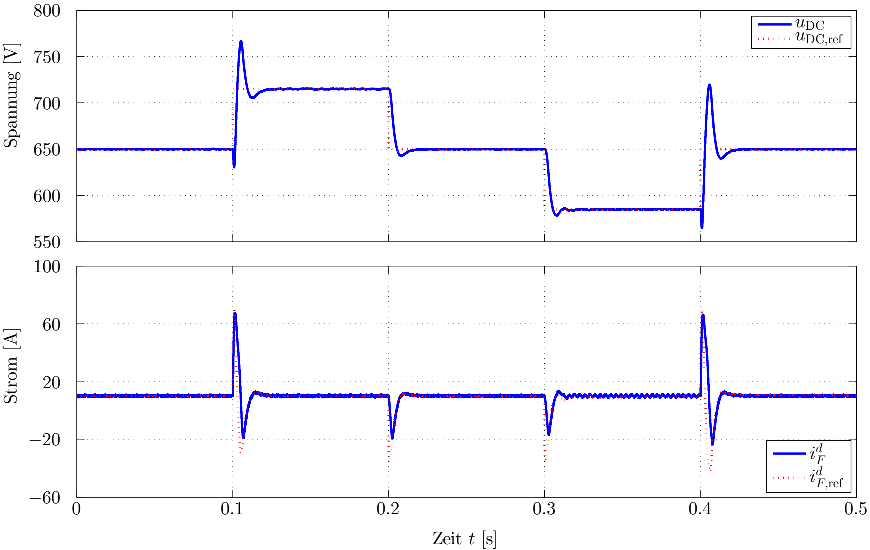
Simulation: Control performance ($L_F = L_0$)



Simulation: Disturbance rejection ($L_F = L_0$)



Simulation: Control performance ($L_F = 3L_0$)



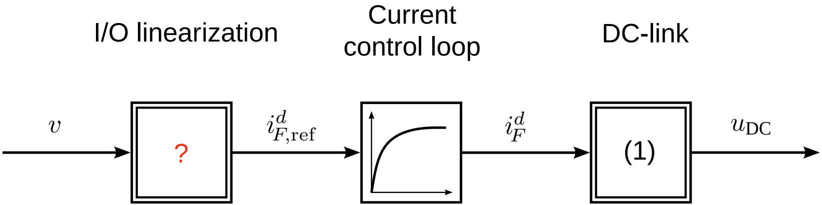
Basic idea

Nonlinear system

$$\frac{\mathrm{d}}{\mathrm{d}t}\, u_{\mathrm{DC}} = \frac{1}{C_{\mathrm{DC}} u_{\mathrm{DC}}} \left[-p_L - \overbrace{\frac{3}{2}\left(R_F \left\| \boldsymbol{i}_F^k \right\|^2 + L_F \left(\frac{\mathrm{d}}{\mathrm{d}t}\, \boldsymbol{i}_F^k \right)^\top \boldsymbol{i}_F^k + \hat{u}_0 i_F^d \right)}^{p_N} \right] \tag{1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\, \boldsymbol{i}_F^k = \frac{1}{T_{\mathrm{ers}}} \left(-\boldsymbol{i}_F^k + \boldsymbol{i}_{F,\mathrm{ref}}^k \right), \qquad \left(u_{\mathrm{DC}}(0), \boldsymbol{i}_F^k(0)^\top\right) = \left(u_{\mathrm{DC}}^0, (\boldsymbol{i}_F^{k,0})^\top\right) \tag{2}$$

$$y = u_{\mathrm{DC}}$$



Simplifying assumptions

For

(A_1) fast current controllers and small inductance,
$\Longrightarrow L_F \frac{\mathrm{d}}{\mathrm{d}t} \boldsymbol{i}_F^k = \frac{L_F}{T_{\mathrm{ers}}} \left(-\boldsymbol{i}_F^k + \boldsymbol{i}_{F,\mathrm{ref}}^k \right) \approx 0$

(A_2) no reactive power, i.e. $Q_{\mathrm{PPC,ref}} = 0 \Longrightarrow i_{F,\mathrm{ref}}^q = 0 \Longrightarrow i_F^q \approx 0$

(A_3) load power known, i.e. p_L is available as measurement.

