PI-funnel control with Anti-windup

Christoph Hackl

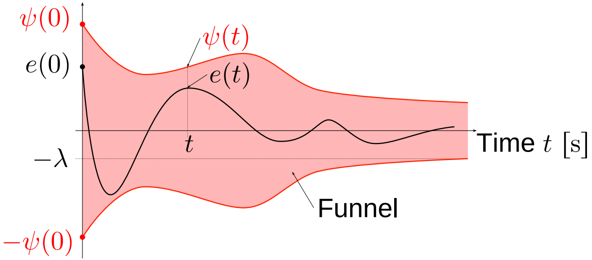
Technische Universität München
Institute for Electrical Drive Systems and Power Electronics

52^{nd} IEEE Conference on Decision and Control, Florence
13^{th} December 2013

Outline

1. PI-funnel control
2. PI-funnel control with Anti-windup
3. Conclusions and future work

Funnel control [Ilchmann et al., 2002]



- Tracking with prescribed transient accuracy by design of $\psi(\cdot)$, i.e.

$$\forall\, t \geqslant 0: \qquad |e(t)| = |y_{\mathrm{ref}}(t) - y(t)| < \psi(t)$$

- Controller

$$u(t) = \underbrace{\frac{1}{\psi(t) - |e(t)|}}_{=:k(t)}\, e(t)$$

Admissible system class

Structural knowledge sufficient:

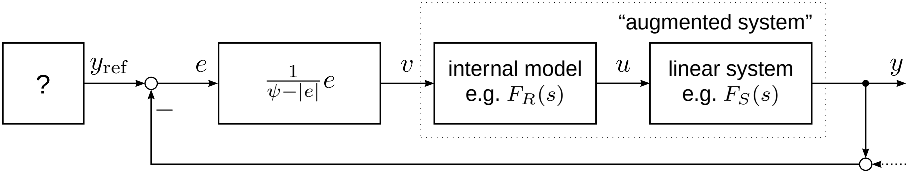
- relative degree one (or known relative degree [Ilchmann et al., 2007])
- BIBO stable zero dynamics (or minimum phase for linear systems)
- known sign of the high-frequency gain

Example (LTI SISO system in Byrnes-Isidori form)

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} y(t) \\ \boldsymbol{z}(t) \end{pmatrix} = \begin{bmatrix} a_1 & \boldsymbol{a}_2^\top \\ \boldsymbol{a}_3 & \boldsymbol{A}_4 \end{bmatrix}\begin{pmatrix} y(t) \\ \boldsymbol{z}(t) \end{pmatrix} + \begin{pmatrix} \gamma_0 \\ \boldsymbol{0}_{n-1} \end{pmatrix} u(t) \,, \qquad \begin{pmatrix} y(0) \\ \boldsymbol{z}(0) \end{pmatrix} = \begin{pmatrix} y_0 \\ \boldsymbol{z}_0 \end{pmatrix} \in \mathbb{R}^n$$

- $\gamma_0 \neq 0$
- $\mathrm{spec}(\boldsymbol{A}_4) \in \mathbb{C}_{<0}$
- $\mathrm{sign}(\gamma_0)$ known

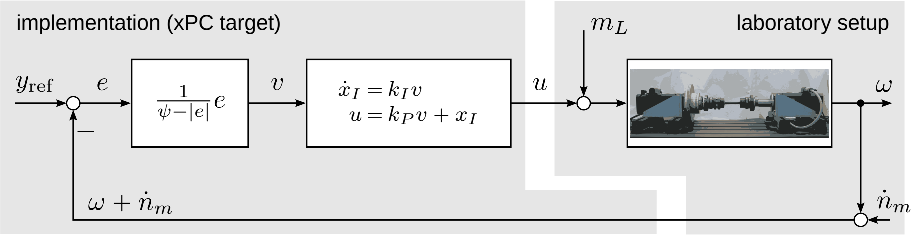
FC with internal model [Ilchmann and Ryan, 2006]



Example ("Augmented system" in the frequency domain)

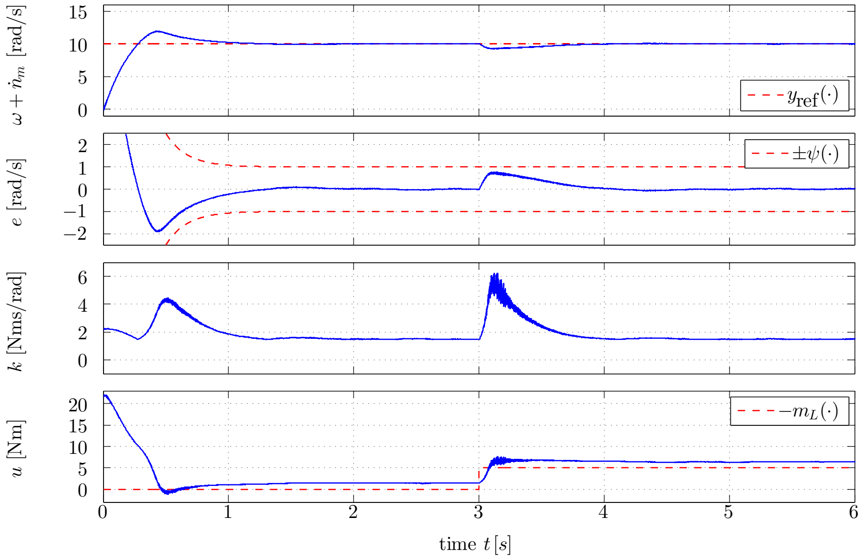
- PI controller: $F_R(s) = k_P + \frac{k_I}{s} = \frac{k_P\, s + k_I}{s}, \qquad k_P,\, k_I > 0$
- system: $F_S(s) = \frac{1+s}{s^2 - s + 1}$
- "augmented system": $F_R(s)F_S(s) = \frac{k_P\, s + k_I}{s} \cdot \frac{1+s}{s^2 - s + 1}$
- $\Longrightarrow \lim_{t\to\infty} e(t) = 0$ for $y_{\text{ref}}(\cdot) =$const.

