Funnel control with disturbance observer for two-mass systems

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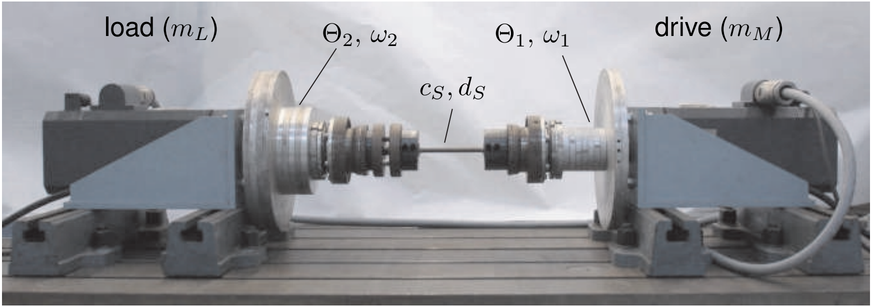
52$^{\mathrm{nd}}$ IEEE Conference on Decision and Control, Florence
13$^{\mathrm{th}}$ December 2013

Outline

Problem: Speed control of a two-mass system (2MS)

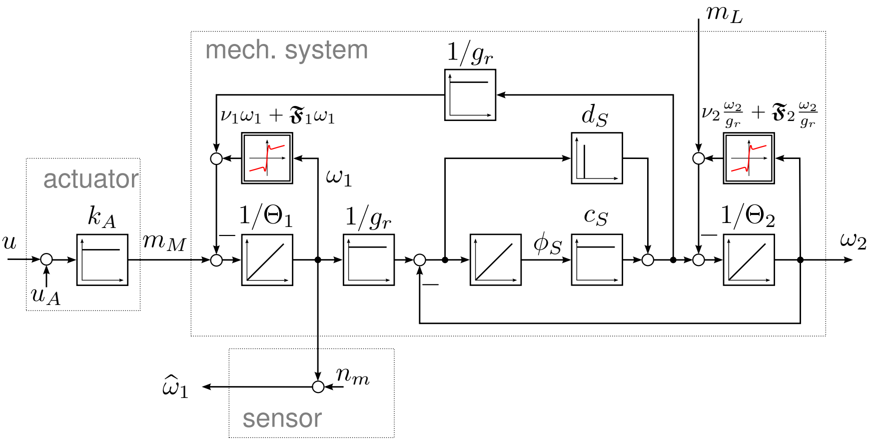


- reference tracking $\omega_2 \to \omega_{2,\mathrm{ref}}$ (or $\omega_1 \to \omega_{1,\mathrm{ref}}$)
- active damping of natural frequency

$$\omega_0 = \sqrt{\frac{c_S(\Theta_1 + \Theta_2)}{\Theta_1\Theta_2}} \approx 61\,[\mathrm{rad/s}] \qquad \Longrightarrow \qquad f_0 \approx 9.7\,[\mathrm{Hz}]$$

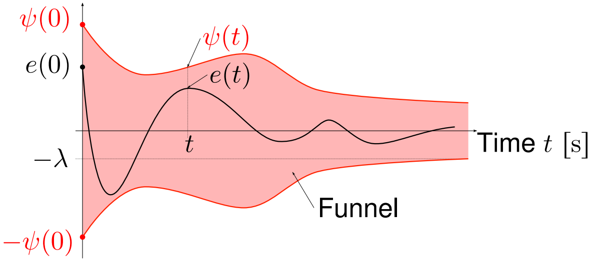
- ω_1 as feedback signal only

Two-mass system model



$$\left.\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\,\boldsymbol{x}_\mathrm{S}(t) &= \boldsymbol{A}_\mathrm{S}\boldsymbol{x}_\mathrm{S}(t) + \boldsymbol{b}_\mathrm{S}\big(u(t)+u_A(t)\big) + \boldsymbol{B}_\mathrm{S}\begin{pmatrix}(\mathfrak{F}_1\omega_1)(t)\\ m_L(t)+(\mathfrak{F}_2\omega_2)(t)\end{pmatrix}, \\ y(t) &= (1,\,0,\,0)\,\boldsymbol{x}_\mathrm{S}(t), \qquad \boldsymbol{x}_\mathrm{S}(0) = (\omega_1^0,\,\phi_S^0,\omega_2^0)\in\mathbb{R}^3 \end{aligned}\right\} \quad \text{(2MS)}$$

