Non-identifier based adaptive control in mechatronics: An overview

Christoph Hackl

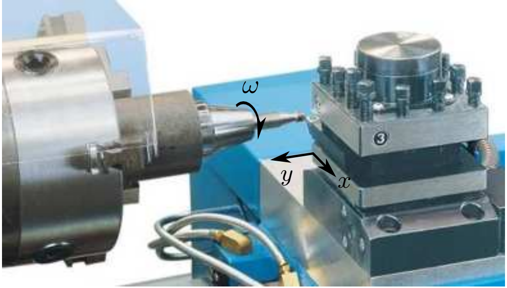
Technische Universität München (TUM)
Munich School of Engineering (MSE)

IEEE Guest Lecture sponsored by IEEE IAS/IES/PELS Joint SA Chapter
25th April 2014, University of Cape Town

Outline

1. Problem statement and motivation
   - Reference tracking
   - High-gain control: An intuition
2. Funnel control
   - Relative-degree-one systems
   - Relative-degree-two systems
   - Steady-state accuracy with internal models
3. Application
   - Speed and position control of stiff servo-systems
   - Funnel control with linear internal model
   - Funnel control for robotic systems
   - Funnel control with saturation
   - PI-funnel control with Anti-windup
   - Funnel control for elastic systems
   - Funnel control for wind turbine systems
4. Conclusion

Reference tracking under load (CNC turning machine)



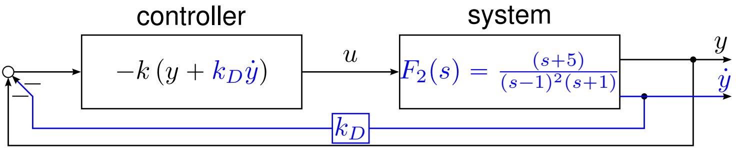
(http://www.knuth.de)

- Problem statement
  - reference $(\omega_{\mathrm{ref}}, x_{\mathrm{ref}}, y_{\mathrm{ref}}) : \mathbb{R}_{\geqslant 0} \to \mathbb{R}^3$
  - precise position and speed control, e.g. for prescribed $\lambda > 0$

$$\boxed{\forall\, t \geqslant t_0 : \qquad |e(t)| = |y_{\mathrm{ref}}(t) - y(t)| \leqslant \lambda.}$$

- Challenges
  - nonlinear effects (e.g. actuator saturation, friction)
  - friction and loads (disturbances) unknown and varying
  - system parameters (solely) roughly known

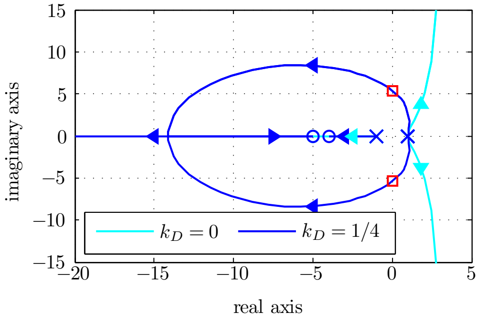
High-gain control: An intuition



'structural properties' of $F_2(s)$

- relative degree (pole excess): $r = 3 - 1 = 2$
- positive high-frequency gain ($\lim_{s\to\infty} s^2 F_1(s) = 1$)
- minimum-phase (numerator is Hurwitz)

Root-locus ($\times$ poles, $\circ$ zeros)



Admissible system class $\mathcal{S}_1$ (simplified)

$$\left.\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}\big(u(t) + u_d(t)\big) + \boldsymbol{B}_{\mathfrak{T}}\Big(\boldsymbol{d}(t) + (\mathfrak{T}\boldsymbol{x})(t)\Big) \\ y(t) &= \boldsymbol{c}^\top \boldsymbol{x}(t), \end{aligned}\right\} \tag{1}$$

with "structural properties":

(i) $\gamma_0 := \boldsymbol{c}^\top \boldsymbol{b} \neq 0$ and $\mathrm{sign}(\gamma_0)$ known;

(ii) $\forall\, s \in \mathbb{C}$ with $\Re(s) \geqslant 0$: $\quad \det \begin{bmatrix} s\boldsymbol{I}_n - \boldsymbol{A} & \boldsymbol{b} \\ \boldsymbol{c}^\top & 0 \end{bmatrix} \neq 0;$

(iii) operator $\mathfrak{T}$ is of class $\mathcal{T}$ (see Def. 1 in [14]) and $M_{\mathfrak{T}} := \sup\left\{ \|(\mathfrak{T}\boldsymbol{\xi})(t)\| \;\middle|\; t \geqslant 0,\ \boldsymbol{\xi}(\cdot) \in \mathcal{C}([-h,\infty),\mathbb{R}^n) \right\} < \infty;$

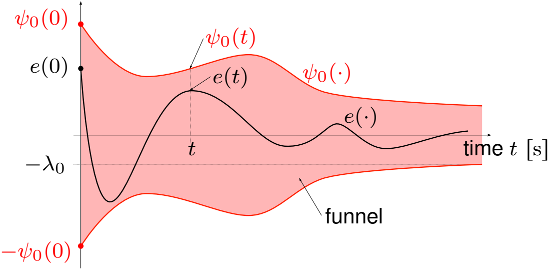
(iv) $u_d(\cdot) \in \mathcal{L}^\infty([-h,\infty);\mathbb{R})$ and $\boldsymbol{d}(\cdot) \in \mathcal{L}^\infty([-h,\infty);\mathbb{R}^m)$.

Example (Linear system in the frequency domain)

$F_1(s) = \boldsymbol{c}^\top (s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b} = \frac{(s+4)(s+5)}{(s-1)^2(s+1)}$

Control objective and funnel controller for $\mathcal{S}_1$ [14]

- objective: 'tracking with prescribed transient accuracy' of reference $y_{\mathrm{ref}}(\cdot)$



where
$e(t) = y_{\mathrm{ref}}(t) - y(t)$,
$y_{\mathrm{ref}}(\cdot) \in \mathcal{C}(\mathbb{R}_{\geqslant 0}; \mathbb{R})$,
$\psi_0(\cdot) \in \mathcal{C}(\mathbb{R}_{\geqslant 0}; [\lambda_0, \infty))$
with $\lambda_0 > 0$

- funnel controller:

$$\boxed{u(t) = k_0(t) e(t) \quad \text{where} \quad k_0(t) = \frac{\varsigma_0(t)}{\psi_0(t) - |e(t)|}} \qquad (\mathrm{FC}_1)$$

- with gain scaling $\varsigma_0(\cdot) \in \mathcal{C}(\mathbb{R}_{\geqslant 0}; \mathbb{R}_{>0})$
(e.g. to fix minimal gain: $k_0(t) \geqslant \varsigma_0(t)/\psi_0(t)$ for all $t \geqslant 0$)

Admissible system class $\mathcal{S}_2$ (simplified)

$$\left.\begin{aligned}\dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}\big(u(t) + u_d(t)\big) + \boldsymbol{B}_{\mathfrak{T}}\Big(\boldsymbol{d}(t) + (\mathfrak{T}\boldsymbol{x})(t)\Big)\\ y(t) &= \boldsymbol{c}^\top \boldsymbol{x}(t),\end{aligned}\right\} \tag{2}$$

with “structural properties”:

(i) $\boldsymbol{c}^\top \boldsymbol{b} = 0$, $\gamma_0 := \boldsymbol{c}^\top \boldsymbol{A}\boldsymbol{b} \neq 0$ and $\operatorname{sign}(\gamma_0)$ known;

(ii) $\forall\, s \in \mathbb{C}$ with $\Re(s) \geqslant 0$: $\quad \det \begin{bmatrix} s\boldsymbol{I}_n - \boldsymbol{A} & \boldsymbol{b} \\ \boldsymbol{c}^\top & 0 \end{bmatrix} \neq 0;$

(iii) operator $\mathfrak{T}$ is of class $\mathcal{T}$ (see Def. 1 in [14]) and
$M_{\mathfrak{T}} := \sup\left\{\|(\mathfrak{T}\boldsymbol{\xi})(t)\| \;\middle|\; t \geqslant 0,\ \boldsymbol{\xi}(\cdot) \in \mathcal{C}([-h,\infty),\mathbb{R}^n)\right\} < \infty;$

(iv) $u_d(\cdot) \in \mathcal{L}^\infty([-h,\infty);\mathbb{R})$ and $\boldsymbol{d}(\cdot) \in \mathcal{L}^\infty([-h,\infty);\mathbb{R}^m)$.

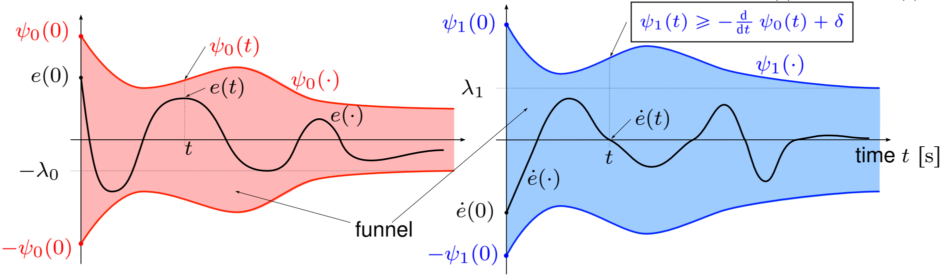
(v) $y(\cdot)$ and $\dot{y}(\cdot)$ are available for feedback.