PI-funnel control for electrical drives [Ilchmann and Schuster, 2009]

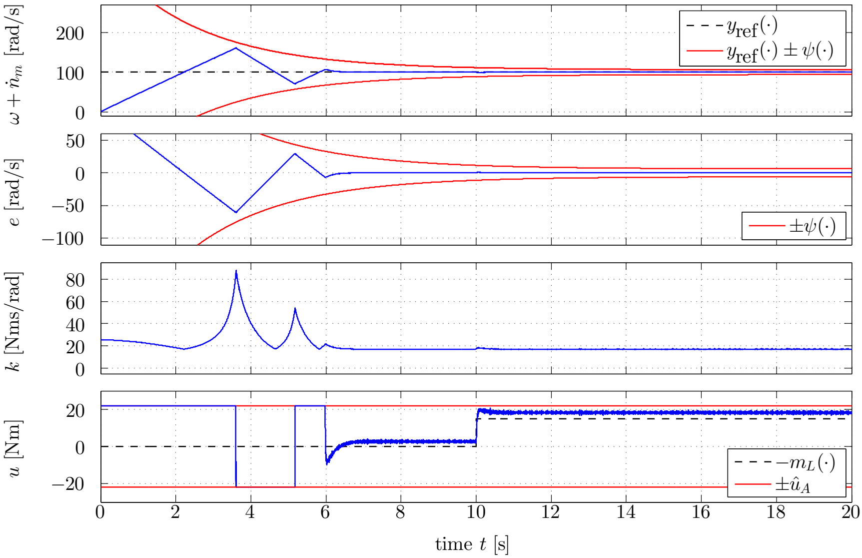


- Scenario 1: $y_{\mathrm{ref}}(\cdot) = 10$ [rad/s]
- Scenario 2: $y_{\mathrm{ref}}(\cdot) = 100$ [rad/s]

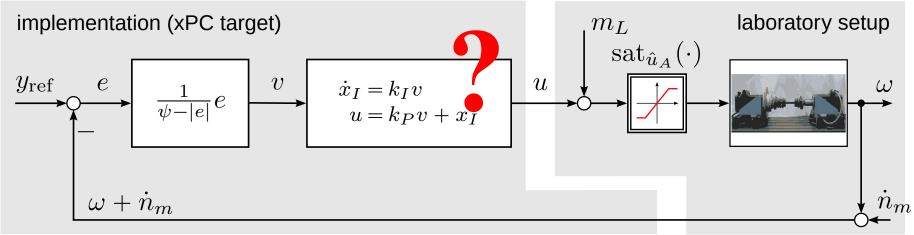
Measurement results for $y_{\mathrm{ref}}(\cdot) = 10$: —— PI-FC



Measurement results for $y_{\text{ref}}(\cdot) = 100$: —— PI-FC



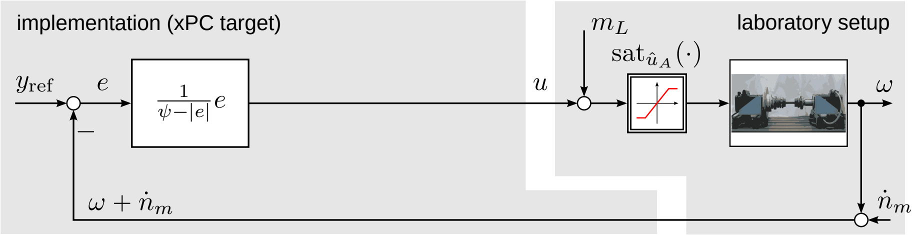
PI-funnel control with saturation



- Tracking with prescribed transient accuracy endangered, i.e.