then

$$\frac{\mathrm{d}}{\mathrm{d}t} u_{\mathrm{DC}} \approx \frac{1}{C_{\mathrm{DC}} u_{\mathrm{DC}}} \left[-p_L - \frac{3}{2} \left(R_F (i_F^d)^2 + \hat{u}_0 i_F^d \right) \right] \neq f(i_{F,\mathrm{ref}}^d) \tag{3}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} i_F^d = \frac{1}{T_{\mathrm{ers}}} \left(-i_F^d + i_{F,\mathrm{ref}}^d \right), \qquad (u_{\mathrm{DC}}(0), i_F^d(0)) = (u_{\mathrm{DC}}^0, i_F^{d,0}) \tag{4}$$

$$y = u_{\mathrm{DC}}$$

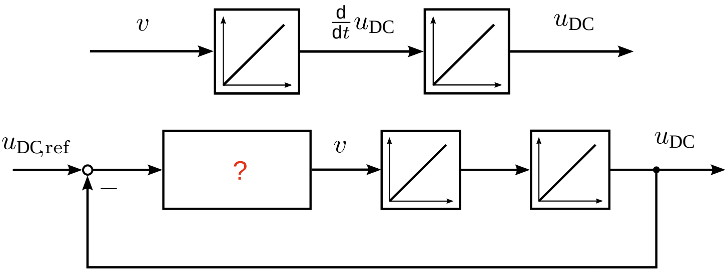
Input and linearization

$$
\begin{aligned}
\text{Input} \quad & \left(\dot y = \tfrac{\mathrm{d}}{\mathrm{d}t}\, u_{\mathrm{DC}} \neq f(i^d_{F,\mathrm{ref}})\right) \\
\ddot y &= \frac{\mathrm{d}^2}{\mathrm{d}t^2}\, u_{\mathrm{DC}} \\
&= -\frac{1}{C_{\mathrm{DC}}^2 u_{\mathrm{DC}}^3}\left[-p_L - \tfrac{3}{2}\left(R_F (i^d_F)^2 + \hat u_0 i^d_F\right)\right]^2 \\
&\quad + \frac{1}{C_{\mathrm{DC}} u_{\mathrm{DC}}}\left[-\dot p_L - \tfrac{3}{2}\left(2R_F(\tfrac{\mathrm{d}}{\mathrm{d}t} i^d_F) i^d_F + \hat u_0 \tfrac{\mathrm{d}}{\mathrm{d}t} i^d_F\right)\right] \\
&\overset{(4)}{=} -\frac{1}{C_{\mathrm{DC}}^2 u_{\mathrm{DC}}^3}\left[-p_L - \tfrac{3}{2}\left(R_F (i^d_F)^2 + \hat u_0 i^d_F\right)\right]^2 \\
&\quad + \frac{1}{C_{\mathrm{DC}} u_{\mathrm{DC}}}\Bigg[-\dot p_L - \tfrac{3}{2}\left(-2\frac{R_F}{T_{\mathrm{ers}}}(i^d_F)^2 + 2\frac{R_F}{T_{\mathrm{ers}}} i^d_F i^d_{F,\mathrm{ref}}\right) \\
&\qquad - \tfrac{3}{2}\frac{\hat u_0}{T_{\mathrm{ers}}}\left(-i^d_F + i^d_{F,\mathrm{ref}}\right)\Bigg] \overset{!}{=} v
\end{aligned}
$$

Linearization

$$
\begin{aligned}
\Longrightarrow\; i^d_{F,\mathrm{ref}} &= \frac{T_{\mathrm{ers}}}{2R_F i^d_F + \hat u_0}\Bigg\{-\tfrac{2}{3} C_{\mathrm{DC}} u_{\mathrm{DC}} v - \cancel{\tfrac{2}{3}\dot p_L} + \frac{2R_F i^d_F + \hat u_0}{T_{\mathrm{ers}}} i^d_F \\
&\qquad - \frac{2}{3C_{\mathrm{DC}} u_{\mathrm{DC}}}\left[-p_L - \tfrac{3}{2}\left(R_F (i^d_F)^2 + \hat u_0 i^d_F\right)\right]^2 \Bigg\}
\end{aligned}
$$