Example (Linear system in the frequency domain)

$F_2(s) = \boldsymbol{c}^\top (s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b} = \frac{(s+5)}{(s-1)^2(s+1)}$

Control objective and funnel controller for $\mathcal{S}_2$ [9]

- objective: 'tracking with prescribed transient accuracy' of $y_{\mathrm{ref}}(\cdot)$ and $\dot{y}_{\mathrm{ref}}(\cdot)$



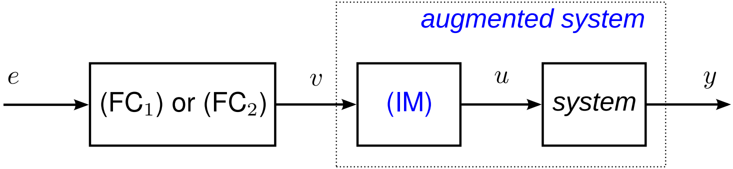
- funnel controller (with derivative feedback)

$$\boxed{u(t) = k_0(t)^2\left(e(t) + \frac{k_1(t)}{k_0(t)}\dot{e}(t)\right), \qquad e(t) = y_{\mathrm{ref}}(t) - y(t)} \qquad (\mathrm{FC}_2)$$

- where $k_0(t) = \dfrac{\varsigma_0(t)}{\psi_0(t) - |e(t)|}$ and $k_1(t) = \dfrac{\varsigma_1(t)}{\psi_1(t) - |\dot{e}(t)|}$
- scaling functions $\varsigma_i(\cdot) \in \mathcal{C}(\mathbb{R}_{\geqslant 0}; \mathbb{R}_{>0})$, $i = 0, 1$

Steady-state accuracy with internal models [1, 6, 12]

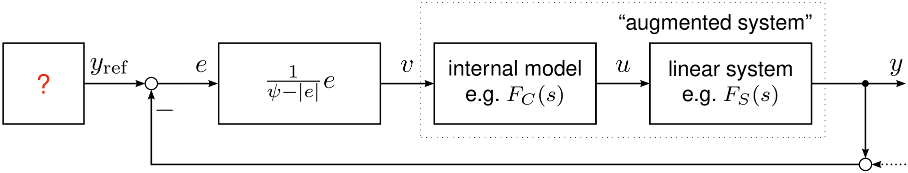
- asymptotic accuracy (i.e. $\lim_{t\to\infty} e(t) = 0$), cannot be guaranteed, since $\psi_0(t) \geqslant \lambda_0 > 0$ for all $t \geqslant 0$
- typical solution: use of internal model (IM) to achieve asymptotic tracking



- if (IM) has
  - relative degree zero
  - positive high-frequency gain
  - and is minimum-phase

then *augmented system* has identical 'structural properties' as *system*

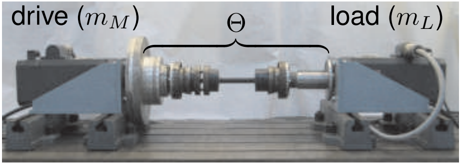
Example: PI-funnel control [12, 13]



Example ("Augmented system" in the frequency domain)

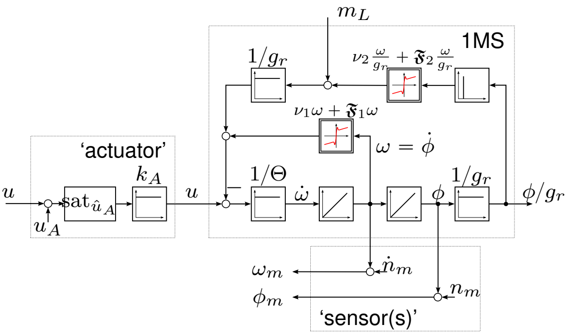
- PI controller: $F_C(s) = k_P + \frac{k_I}{s} = \frac{k_P\,s+k_I}{s}, \qquad k_P,\, k_I > 0$
- system: $F_S(s) = \frac{1+s}{s^2-s+1}$
- "augmented system": $F_C(s)F_S(s) = \frac{k_P\,s+k_I}{s} \cdot \frac{1+s}{s^2-s+1}$
- $\implies \lim_{t\to\infty} e(t) = 0$ for $y_{\mathrm{ref}}(\cdot)$ =const.

Stiff servo-systems



- drive torque $m_M(\cdot) \in \mathcal{C}(\mathbb{R}_{\geqslant 0}; \mathbb{R})$ $[\mathrm{Nm}]$ (control input)
- load torque $m_L(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geqslant 0}; \mathbb{R})$ $[\mathrm{Nm}]$ (disturbance)
- inertia $\Theta > 0$ $\left[\mathrm{kg\,m^2}\right]$
- state $\boldsymbol{x} = (\phi, \omega)^\top$: position ϕ $[\mathrm{rad}]$, speed ω $[\mathrm{rad/s}]$
- friction (on drive & load side)
- gear with ratio $g_r \in \mathbb{R}\setminus\{0\}$ $[1]$ (neglecting dynamics & backlash)
- signals available for feedback: position ϕ and/or speed $\omega = \dot{\phi}$ (deteriorated by $n_m(\cdot)$ and/or $\dot{n}_m(\cdot)$, resp.)

Nonlinear model of stiff servo-systems [7, 8]

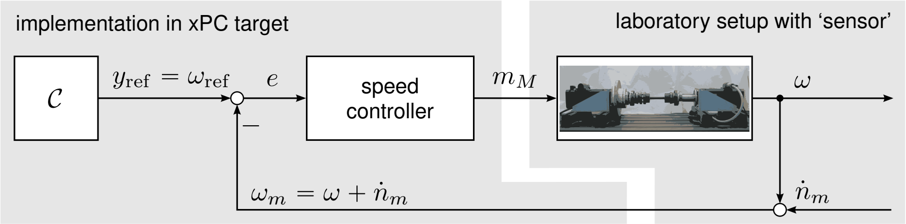


$$\left.\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}\,\mathrm{sat}_{\hat u_A}\big(m_M(t) + u_A(t) - (\mathfrak{F}_1\omega)(t)\big) + \boldsymbol{b}_L\big(m_L(t) + (\mathfrak{F}_2\tfrac{\omega}{g_r})(t)\big) \\ y(t) &= \boldsymbol{c}^\top\boldsymbol{x}(t), \qquad \boldsymbol{x}(0) = (\phi^0,\,\omega^0)^\top \end{aligned}\right\} \qquad \text{(1MS)}$$

where $k_A > 0$, $\hat u_A > 0$, $u_A \in \mathcal{L}^\infty(\mathbb{R}_{\geqslant 0};\mathbb{R})$, $g_r \in \mathbb{R}\backslash\{0\}$, $\Theta > 0$, $\nu_1, \nu_2 > 0$, and for all $i \in \{1,2\}$: $\mathfrak{F}_i \in \mathcal{T}$, $M_{\mathfrak{F}_i} := \sup\{\|(\mathfrak{F}_i\beta)(t)\| \mid t \geqslant 0,\ \beta(\cdot) \in \mathcal{C}(\mathbb{R}_{\geqslant 0},\mathbb{R})\} < \infty$ and

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{\nu_1 + \nu_2/g_r^2}{\Theta} \end{bmatrix}, \boldsymbol{b} = \begin{pmatrix} 0 \\ \frac{1}{\Theta} \end{pmatrix}, \boldsymbol{b}_L = \begin{pmatrix} 0 \\ \frac{-1}{g_r\Theta} \end{pmatrix} \text{ and e.g. } \boldsymbol{c} = \begin{pmatrix} \frac{1}{g_r} \\ 0 \end{pmatrix}.$$

Speed control implementation



- Standard PI controller

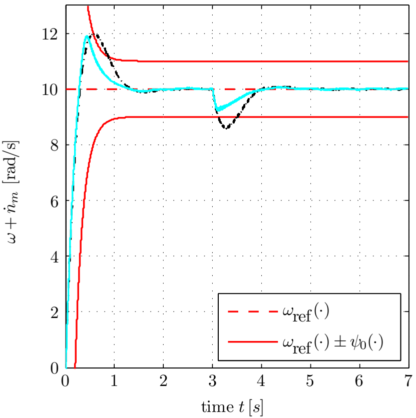
$$m_M(t) = k_P\, e(t) + k_I \int_0^t e(\tau)\,\mathrm{d}\tau$$

- PI-funnel controller

$$m_M(t) = k_0(t)\, e(t) + k_I \int_0^t k_0(\tau) e(\tau)\,\mathrm{d}\tau \qquad \text{where} \qquad k_0(t) = \frac{\varsigma_0(t)}{\psi_0(t) - |e(t)|}$$

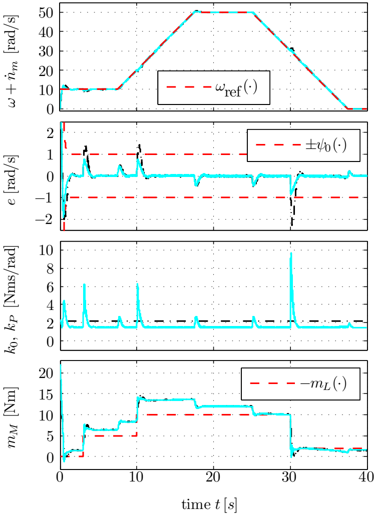
Speed control measurement results

Set-point tracking

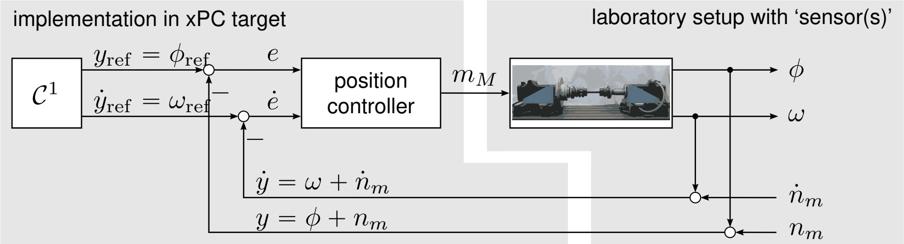


PI-funnel controller
PI controller

Reference tracking



Position control implementation



- Standard PID controller (with feedforward)

$$m_M(t) = k_P\, e(t) + k_I \int_0^t e(\tau)\, \mathrm{d}\tau \; + k_D \dot{e}(t) + u_F(t)$$

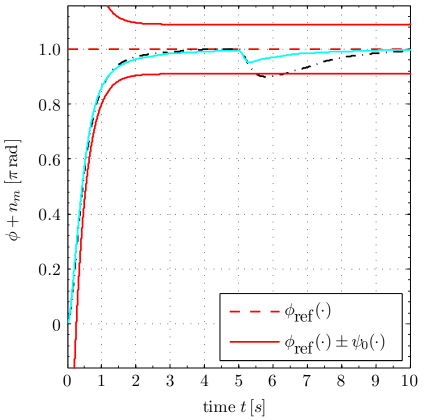
- PID-funnel controller

$$m_M(t) = k_0(t)^2 \left(e(t) + \frac{k_1(t)}{k_0(t)} \dot{e}(t) \right) + k_I \int_0^t k_0(\tau)^2 \left(e(\tau) + \frac{k_1(\tau)}{k_0(\tau)} \dot{e}(\tau) \right) \mathrm{d}\tau$$

where $k_i(t) = \frac{\varsigma_i(t)}{\psi_i(t) - |e^{(i)}(t)|}$ for $i = 0, 1$.