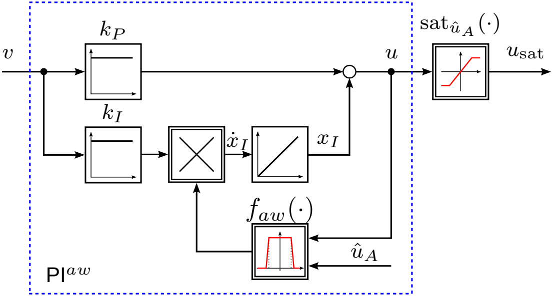
$$\cancel{\forall\, t \geqslant 0: \quad |e(t)| < \psi(t)} \qquad ?$$

Funnel control with saturation [Hopfe et al., 2010]



- $\hat{u}_A \geqslant u_{\mathrm{feas}} = u_{\mathrm{feas}}(y_{\mathrm{ref}}, \psi, \text{system}, \dots) \implies \forall\, t \geqslant 0\colon\ \psi(t) - |e(t)| \geqslant \varepsilon$
- $\varepsilon \leqslant \min\left\{\frac{\lambda}{2},\, \psi(0) - |e(0)|,\, \frac{\lambda}{2(\hat{u}_A + \|m_L\|_\infty)}\right\}$
- u_{feas} conservative (for setup $u_{\mathrm{feas}} \approx 63\ [\mathrm{Nm}]$)

PI controller with Anti-windup ("conditional integration")



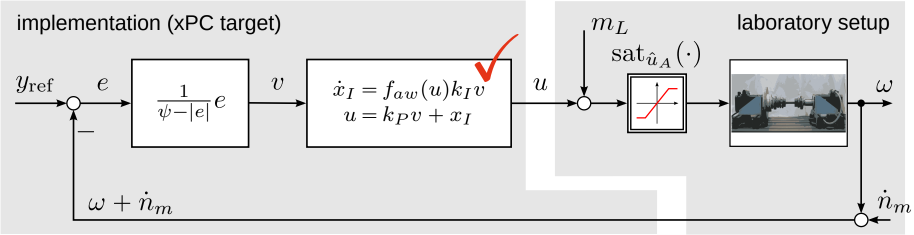
System theoretical interpretation:

Lemma

$$\forall\, v(\cdot) \in \mathcal{C}(\mathbb{R}_{\geqslant 0};\mathbb{R}) \quad \Longrightarrow \quad \forall\, t \geqslant 0: \quad |x_I(t)| \leqslant \max\{\hat{u}_A,\, x_I(0)\} =: x_I^{\max}$$

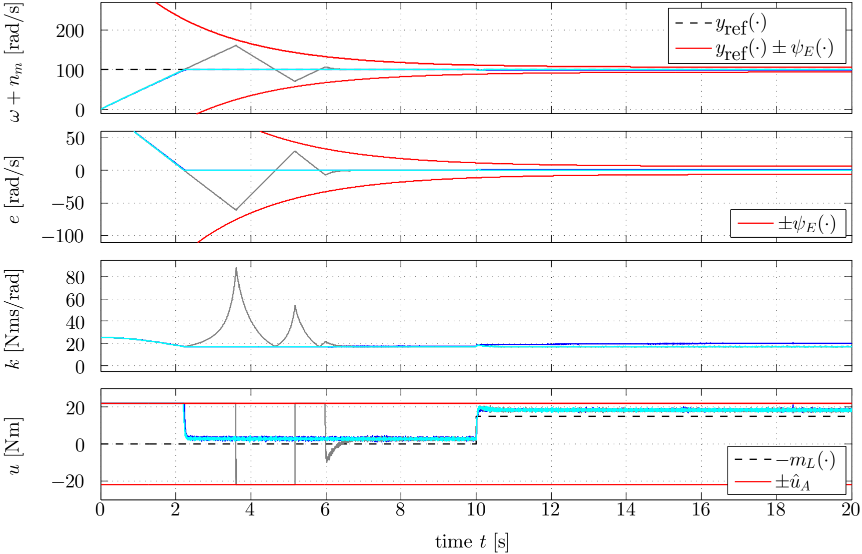
$\Longrightarrow$ Anti-windup: $x_I(\cdot)$ acts as **bounded "input disturbance"**

PI-funnel control with saturation



- $\hat{u}_A \geqslant u_{\text{feas}} = u_{\text{feas}}(y_{\text{ref}}, \psi, \text{system}, \ldots) \implies \forall\, t \geqslant 0\colon\ \psi(t) - |e(t)| \geqslant \varepsilon$
- $\varepsilon \leqslant \min\left\{ \frac{\lambda}{2},\ \psi(0) - |e(0)|,\ \frac{k_P \lambda}{2(\hat{u}_A + x_I^{\max} + \|m_L\|_\infty)} \right\}$
- $u_{\text{feas}} \approx 63\ [\text{Nm}]$

Results for $y_{\mathrm{ref}}(\cdot) = 100$: —— PI-FC, —— PI^{aw}-FC



Conclusions and future work

To take home:

- PI-funnel control: windup possible
- use of anti-windup (“conditional integration”)
  - windup eliminated
  - improved control performance (no “saw tooth response”)
  - integral control action acts as “bounded input disturbance”

$\Longrightarrow$ PI$^{\mathrm{aw}}$-funnel control admissible for systems with saturation

Open questions:

- extension feasible?
  - ... PI-funnel control for systems with relative degree two?
  - ... funnel control with internal model?
- ... different anti-windup strategies beneficial?

References

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