Virtual system (ideal) and controller design

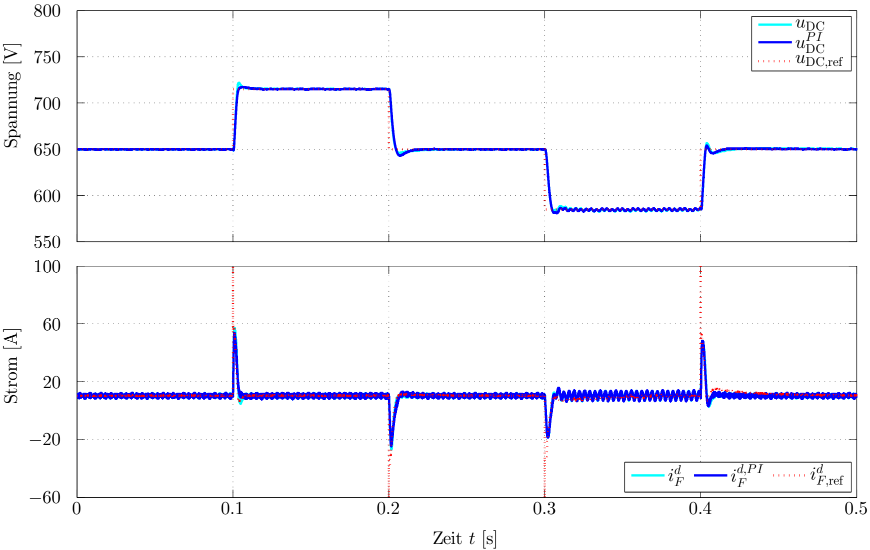


PID controller design

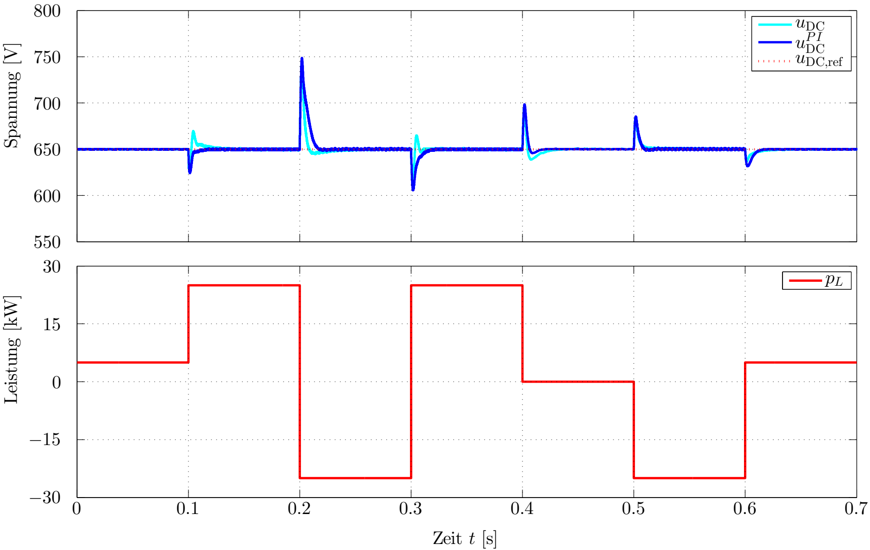
$$v(t) = k_P e(t) + k_D \dot{e}(t) + k_I \int_0^t e(\tau)\,\mathrm{d}\tau\,, \qquad e(t) = u_{\mathrm{DC,ref}}(t) - u_{\mathrm{DC}}(t)$$

$$\Longrightarrow \quad F_{RK}(s) = \frac{u_{\mathrm{DC}}(s)}{u_{\mathrm{DC,ref}}(s)} = \frac{s^2 k_D + s k_P + k_I}{s^3 + s^2 k_D + s k_P + k_I}$$

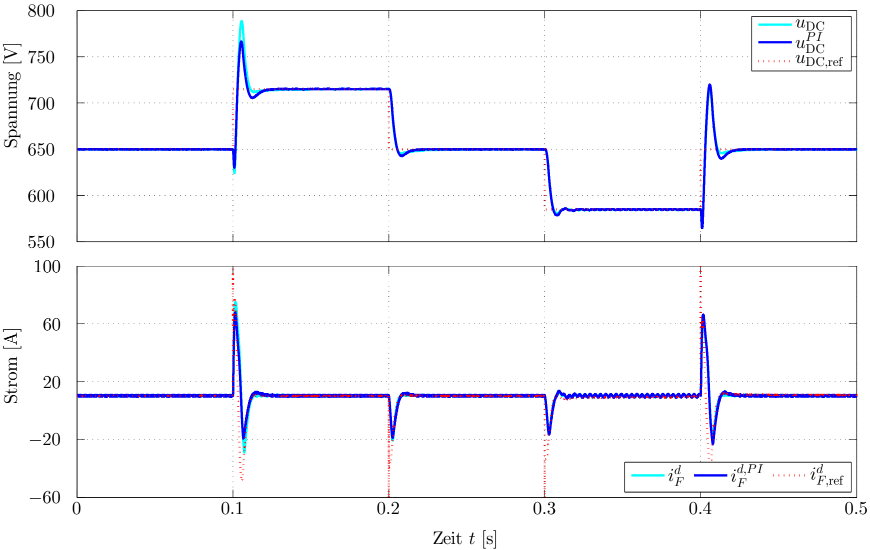
Simulation: Control performance ($L_F = L_0$)



Simulation: Disturbance rejection ($L_F = L_0$)



Simulation: Control performance ($L_F = 3L_0$)



Conclusion and future work

Conclusion:

- DC-link control problem is **not** trivial, since
  - system is nonlinear
  - system has varying structure (depends on equilibrium, e.g. **non** minimum-phase system possible!)
- PI controller and I/O-Linearization
  → comparable performance / local stability results
- I/O-Linearization: higher implementation effort (e.g. unknown load)

Future work:

- Disturbance observer (for p_L & $\dot{p}_L$)
  - ... reasonable for I/O-Linearization?
  - ... reasonable for Model Predictive Control (MPC)?
- actuator saturation?
- Robustness and stability analysis (for parameter uncertainties)?
- Validation in laboratory (measurements)?

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