Position control measurement results

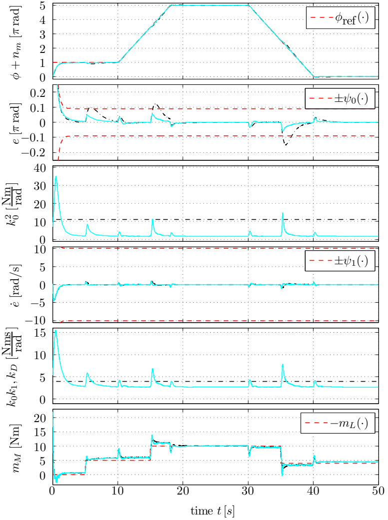
Set-point tracking



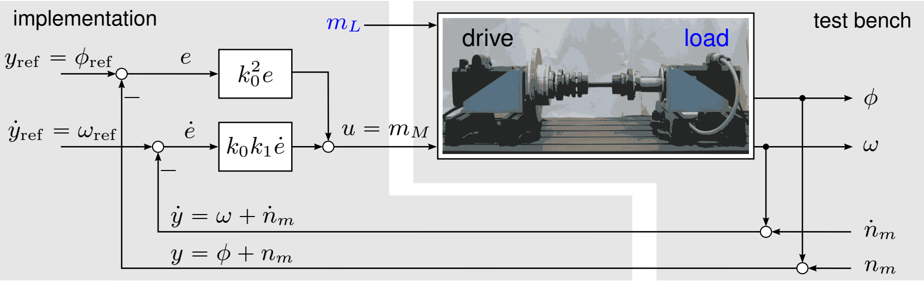
PID-funnel controller

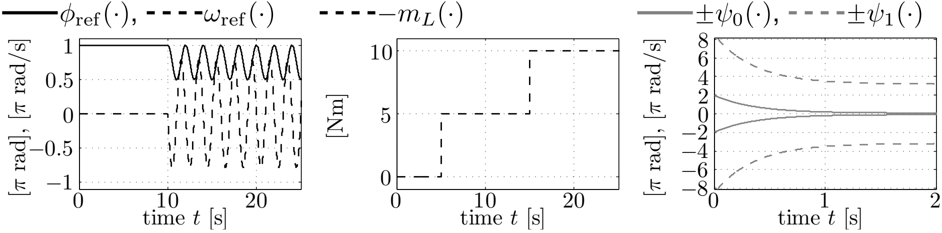
PID controller (with feedforward)

Reference tracking

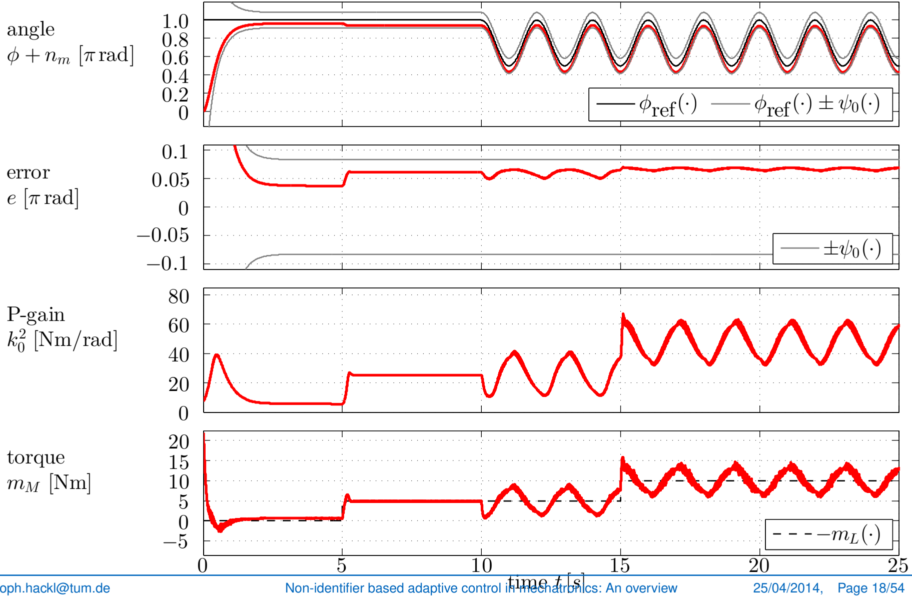


Funnel control with linear internal model [6]

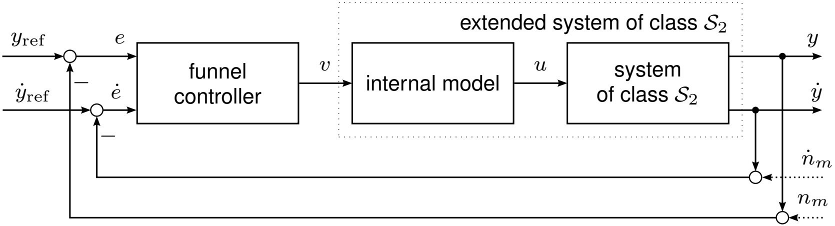




Measurements: ——(FC$_2$)



Design and combination of internal models



- reference

$$y_{\text{ref}}(t) := \begin{cases} y_0 & , 0 \leqslant t < t_0 \\ \frac{y_0}{4}\cos(2\pi f_0(t-t_0)) + \frac{3y_0}{4} & , t_0 \leqslant t \leqslant t_{end} \end{cases}, \qquad \begin{array}{l} y_0 \in \mathbb{R}\ [\text{rad}] \\ f_0 \geqslant 0\ [1/\text{s}] \end{array}$$

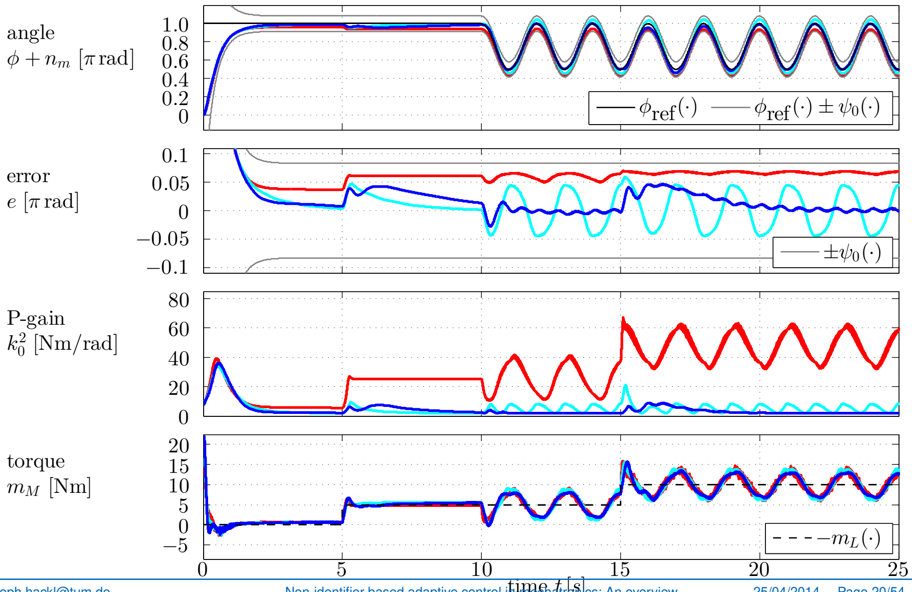
- design: (i) $y_0 \circ\!\!-\!\!\bullet \frac{y_0}{s} \implies F_{PI}(s) = \frac{k_P\, s + k_I}{s}$ and (ii)

$$\cos(2\pi f_0 t) \circ\!\!-\!\!\bullet \frac{1}{s^2 + (2\pi f_0)^2} \implies F_{COS}(s) = \frac{(s+z_0)^2}{s^2 + (2\pi f_0)^2}, \quad z_0, f_0 > 0.$$

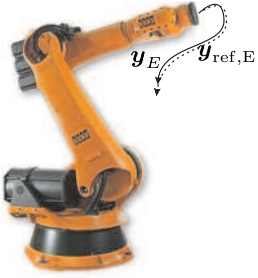
- combination

$$F_{IM}(s) = \frac{u(s)}{v(s)} = k_P \cdot \frac{s + k_I/k_P}{s} \cdot \frac{(s+z_0)^2}{s^2 + (2\pi f_0)^2}, \quad k_P, k_I, z_0, f_0 > 0. \qquad \text{(IM)}$$

Measur.: —— (FC_2), —— $(FC_2)+(PI)$, —— $(FC_2)+(IM)$



Funnel control for robotic systems [5]



Kuka KR 150-2 (Series 2000)
(http://www.kuka-robotics.com)

Applications

- (spot-)welding
- painting/enameling
- mounting/assembling
- laser-beam cutting
- ...

Goals:

- Accurate position control of end effector , i.e. for given $\lambda > 0$:

$$\boxed{\forall\, t \geqslant t_0: \qquad \|\boldsymbol{e}_E(t)\| = \left\|\boldsymbol{y}_{\mathrm{ref,E}}(t) - \boldsymbol{y}_E(t)\right\| \leqslant \lambda.}$$

- controller: simple and 'intuitive' to tune

Rigid-link revolute-joint robotic systems

Model of robotic systems (in joint space)

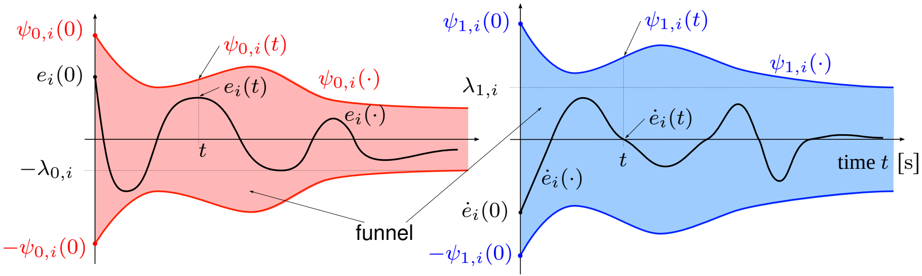
$$\left.\begin{array}{c} \boldsymbol{M}(\boldsymbol{y}(t))\ddot{\boldsymbol{y}}(t) + \boldsymbol{C}(\boldsymbol{y}(t), \dot{\boldsymbol{y}}(t))\dot{\boldsymbol{y}}(t) + (\boldsymbol{\mathfrak{F}}\dot{\boldsymbol{y}})(t) + \boldsymbol{g}(\boldsymbol{y}(t)) + \boldsymbol{d}(t) = \boldsymbol{u}(t), \\ (\boldsymbol{y}(0),\, \dot{\boldsymbol{y}}(0))^\top = (\boldsymbol{y}_0,\, \boldsymbol{y}_1)^\top \in \mathbb{R}^{2n} \end{array}\right\} \tag{ROBOT}$$