Funnel control [Ilchmann et al., 2002]



- Tracking with prescribed transient accuracy by design of $\psi(\cdot)$, i.e.

$$\forall\, t \geqslant 0: \qquad |e(t)| = |y_{\mathrm{ref}}(t) - y(t)| < \psi(t)$$

- Controller

$$u(t) = \underbrace{\frac{1}{\psi(t) - |e(t)|}}_{=:k(t)}\, e(t)$$

Admissible system class $\mathcal{S}$ (simplified)

System

$$\left.\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}\big(u(t) + u_d(t)\big) + \boldsymbol{B}_{\mathfrak{T}}\Big(\boldsymbol{d}(t) + (\mathfrak{T}\boldsymbol{x})(t)\Big) \\ y(t) &= \boldsymbol{c}^\top \boldsymbol{x}(t), \end{aligned}\right\} \tag{1}$$

satisfying

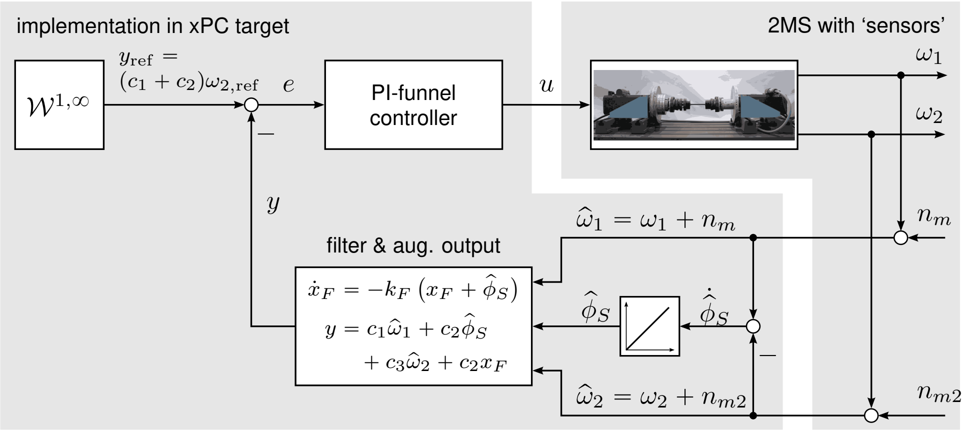
(i) $\gamma_0 := \boldsymbol{c}^\top \boldsymbol{b} \neq 0$ and $\operatorname{sign}(\gamma_0)$ known;

(ii) $\forall\, s \in \mathbb{C}$ with $\Re(s) \geqslant 0$: $\quad \det \begin{bmatrix} s\boldsymbol{I}_n - \boldsymbol{A} & \boldsymbol{b} \\ \boldsymbol{c}^\top & 0 \end{bmatrix} \neq 0;$

(iii) operator $\mathfrak{T}$ is of class $\mathcal{T}$ (see Def. 1 in Ilchmann et al. [2002]) and $M_{\mathfrak{T}} := \sup\left\{ \|(\mathfrak{T}\boldsymbol{\xi})(t)\| \;\middle|\; t \geqslant 0,\ \boldsymbol{\xi}(\cdot) \in \mathcal{C}([-h, \infty), \mathbb{R}^n) \right\} < \infty;$

(iv) $u_d(\cdot) \in \mathcal{L}^\infty([-h, \infty); \mathbb{R})$ and $\boldsymbol{d}(\cdot) \in \mathcal{L}^\infty([-h, \infty); \mathbb{R}^m)$.

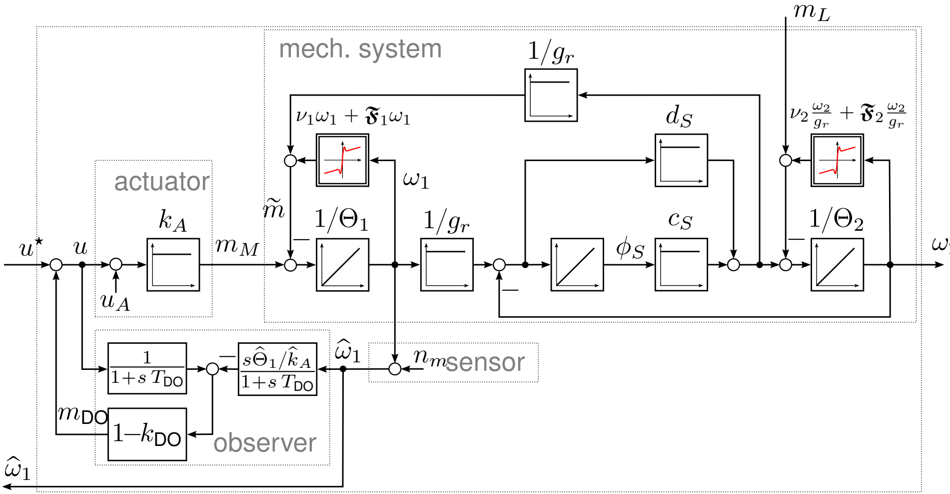
Funnel control for 2MS [Ilchmann and Schuster, 2009]



