# — Block seminar —

# Modeling and control of wind turbine systems: An introduction

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**29.04.2014 – 3. Lecture at Stellenbosch University:**
Controller design for the DC-link and power flow

# Outline of the block course: Modeling and control of wind turbine systems: An introduction

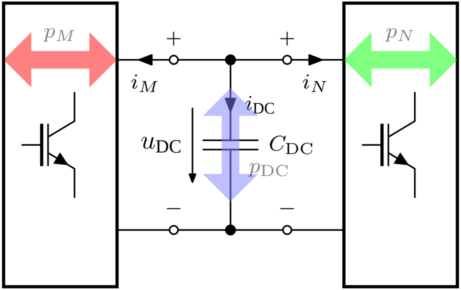
| Date | Time | Content |
|---|---|---|
| 22.04.2014 | 14:00–15:30 | Modeling of core components on machine and grid side |
| 23.04.2014 | 10:00–11:30 | Controller design on machine and grid side |
| 29.04.2014 | 14:00–15:30 | Controller design for the DC-link and power flow |

# Outline of 3. Lecture (29.04.2014)

1. Problem: DC-link control
   - Motivation
   - Modeling (revisited)
   - Block diagram of DC-link control

2. PI-controller design
   - Linearization at equilibrium
   - Structurally varying system
   - Root locus
   - Stability analysis (Hurwitz criterion)
   - Simulation results

# Motivation: DC-link control



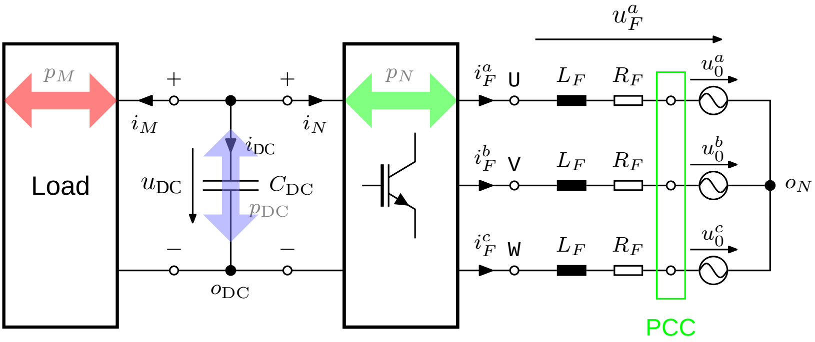

Power balance

$$p_{DC} = u_{DC} C_{DC} \frac{\mathrm{d}}{\mathrm{d}t} u_{DC} = -p_M - p_G \quad \Longrightarrow \quad \boxed{\frac{\mathrm{d}}{\mathrm{d}t} u_{DC} = \frac{1}{u_{DC} C_{DC}} \big( -p_M - p_G \big)}$$

such that

$$\boxed{u_{DC} = u_{DC,\mathrm{ref}} > 0}$$

# Modeling of grid-side network (revisited)



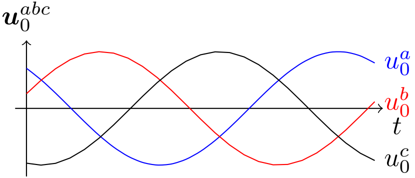

## Network

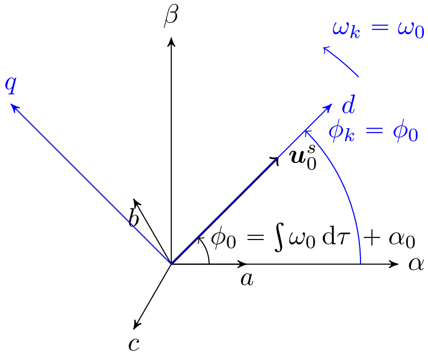
$$\boldsymbol{u}_F^{abc} = \left(u_F^a, \quad u_F^b, \quad u_F^c\right)^\top = R_F \boldsymbol{i}_F^{abc} + L_F \frac{\mathrm{d}}{\mathrm{d}t} \boldsymbol{i}_F^{abc} + \boldsymbol{u}_0^{abc}$$