- Standard assumptions
  - $\exists\, \overline{c}_{\boldsymbol{M}}, \underline{c}_{\boldsymbol{M}} > 0\ \forall\, \boldsymbol{y} \in \mathbb{R}^n : \quad 0 < \underline{c}_{\boldsymbol{M}} \boldsymbol{I}_n \leqslant \boldsymbol{M}(\boldsymbol{y}) = \boldsymbol{M}(\boldsymbol{y})^\top \leqslant \overline{c}_{\boldsymbol{M}} \boldsymbol{I}_n$
  - $\exists\, c_{\boldsymbol{C}} > 0\ \forall\, \boldsymbol{y}, \boldsymbol{v} \in \mathbb{R}^n : \quad \|\boldsymbol{C}(\boldsymbol{y}, \boldsymbol{v})\boldsymbol{v}\| \leqslant c_{\boldsymbol{C}} \|\boldsymbol{v}\|^2$
  - $\boldsymbol{\mathfrak{F}} : \mathcal{C}(\mathbb{R}_{\geqslant 0}; \mathbb{R}) \to \mathcal{L}^\infty_{\mathrm{loc}}(\mathbb{R}_{\geqslant 0}; \mathbb{R})$ (causal friction operator)
  - $\exists\, c_{\boldsymbol{g}} > 0\ \forall\, \boldsymbol{y} \in \mathbb{R}^n : \quad \|\boldsymbol{g}(\boldsymbol{y})\| \leqslant c_{\boldsymbol{g}}$
  - $\boldsymbol{d}(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geqslant 0}; \mathbb{R}^n)$ (disturbance)
  - $\boldsymbol{u}(\cdot) \in \mathcal{C}(\mathbb{R}_{\geqslant 0}; \mathbb{R}^n)$ (control input)
  - Measurements: $\boldsymbol{y}(\cdot)$ and $\dot{\boldsymbol{y}}(\cdot)$
- ‘Structural properties’
  - (strict) relative degree $r = 2$
  - positive (definite) high-frequency gain (matrix)
  - no internal dynamics (minimum-phase)
  - ‘derivative feedback’ admissible

Control objective

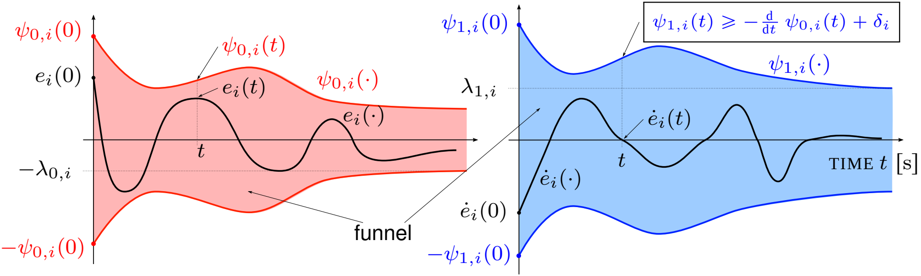
- '**Tracking with prescribed transient accuracy**' for **each joint** $i \in \{1, \ldots, n\}$:

$$\boxed{\forall\, t \geqslant 0: \quad |e_i(t)| = |y_{\mathrm{ref,i}}(t) - y_i(t)| < \psi_{0,i}(t) \quad \text{and} \quad |\dot{e}_i(t)| < \psi_{1,i}(t)}$$



- (joint-)reference $y_{\mathrm{ref,i}}(\cdot) \in \mathcal{W}^{2,\infty}(\mathbb{R}_{\geqslant 0}; \mathbb{R})$
- (joint-)boundary $(\psi_{0,i}(\cdot),\, \psi_{1,i}(\cdot))$:
  - $|e_i(0)| < \psi_{0,i}(0)$ und $|\dot{e}_i(0)| < \psi_{1,i}(0)$
  - $\psi_{0,i}(\cdot),\, \psi_{1,i}(\cdot) \in \mathcal{L}^{\infty}(\mathbb{R}_{\geqslant 0}; \mathbb{R}_{>0})$ and globally Lipschitz continuous
  - $\exists\, \delta_i > 0\; \forall t \geqslant 0: \;\; \psi_{1,i}(t) \geqslant -\frac{\mathrm{d}}{\mathrm{d}t}\, \psi_{0,i}(t) + \delta_i$

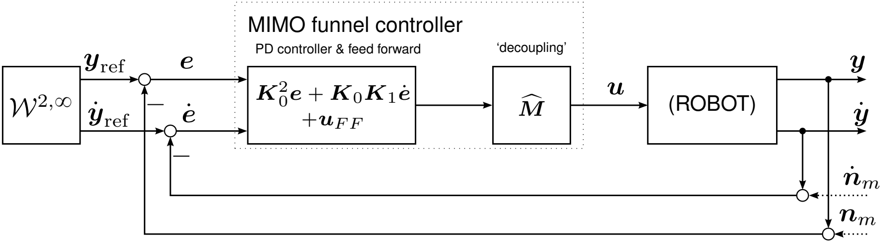
MIMO funnel controller



$$\boxed{\boldsymbol{u}(t) = \widehat{\boldsymbol{M}}(\boldsymbol{y}(t))\Big(\underbrace{\boldsymbol{K}_0(t)^2\boldsymbol{e}(t)}_{\textbf{P}} + \underbrace{\boldsymbol{K}_0(t)\boldsymbol{K}_1(t)\dot{\boldsymbol{e}}(t)}_{\textbf{D}} + \boldsymbol{u}_{FF}(t)\Big)} \qquad (\mathrm{FC}_n)$$

- $\boldsymbol{K}_0(t) = \mathrm{diag}\left\{k_{0,1}(t),\,\ldots,\,k_{0,n}(t)\right\}$ und
  $\boldsymbol{K}_1(t) = \mathrm{diag}\left\{k_{1,1}(t),\,\ldots,\,k_{1,n}(t)\right\}$
- $\forall\, i \in \{1,\ldots,n\}: \quad k_{0,i}(t) = \dfrac{1}{\psi_{0,i}(t) - |e_i(t)|}$ and $k_{1,i}(t) = \dfrac{1}{\psi_{1,i}(t) - |\dot{e}_i(t)|}$
- $\boldsymbol{u}_{FF}(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geqslant 0}; \mathbb{R}^n)$

Properties of the closed-loop system



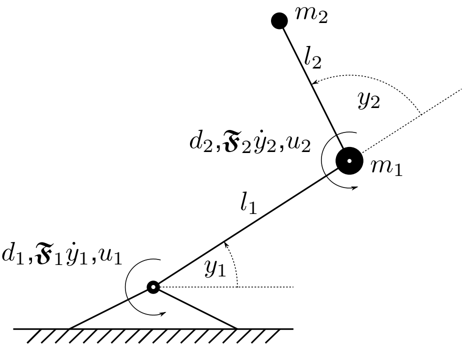
For

- $\widehat{\boldsymbol{M}}(\boldsymbol{y}) = \boldsymbol{M}(\boldsymbol{y})\ \mathrm{diag}\left\{\Delta_1(\boldsymbol{y}), \ldots, \Delta_n(\boldsymbol{y})\right\}$
- $\boldsymbol{u}_{FF}(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geqslant 0}; \mathbb{R}^n)$
- $\boldsymbol{n}_m(\cdot) \in \mathcal{W}^{2,\infty}(\mathbb{R}_{\geqslant 0}; \mathbb{R}^n)$ (measurement noise admissible)

we have

- $\forall i \in \{1, \ldots, n\}\ \exists \varepsilon_i > 0\ \forall\, t \geqslant 0\colon\ \ \psi_{0,i}(t) - |e_i(t)| \geqslant \varepsilon_i$ and $\psi_{1,i}(t) - |\dot{e}_i(t)| \geqslant \varepsilon_i$
- $\boldsymbol{K}_0(\cdot), \boldsymbol{K}_1(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geqslant 0}; \mathbb{R}^n_{>0})$
- $\boldsymbol{u}(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geqslant 0}; \mathbb{R}^n)$

Example: Planar robot (2DOF)

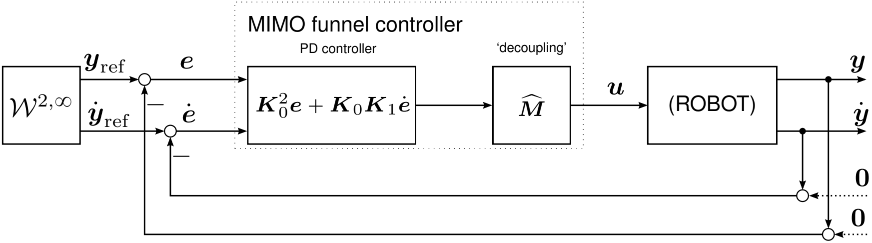


- (point-)masses m_1, m_2 $[\mathrm{kg}]$
- length of joints l_1, l_2 $[\mathrm{m}]$
- joint angle $\boldsymbol{y} := \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ $[\mathrm{rad}]^2$
- control input $\boldsymbol{u} := \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ $[\mathrm{Nm}]^2$
- friction $\mathfrak{F}\dot{\boldsymbol{y}} := \begin{pmatrix} \mathfrak{F}_1\dot{y}_1 \\ \mathfrak{F}_2\dot{y}_2 \end{pmatrix}$ $[\mathrm{Nm}]^2$
- disturbance $\boldsymbol{d} := \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$ $[\mathrm{Nm}]^2$

$$\boxed{\left.\begin{aligned} \boldsymbol{M}(\boldsymbol{y}(t))\ddot{\boldsymbol{y}}(t) + \boldsymbol{C}(\boldsymbol{y}(t), \dot{\boldsymbol{y}}(t))\dot{\boldsymbol{y}}(t) + (\mathfrak{F}\dot{\boldsymbol{y}})(t) + \boldsymbol{g}(\boldsymbol{y}(t)) + \boldsymbol{d}(t) = \boldsymbol{u}(t), \\ (\boldsymbol{y}(0),\, \dot{\boldsymbol{y}}(0))^\top = (\mathbf{0},\, \mathbf{0})^\top \in \mathbb{R}^4 \end{aligned}\right\}}$$

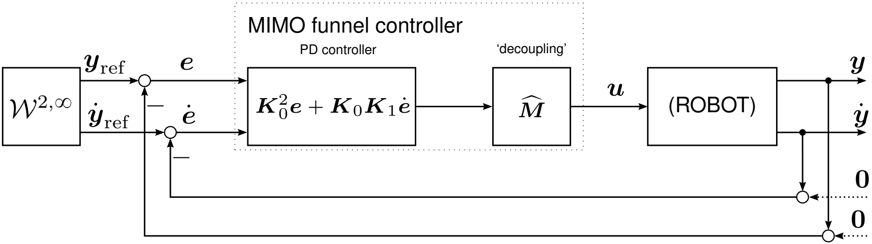
(ROBOT)

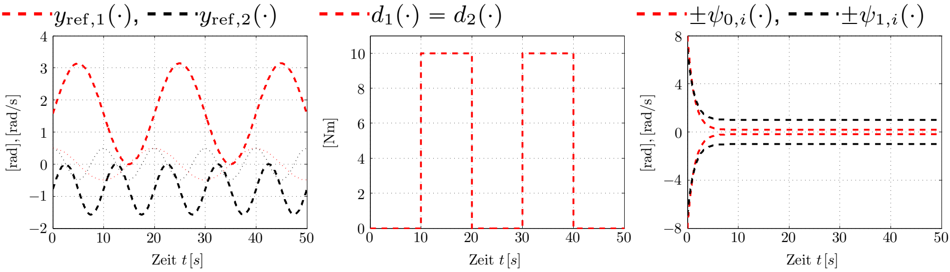
Implementation: MIMO funnel controller



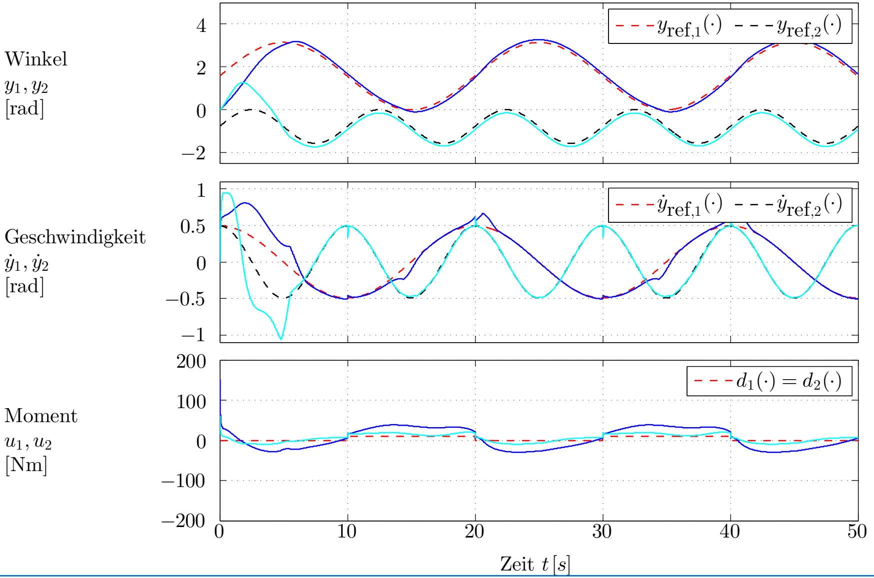
- $$\boldsymbol{u}(t) = \overbrace{\boldsymbol{M}(\boldsymbol{y}(t)) \cdot \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}}^{\widehat{\boldsymbol{M}}(\boldsymbol{y}(t))} \cdot \left(\begin{bmatrix} k_{0,1}(t)^2 & 0 \\ 0 & k_{0,2}(t)^2 \end{bmatrix} \boldsymbol{e}(t) + \begin{bmatrix} k_{0,1}(t)k_{1,1}(t) & 0 \\ 0 & k_{0,2}(t)k_{1,2}(t) \end{bmatrix} \dot{\boldsymbol{e}}(t) \right)$$
- $$\forall\, i \in \{1,2\}: \quad k_{0,i}(t) = \frac{\psi_{0,i}(t)}{\psi_{0,i}(t) - |e_i(t)|} \geqslant 1 \;\&\; k_{1,i}(t) = \frac{5\psi_{1,i}(t)}{\psi_{1,i}(t) - |\dot{e}_i(t)|} \geqslant 5$$

Implementation: References, disturbance & boundary

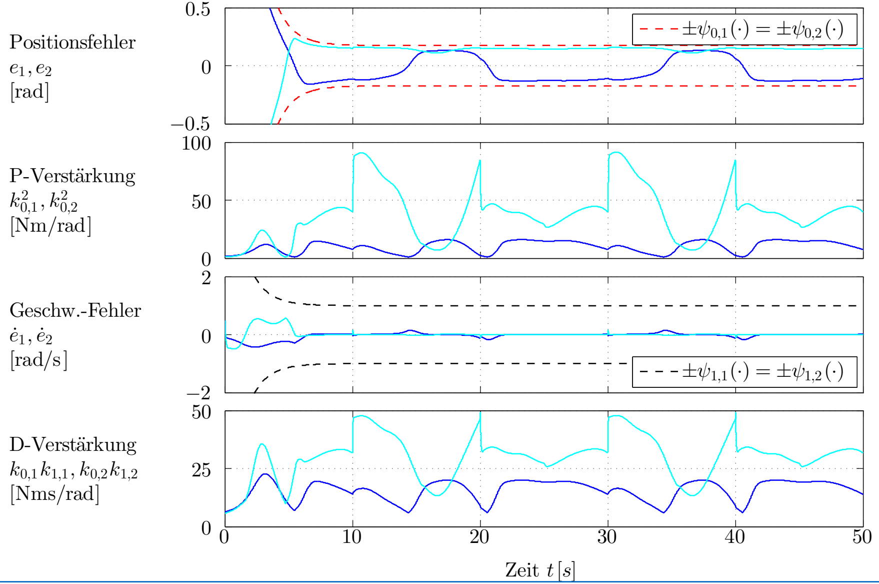




Simulation results (joints: —— #1, —— #2)



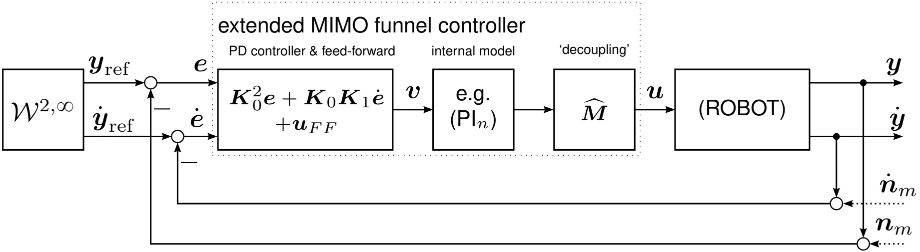
Simulation results (joints: —— #1, —— #2)



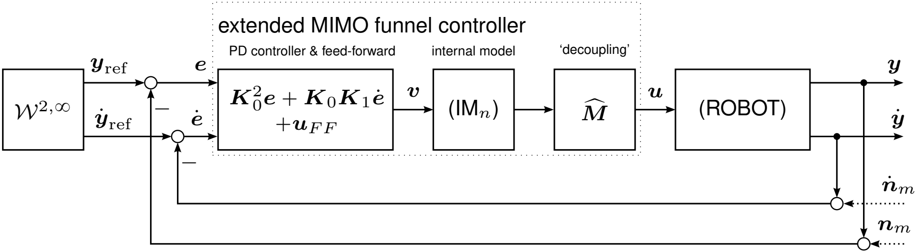
Funnel control with internal model for robotic systems

- $\lim_{t\to\infty} e_i(t) = 0$ not guaranteed (since $\psi_{0,i}(t) \geqslant \lambda_{0,i} > 0$ for all $t \geqslant 0$)
- application of (linear) internal model admissible, if
  - relative degree zero
  - positive high-frequency gain
  - control- and observable ($\Longrightarrow$ minimum-phase)
- 'Standard internal model' in industry (SISO):

$$\left.\begin{aligned} \dot{x}_I(t) &= u(t), \quad x_I(0) = 0 \\ v(t) &= k_P u(t) + k_I x_I(t) \end{aligned}\right\} \circ\!\!-\!\!\bullet\; v(s) = \frac{k_P\, s + k_I}{s}\, u(s), \quad k_P, k_I > 0 \quad \text{(PI)}$$



Combination of internal models



- References (for each joints)

$$y_{\rm ref,i}\colon \mathbb{R}_{\geqslant 0} \to \mathbb{R}, \quad t \mapsto y_{\rm ref,i}(t) := a_i \sin\big(2\pi\, f_i\, t\big) + b_i, \qquad a_i > 0,\ b_i \neq 0$$

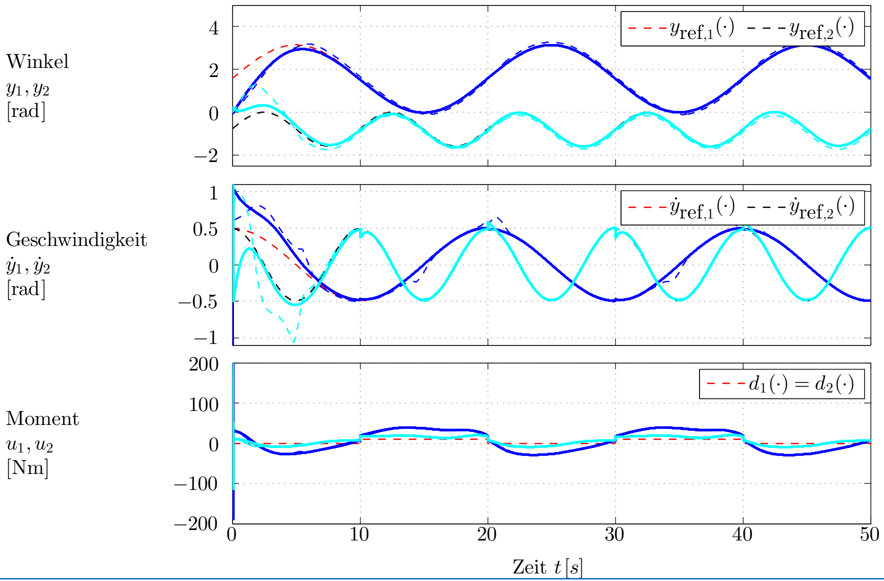
- Internal models (SISO)

$$F_{PI}(s) = \frac{k_P\, s + k_I}{s}, \ k_P, k_I > 0 \quad \text{and} \quad F_{SIN}(s) = \frac{(s+\gamma)^2}{s^2 + (2\pi f_i)^2}, \ \gamma, f_i > 0$$

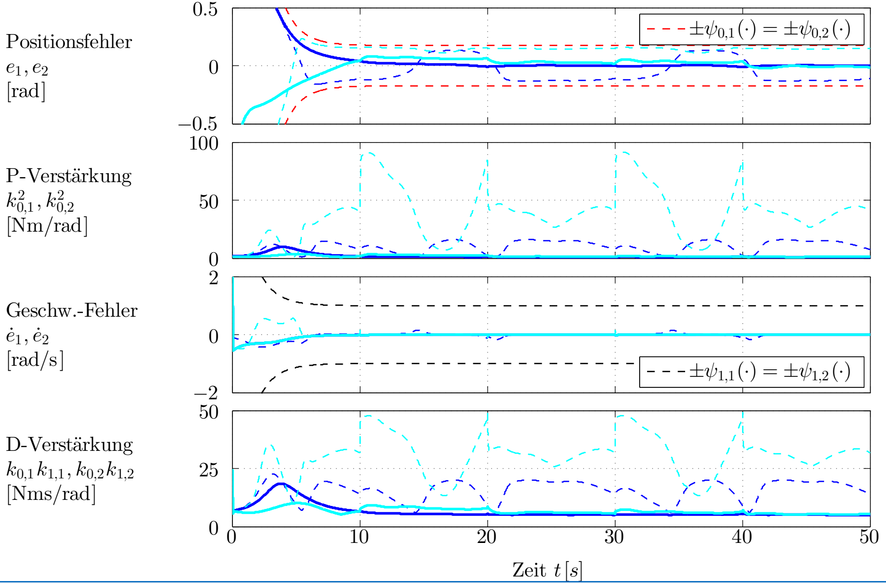
- Serial interconnection

$$F_{IM}(s) = \frac{k_P\, s + k_I}{s} \cdot \frac{(s+\gamma)^2}{s^2 + (2\pi f_i)^2}, \qquad k_P, k_I, \gamma, f_i > 0 \qquad \text{(IM)}$$