## Power (lossless inverter)

$$p_N = i_N u_{\mathrm{DC}} = u_F^a i_F^a + u_F^b i_F^b + u_F^c i_F^c = (\boldsymbol{u}_F^{abc})^\top \boldsymbol{i}_F^{abc}$$

# Modeling of grid (revisited)




$$\boldsymbol{u}_0^{abc} = (u_0^a,\, u_0^b,\, u_0^c)^\top = \hat{u}_0 \begin{pmatrix} \cos(\omega_0 t + \alpha_0) \\ \cos(\omega_0 t - 2/3\pi + \alpha_0) \\ \cos(\omega_0 t - 4/3\pi + \alpha_0) \end{pmatrix}$$

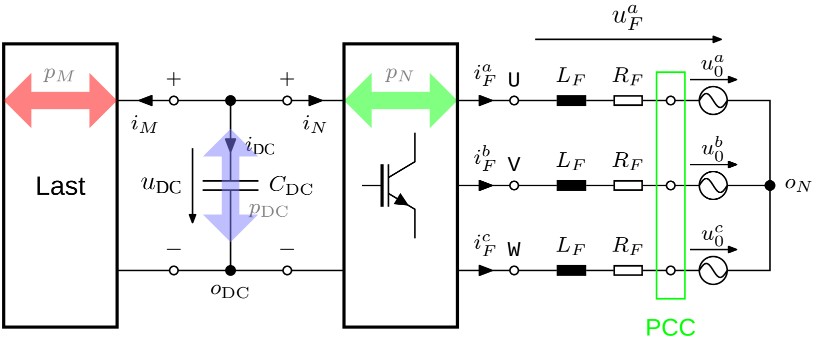
where

- $\hat{u}_0 = \sqrt{2} \cdot 230 \ [\mathrm{V}]$
- $\omega_0 = 2\pi \cdot 50 \ [\mathrm{rad/s}]$
- (here: $\alpha_0 = \pi/4 \ [\mathrm{rad}]$)

$$\|\boldsymbol{u}_0^s\| = \hat{u}_0 \ \text{ und } \ \phi_0 = \mathrm{atan2}\begin{pmatrix} \dfrac{u_0^\alpha}{u_0^\beta} \end{pmatrix}$$

$$\Longrightarrow \quad \boxed{u_0^d = \hat{u}_0 \quad \text{und} \quad u_0^q = 0}$$

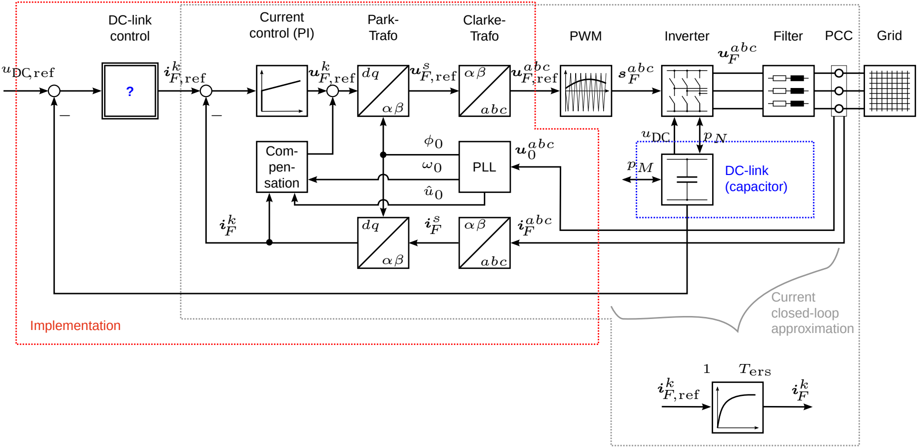
# Modeling of grid-side network (voltage orientation)




$$\boldsymbol{u}_F^k = R_F \boldsymbol{i}_F^k + L_F \frac{\mathrm{d}}{\mathrm{d}t} \boldsymbol{i}_F^k + \omega_0 L_F \begin{pmatrix} -i_F^q \\ i_F^d \end{pmatrix} + \begin{pmatrix} \hat{u}_0 \\ 0 \end{pmatrix}$$