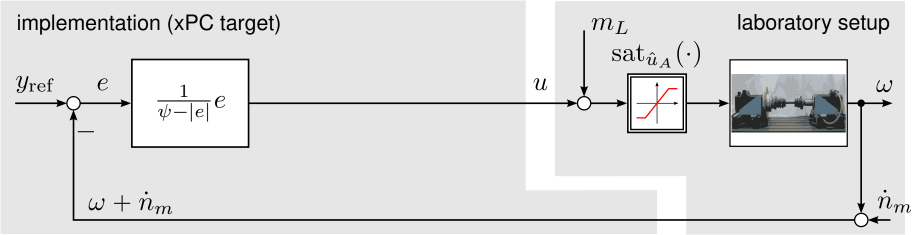
Simulation results with (IM_n) (——#1, ——#2)



Simulation results with (IM_n) (——#1, ——#2)

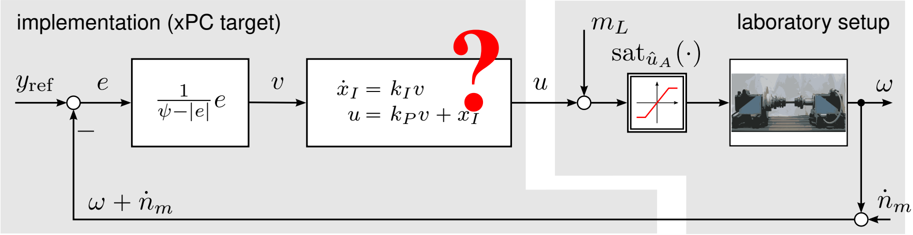


Funnel control with saturation [9, 10]



- $\hat{u}_A \geqslant u_{\mathrm{feas}} = u_{\mathrm{feas}}(y_{\mathrm{ref}}, \psi, \text{system}, \dots) \implies \forall\, t \geqslant 0\colon\ \psi(t) - |e(t)| \geqslant \varepsilon$
- $\varepsilon \leqslant \min\left\{\frac{\lambda}{2},\ \psi(0) - |e(0)|,\ \frac{\lambda}{2(\hat{u}_A + \|m_L\|_\infty)}\right\}$
- u_{feas} conservative (for setup $u_{\mathrm{feas}} \approx 63$ [Nm] and $\hat{u}_A = 22$ [Nm])

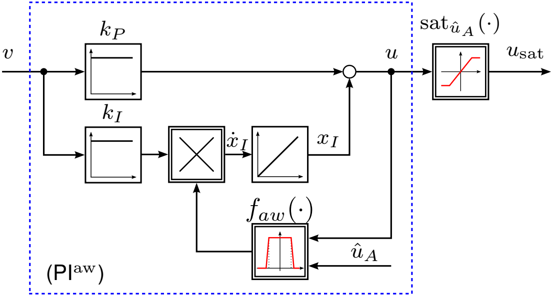
PI-funnel control with saturation feasible?



- Tracking with prescribed transient accuracy endangered, i.e.

$$\cancel{\forall\, t \geqslant 0:\quad |e(t)| < \psi(t)} \qquad ?$$

PI controller with Anti-windup ("conditional integration")



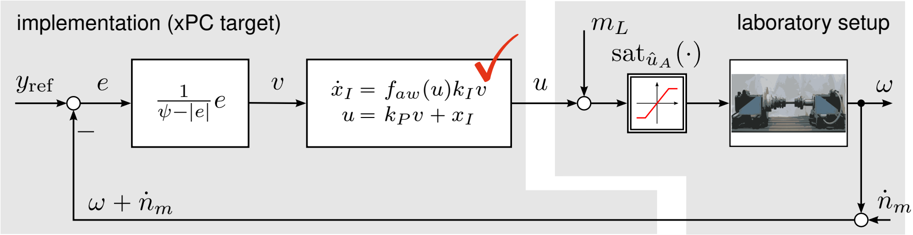
System theoretical interpretation:

Lemma (see [3])

$$\forall\, v(\cdot) \in \mathcal{C}(\mathbb{R}_{\geqslant 0}; \mathbb{R}) \quad \Longrightarrow \quad \forall\, t \geqslant 0: \quad |x_I(t)| \leqslant \max\{\hat{u}_A,\, x_I(0)\} =: x_I^{\max}$$

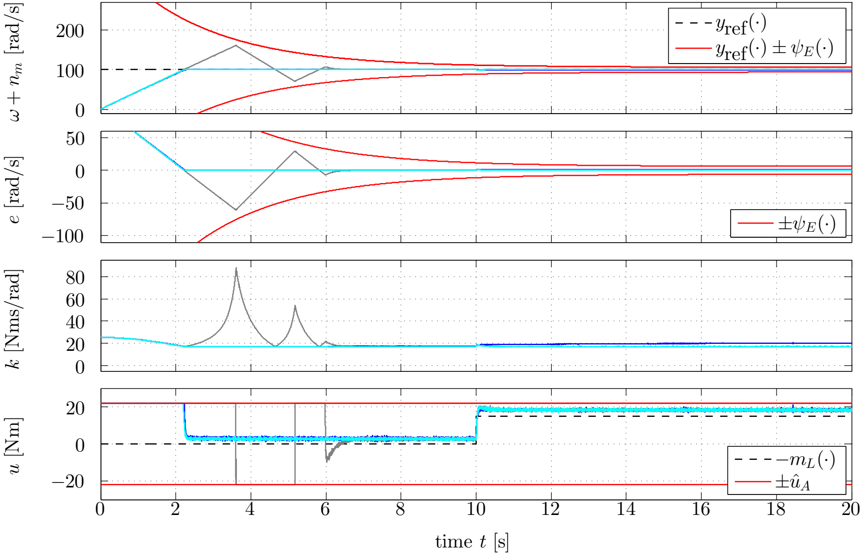
$\Longrightarrow$ Anti-windup: $x_I(\cdot)$ acts as **bounded "input disturbance"**

PI-funnel control with saturation [3]

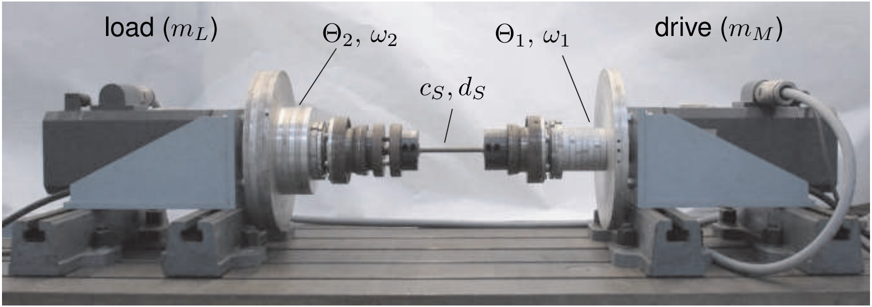


- $\hat{u}_A \geqslant u_{\text{feas}} = u_{\text{feas}}(y_{\text{ref}}, \psi, \text{system}, \ldots) \implies \forall\, t \geqslant 0\colon\ \psi(t) - |e(t)| \geqslant \varepsilon$
- $\varepsilon \leqslant \min\left\{\frac{\lambda}{2},\ \psi(0) - |e(0)|,\ \frac{k_P\lambda}{2(\hat{u}_A + x_I^{\max} + \|m_L\|_\infty)}\right\}$
- $u_{\text{feas}} \approx 63\ [\text{Nm}]$

Measur. results: —— (FC$_1$)+(PI), —— (FC$_1$)+(PI$^{\mathrm{aw}}$)



Funnel control for elastic systems [2]

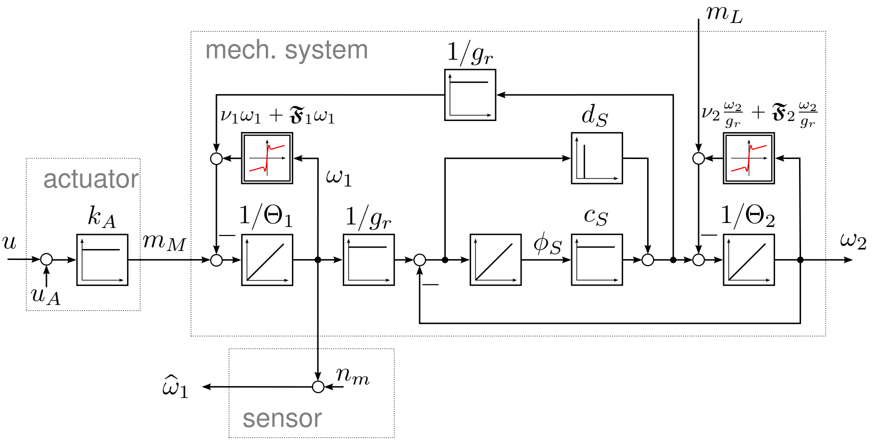


- Goal 1: Reference tracking $\omega_2 \to \omega_{2,\text{ref}}$ (or $\omega_1 \to \omega_{1,\text{ref}}$)
- Goal 2: Active damping of natural frequency

$$\omega_0 = \sqrt{\frac{c_S(\Theta_1+\Theta_2)}{\Theta_1\Theta_2}} \approx 61\,[\text{rad/s}] \qquad \Longrightarrow \qquad f_0 \approx 9.7\,[\text{Hz}]$$

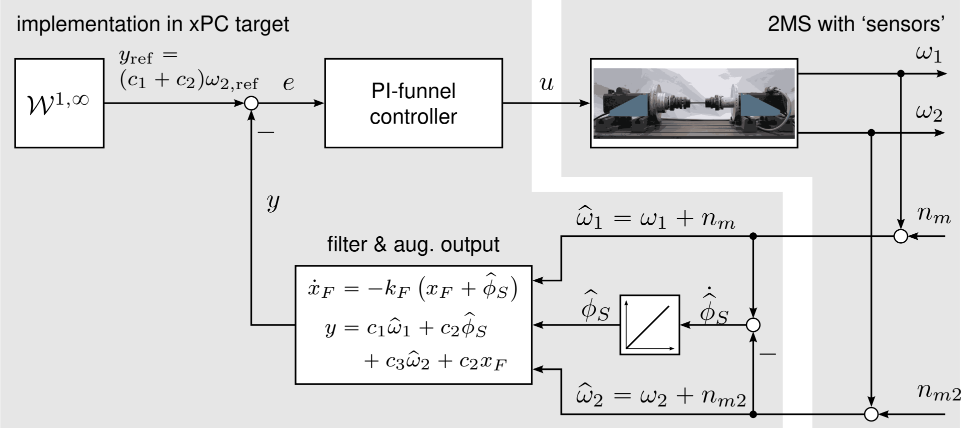
- Constraint: ω_1 as feedback signal only

Elastic system: Two-mass system model



$$\left.\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x}_{\mathrm{S}}(t) &= \boldsymbol{A}_{\mathrm{S}}\boldsymbol{x}_{\mathrm{S}}(t) + \boldsymbol{b}_{\mathrm{S}}\big(u(t) + u_A(t)\big) + \boldsymbol{B}_{\mathrm{S}}\begin{pmatrix} (\mathfrak{F}_1\omega_1)(t) \\ m_L(t) + (\mathfrak{F}_2\omega_2)(t) \end{pmatrix}, \\ y(t) &= (1,\,0,\,0)\,\boldsymbol{x}_{\mathrm{S}}(t), \qquad \boldsymbol{x}_{\mathrm{S}}(0) = (\omega_1^0,\, \phi_S^0, \omega_2^0) \in \mathbb{R}^3 \end{aligned}\right\} \quad \text{(2MS)}$$

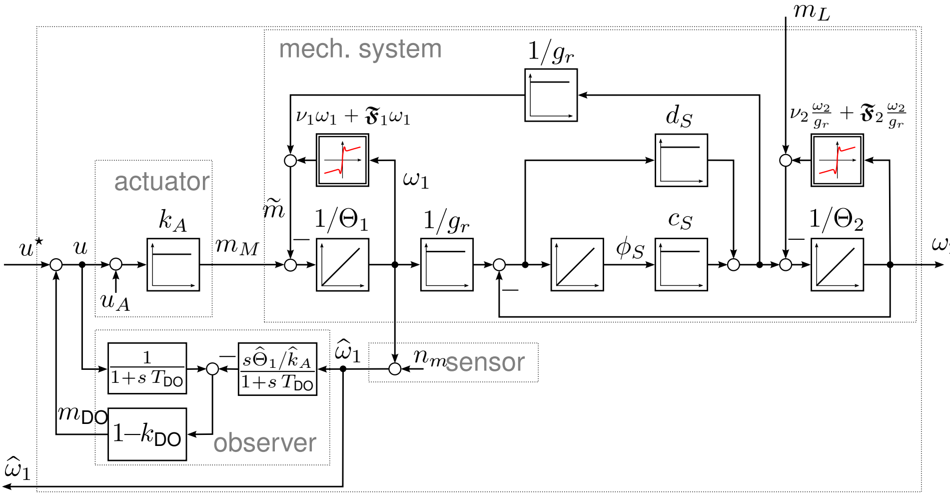
Funnel control for elastic systems with filter and state feedback [13]



$$\boxed{\text{2MS} \in \mathcal{S}_1} \quad \Longleftarrow \quad c_1 > 0 \;\wedge\; \frac{c_2}{g_r} \geqslant 0 \;\wedge\; \frac{c_3}{g_r} > -c_1 \;\wedge\; k_F > 0.$$

For active damping: Feedback of ω_1 and ω_2 necessary!

Elastic system with disturbance observer (DO) [11]



$$\boxed{m_{DO}(s) = \frac{\widetilde{m}(s)}{k_A} - u_A(s)}$$ (for $\widehat{k}_A = k_A,\ \widehat{\Theta}_1 = \Theta_1,\ k_{DO} = T_{DO} = n_m = 0$)

Elastic system with (DO) element of class $\mathcal{S}_1$

Extended two-mass system with $\boldsymbol{x} = (\omega_1,\, \phi_S,\, \omega_2,\, x_F)^\top$

$$\left.\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\,\boldsymbol{x}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}\big(u^\star(t) + u_A(t)\big) + \boldsymbol{B}\begin{pmatrix} (\mathfrak{F}_1\omega_1)(t) \\ m_L(t) + (\mathfrak{F}_2\omega_2)(t) \end{pmatrix}, \\ y(t) &= \boldsymbol{c}^\top \boldsymbol{x}(t), \qquad \boldsymbol{x}(0) = ((\boldsymbol{x}_{\mathrm{S}}^0)^\top,\, 0)^\top \in \mathbb{R}^4 \end{aligned}\right\} \tag{3}$$

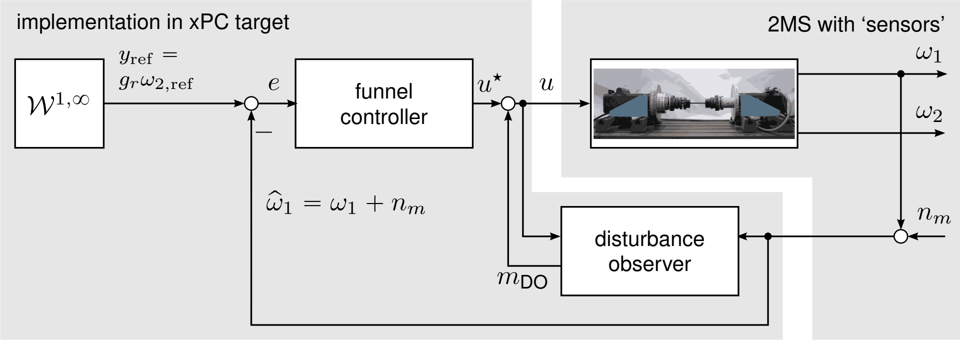
where $\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_S & \star \\ \star & -\frac{k_{\mathsf{DO}}}{T_{\mathsf{DO}}} \end{bmatrix}$, $\boldsymbol{b} = \begin{pmatrix} \boldsymbol{b}_S \\ \frac{1}{T_{\mathsf{DO}}} \end{pmatrix}$, $\boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}_S \\ 0,\; 0 \end{bmatrix}$ and $\boldsymbol{c}^\top = (1,\, 0,\, 0,\, 0)$.

Proposition (see [2])

For $T_{\mathsf{DO}} > 0$, $k_{\mathsf{DO}} \in \mathbb{R}$, $\widehat{k}_A > 0$ and $\widehat{\Theta}_1 > 0$, the extended two-mass system (3) is element of system class $\mathcal{S}_1$.

Proof by checking system properties (i)–(iv) of system class $\mathcal{S}_1$.

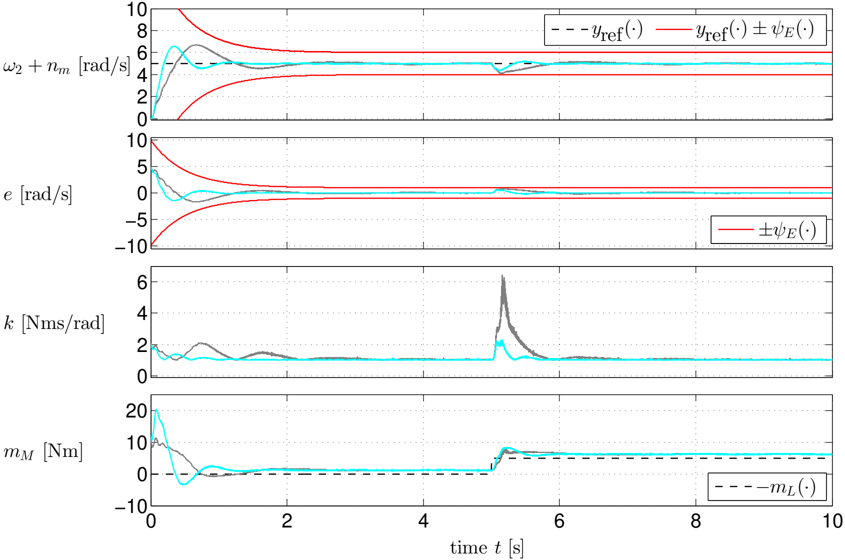
Implementation: Funnel control with (DO)



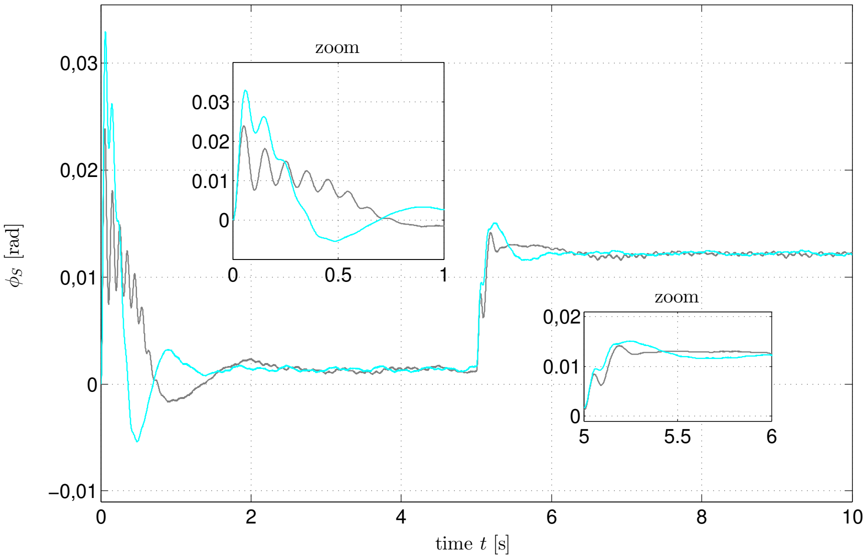
Disturbance observer design

$$k_{\text{DO}} = 0 \quad \wedge \quad T_{\text{DO}} = 0.03\ [\text{s}] \quad \left(\frac{1}{T_{\text{DO}}} = 33.3\,[\text{Hz}] > f_0 = 9.7\,[\text{Hz}]\right)$$

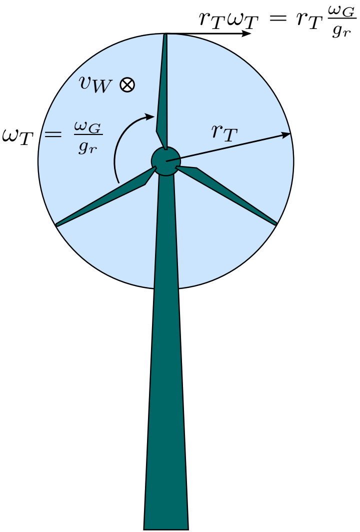
Measur. results: —— (FC_1)+(PI), —— (FC_1)+(DO)



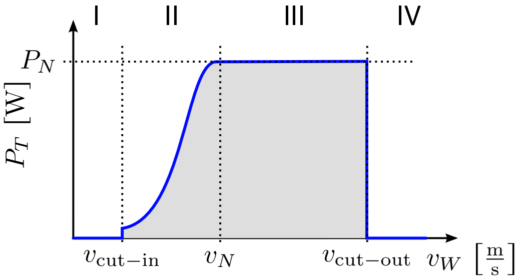
Meas. res. (zoom): —— $(FC_1)+(PI)$, —— $(FC_1)+(DO)$