$$p_N = \underbrace{\frac{3}{2} R_F \left\| \boldsymbol{i}_F^k \right\|^2}_{=:p_{V,R_F}} + \underbrace{\frac{3}{2} L_F (\frac{\mathrm{d}}{\mathrm{d}t} \boldsymbol{i}_F^k)^\top \boldsymbol{i}_F^k}_{=:p_{V,L_F}} + \underbrace{\frac{3}{2} \hat{u}_0 i_F^d}_{\substack{=:P \\ \text{Active power}}} \quad \& \quad \underbrace{Q = -\frac{3}{2} \hat{u}_0 i_F^q}_{\text{reactive power}}$$

# Block diagram of DC-link control



# Linearization at equilibrium $(u_{\mathrm{DC}}^\star, i_F^{d,\star}, i_F^{q,\star})$

$$\frac{\mathrm{d}}{\mathrm{d}t} u_{\mathrm{DC}} = \frac{1}{C_{\mathrm{DC}} u_{\mathrm{DC}}}\left[-p_M - \frac{3}{2}\left(R_F \left\|\boldsymbol{i}_F^k\right\|^2 + L_F\left(\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{i}_F^k\right)^\top \boldsymbol{i}_F^k + \hat{u}_0 i_F^d\right)\right]$$

$$\Longrightarrow \frac{\mathrm{d}}{\mathrm{d}t}\widetilde{u}_{\mathrm{DC}} = -\frac{1}{C_{\mathrm{DC}} u_{\mathrm{DC}}^\star}\left[\widetilde{p}_L + \frac{3}{2}\left(\left(\hat{u}_0 + 2R_F i_F^{d,\star}\right)\widetilde{i}_F^d + L_F i_F^{d,\star}\frac{\mathrm{d}}{\mathrm{d}t}\widetilde{i}_F^d\right) + \frac{3}{2}\left(2R_F i_F^{q,\star}\widetilde{i}_F^q + L_F i_F^{d,\star}\frac{\mathrm{d}}{\mathrm{d}t}\widetilde{i}_F^q\right)\right]$$

$$\circ\!\!-\!\!\bullet\quad s\widetilde{u}_{\mathrm{DC}}(s) = -\frac{1}{C_{\mathrm{DC}} u_{\mathrm{DC}}^\star}\left[\widetilde{p}_L(s) + \frac{3}{2}\left(\left(\hat{u}_0 + 2R_F i_F^{d,\star}\right) + sL_F i_F^{d,\star}\right)\widetilde{i}_F^d(s) + \frac{3}{2}\left(2R_F + sL_F\right) i_F^{q,\star}\widetilde{i}_F^q(s)\right]$$

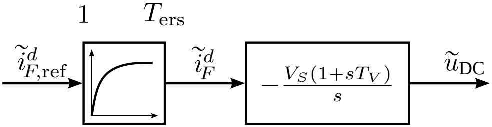
Closed-loop transfer function:

$$\Longrightarrow \boxed{\frac{\widetilde{u}_{\mathrm{DC}}(s)}{\widetilde{i}_F^d(s)} = -V_S\frac{1 + T_V s}{s}} \quad \text{with } T_V := \frac{L_F i_F^{d,\star}}{\hat{u}_0 + 2R_F i_F^{d,\star}}, \quad V_S := \frac{3}{2}\frac{\hat{u}_0 + 2R_F i_F^{d,\star}}{C_{\mathrm{DC}} u_{\mathrm{DC}}^\star}$$

# Structurally varying system

Current closed-loop

Linearized system $F_S$




(S$_1$) $i_F^{d,\star} = 0 \implies T_V = 0 \implies \dfrac{u_{\mathrm{DC}}(s)}{i^d_{F,\mathrm{ref}}(s)} = \dfrac{V_S}{s(1+sT_{\mathrm{ers}})}$ (IT$_1$)