Funnel control for wind turbine systems [4]



Operation regimes

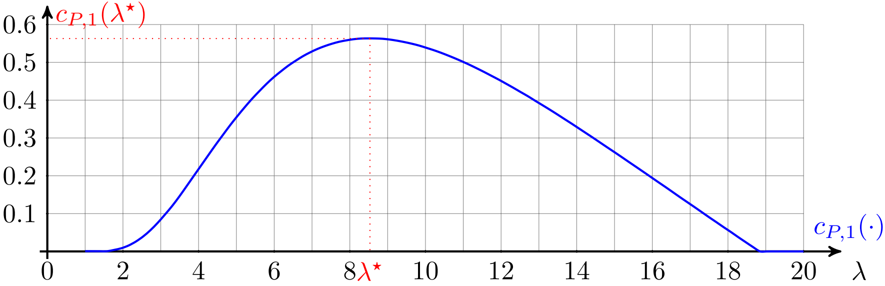


Simple turbine model

$$\frac{\mathrm{d}}{\mathrm{d}t}\omega_G = \frac{1}{\Theta}\left(\frac{m_T(v_W, \beta, \omega_G)}{g_r} + m_{M,\mathrm{ref}}\right)$$

with $\Theta := \Theta_G + \Theta_T/g_r^2$

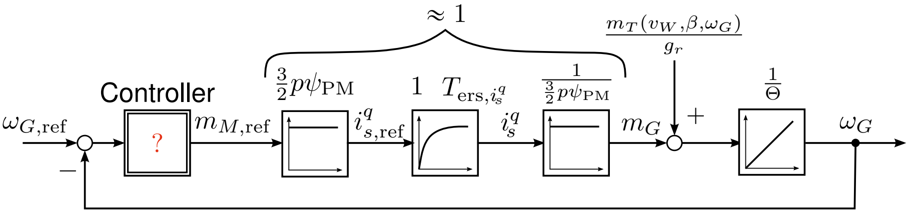
Power coefficient, turbine torque and goal



$$P_T = c_P(v_W, \beta, \omega_G)\underbrace{\frac{1}{2}\varrho r_T^2 \pi v_W^3}_{:=P_W} \implies m_T(v_W, \beta, \omega_G) = \frac{g_r c_P(v_W, \beta, \omega_G) P_W}{\omega_G}$$

Goal for regime II: Maximum power point tracking for fixed $\beta = \beta_0 \geqslant 0$, i.e. $\lambda = \frac{r_T \omega_G}{g_r v_W} \to \lambda^\star$ (or $\omega_G \to \omega_G^\star$) for all $\lambda \in (\lambda_{\min}, \lambda_{\max})$ and $v_{\text{cut-in}} \leqslant v_W < v_N$

Nonlinear controller design



- Standard controller [15]:

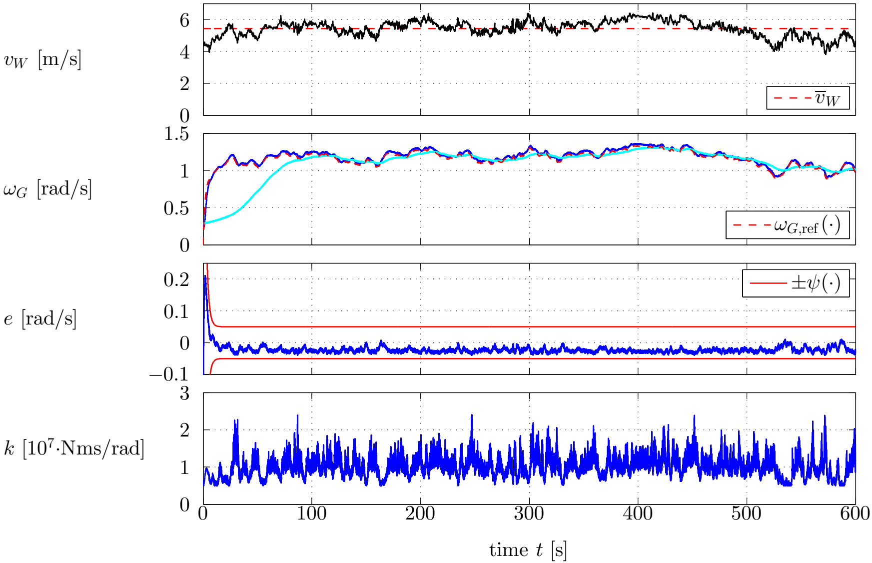
$$m_{G,\mathrm{ref}}(t) = -k_P^\star \omega_G(t)^2 \text{ with } k_P^\star = \frac{1}{2}\varrho r_T^2 \pi \frac{c_P(\beta,\lambda^\star)}{(\lambda^\star)^3} \qquad \text{(WTC)}$$

- Funnel controller:

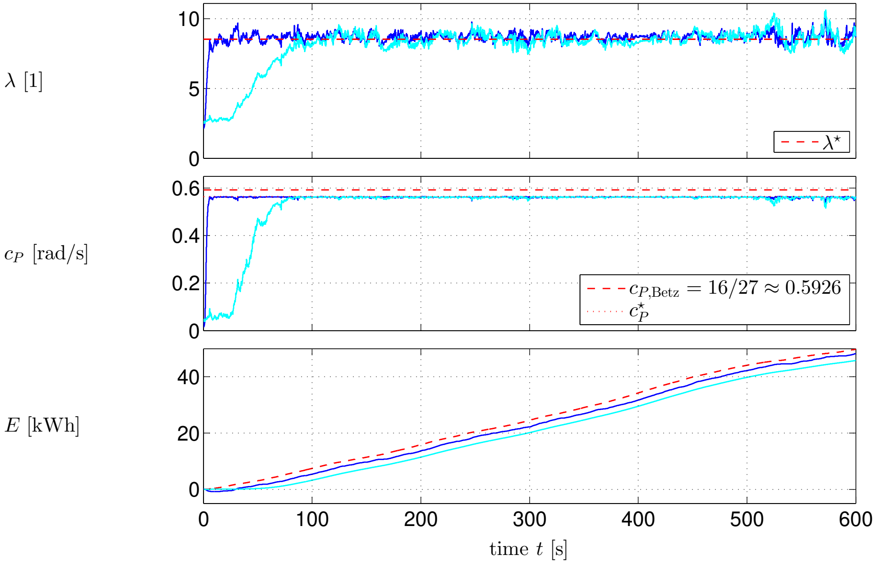
$$m_{G,\mathrm{ref}}(t) = -\underbrace{\frac{\varsigma(t)}{\psi(t) - |e(t)|}}_{=:k(t)} \underbrace{\omega_{G,\mathrm{ref}}(t) - \omega_G(t)}_{=:e(t)} \text{ with } \omega_{G,\mathrm{ref}}(t) = \frac{g_r \lambda^\star}{r_T} v_W(t)$$

$$(\mathrm{FC}_1^{\mathrm{WT}})$$

Simulation results: —— (FC_1^{WT}), —— (WTC)



Simulation results: —— ($\mathrm{FC}_1^{\mathrm{WT}}$), —— (WTC)



Conclusion

To take home

- **funnel control applicable** for speed and position control of
  - **rigid systems**
  - **elastic systems** (active damping by disturbance observer)
  - **rigid-link revolute-joint robotic manipulators** (if inertia matrix is roughly known)
  - **wind turbine systems** is feasible (if rotor radius, optimal tip speed ratio, gear ratio and wind measurements are available)
- **only structural properties** must be checked (robustness)
- **no** system identification or parameter estimation necessary (mainly)
- **time-varying gains** (also decrease possible)
- '**tracking with prescribed transient accuracy**'
- **steady state accuracy** in conjunction with internal model (e.g. PI)
- funnel control in presence of **actuator saturation** is feasible (conservative feasibility condition must be satisfied)
- PI-funnel control **with Anti-windup** feasible (wind-up effect removed)

References

[1] C. M. Hackl. High-gain adaptive position control. *International Journal of Control*, 84(10):1695–1716, 2011.

[2] C. M. Hackl. Funnel control with disturbance observer for two-mass systems. In *Proceedings of the 52nd IEEE Conference on Decision and Control*, pages 6244–6249, 2013.

[3] C. M. Hackl. PI-funnel control with Anti-windup and its application for speed control of electrical drives. In *Proceedings of the 52nd IEEE Conference on Decision and Control*, pages 6250–6255, 2013.

[4] C. M. Hackl. Funnel control for wind turbine systems. In *submitted to the 2014 IEEE Multi-conference on Systems and Control*, 2014.

[5] C. M. Hackl and R. M. Kennel. Position funnel control for rigid revolute joint robotic manipulators with known inertia matrix. In *Proceedings of the 20th Mediterranean Conference on Control and Automation*, pages 615–620, 2012.

[6] C. M. Hackl and R. M. Kennel. Position funnel control with linear internal model. In *Proceedings of 2012 IEEE International Conference on Control Applications*, pages 1334–1339, 2012.

[7] C. M. Hackl, A. G. Hofmann, R. W. De Doncker, and R. M. Kennel. Funnel control for speed & position control of electrical drives: A survey. In *Proceedings of the 19th Mediterranean Conference on Control and Automation*, pages 181–188, 2011.

[8] C. M. Hackl, A. G. Hofmann, and R. M. Kennel. Funnel control in mechatronics: An overview. In *Proceedings of the 50th IEEE Conference on Decision and Control and European Control Conference*, pages 8000–8007, 2011.

[9] C. M. Hackl, N. Hopfe, A. Ilchmann, M. Mueller, and S. Trenn. Funnel control for systems with relative degree two. *SIAM Journal on Control and Optimization*, 51(2):965–995, 2013.

[10] N. Hopfe, A. Ilchmann, and E. P. Ryan. Funnel control with saturation: Nonlinear SISO systems. *IEEE Transactions on Automatic Control*, 55(9): 2177–2182, 2010.

[11] Y. Hori, H. Sawada, and Y. Chun. Slow resonance ratio control for vibration suppression and disturbance rejection in torsional system. *IEEE Transactions on Industrial Electronics*, 46(1):162–168, 1999.

[12] A. Ilchmann and E. P. Ryan. Asymptotic tracking with prescribed transient behaviour for linear systems. *International Journal of Control*, 79(8): 910–917, 2006.

[13] A. Ilchmann and H. Schuster. PI-funnel control for two mass systems. *IEEE Transactions on Automatic Control*, 54(4):918–923, 2009.

[14] A. Ilchmann, E. P. Ryan, and C. J. Sangwin. Tracking with prescribed transient behaviour. *ESAIM: Control, Optimisation and Calculus of Variations*, 7: 471–493, 2002.

[15] L. Y. Pao and K. E. Johnson. Control of wind turbines: Approaches, challenges, and recent developments. *IEEE Control Systems Magazine*, 31(2): 44–62, 2011.