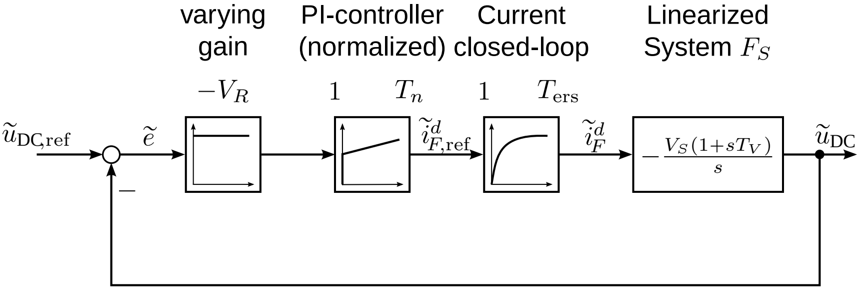
(S$_2$) $i_F^{d,\star} > 0 \implies T_V > 0 \implies \dfrac{u_{\mathrm{DC}}(s)}{i^d_{F,\mathrm{ref}}(s)} = \dfrac{V_S(1+sT_V)}{s(1+sT_{\mathrm{ers}})}$

(IT$_1$ + PD, minimum-phase)

(S$_3$) $i_F^{d,\star} < 0 \implies T_V < 0 \implies \dfrac{u_{\mathrm{DC}}(s)}{i^d_{F,\mathrm{ref}}(s)} = \dfrac{V_S(1-s|T_V|)}{s(1+sT_{\mathrm{ers}})}$

(IT$_1$ + PD, **non**minimum-phase)

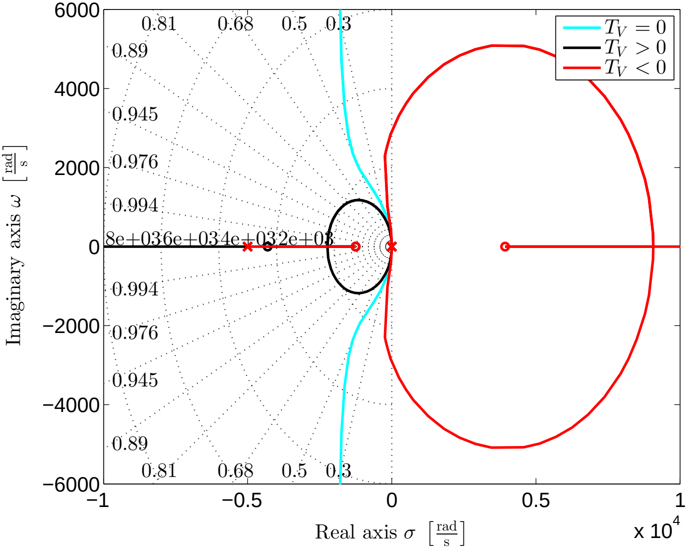
# Root locus of linearized closed-loop system




Open loop

$$\frac{\widetilde{u}_{\mathrm{DC}}(s)}{\widetilde{e}(s)} = \frac{V_R V_S (1+sT_n)(1+sT_V)}{s^2 T_n (1+sT_{\mathrm{ers}})}, \quad \begin{cases} T_{\mathrm{ers}}, T_n > 0,\ V_S > 0 \\ T_V \in \mathbb{R} \ \text{(depending on equilibrium!)} \\ V_R \to \infty \ \text{varying gain} \end{cases}$$

# Root locus for different equilibria



# Stability analysis (Hurwitz criterion)

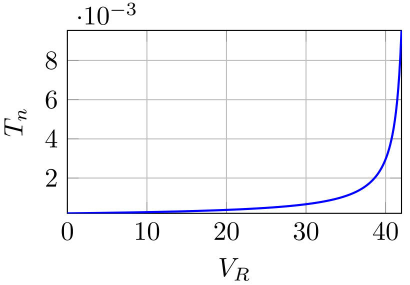
Closed-loop system (linearized) with $T_V := \frac{L_F i_F^{d,\star}}{\hat{u}_0 + 2R_F i_F^{d,\star}}$

$$F_{RK}(s) = \frac{u_{DC}(s)}{u_{DC,ref}(s)} = \frac{V_R V_S(1+sT_{ers})(1+sT_V)}{s^3 \underbrace{T_n T_{ers}}_{=:a_3} + s^2 \underbrace{T_n(1+T_V V_S V_R)}_{=:a_2} + s\underbrace{(T_V+T_n)V_S V_R}_{=:a_1} + \underbrace{V_R V_S}_{=:a_0}}$$

Two conditions (sufficient)

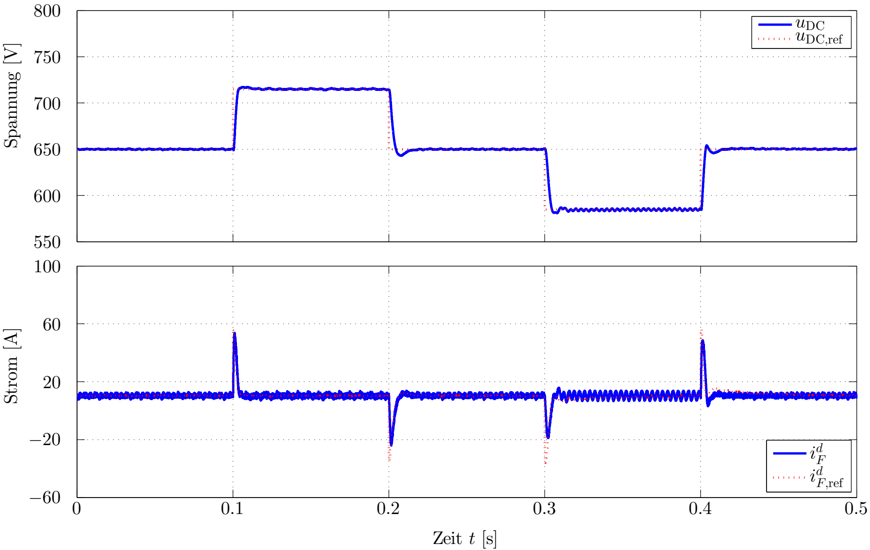
(B$_1$) $0 < V_R < \frac{1}{|T_V| V_S}$

(B$_2$) $T_n > \frac{T_{ers}}{1 - V_R V_S |T_V|} + |T_V| > 0$

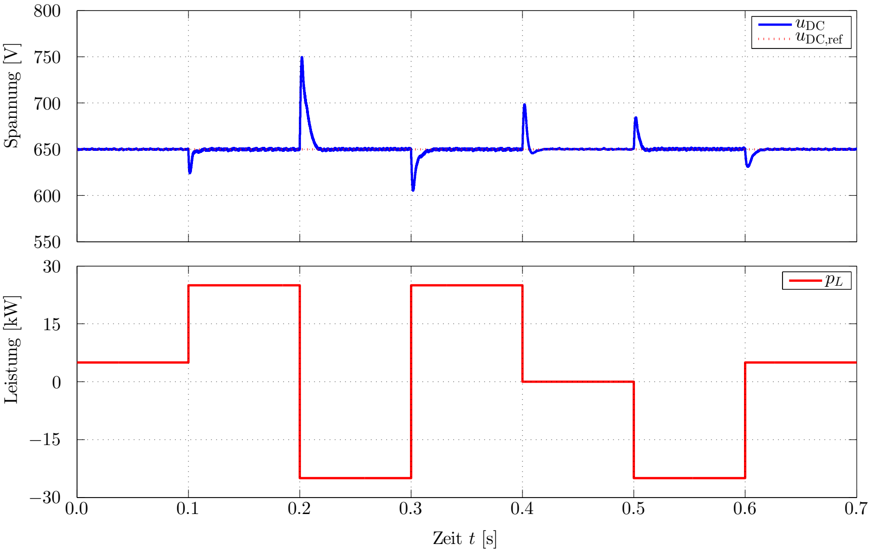



$\Longrightarrow$ **locally stable**, since $a_0, a_1, a_2, a_3 > 0$ and $D_2 = a_2 a_1 - a_3 a_0 > 0$.

# Simulation: Set-point tracking ($L_F = L_0$)



# Simulation: Disturbance rejection ($L_F = 3L_0$)



# Simulation: Set-point tracking ($L_F = 3L_0$)