$$\boxed{\text{2MS} \in \mathcal{S}} \quad \Longleftarrow \quad c_1 > 0 \;\wedge\; \frac{c_2}{g_r} \geqslant 0 \;\wedge\; \frac{c_3}{g_r} > -c_1 \;\wedge\; k_F > 0.$$

For active damping: Feedback of ω_1 and ω_2 necessary!

2MS with disturbance observer (DO) [Hori et al., 1999]



$$\boxed{m_{DO}(s) = \frac{\widetilde{m}(s)}{k_A} - u_A(s)}$$ (for $\widehat{k}_A = k_A, \widehat{\Theta}_1 = \Theta_1, k_{DO} = T_{DO} = n_m = 0$)

2MS with disturbance observer element of class $\mathcal{S}$

Extended two-mass system with $\boldsymbol{x} = (\omega_1,\, \phi_S,\, \omega_2,\, x_F)^\top$

$$\left.\begin{aligned} \tfrac{\mathrm{d}}{\mathrm{d}t}\,\boldsymbol{x}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}\big(u^\star(t) + u_A(t)\big) + \boldsymbol{B}\begin{pmatrix} (\mathfrak{F}_1\omega_1)(t) \\ m_L(t) + (\mathfrak{F}_2\omega_2)(t)\end{pmatrix}, \\ y(t) &= \boldsymbol{c}^\top \boldsymbol{x}(t), \qquad \boldsymbol{x}(0) = ((\boldsymbol{x}_{\mathrm{S}}^0)^\top,\, 0)^\top \in \mathbb{R}^4 \end{aligned}\right\} \tag{2}$$

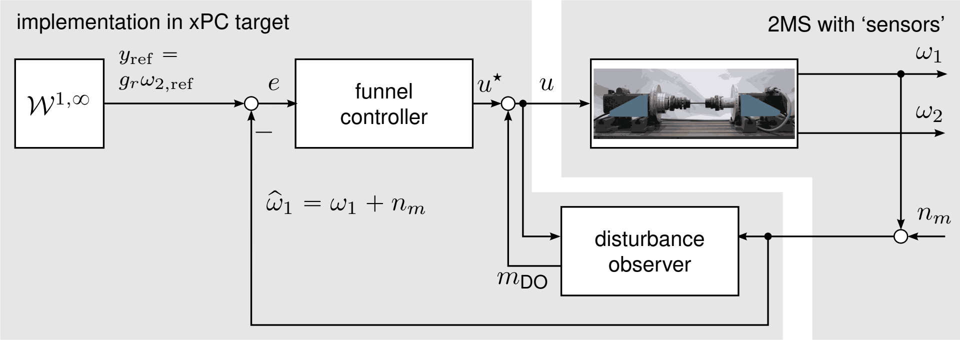
where $\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_S & \star \\ \star & -\frac{k_{\mathrm{DO}}}{T_{\mathrm{DO}}}\end{bmatrix}$, $\boldsymbol{b} = \begin{pmatrix} \boldsymbol{b}_S \\ \frac{1}{T_{\mathrm{DO}}}\end{pmatrix}$, $\boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}_S \\ 0,\ 0\end{bmatrix}$ and $\boldsymbol{c}^\top = (1,\, 0,\, 0,\, 0)$.

Proposition

For $T_{\mathrm{DO}} > 0$, $k_{\mathrm{DO}} \in \mathbb{R}$, $\widehat{k}_A > 0$ and $\widehat{\Theta}_1 > 0$, the extended two-mass system (2) is element of system class $\mathcal{S}$.

Proof by checking system properties (i)–(iv) of system class $\mathcal{S}$.

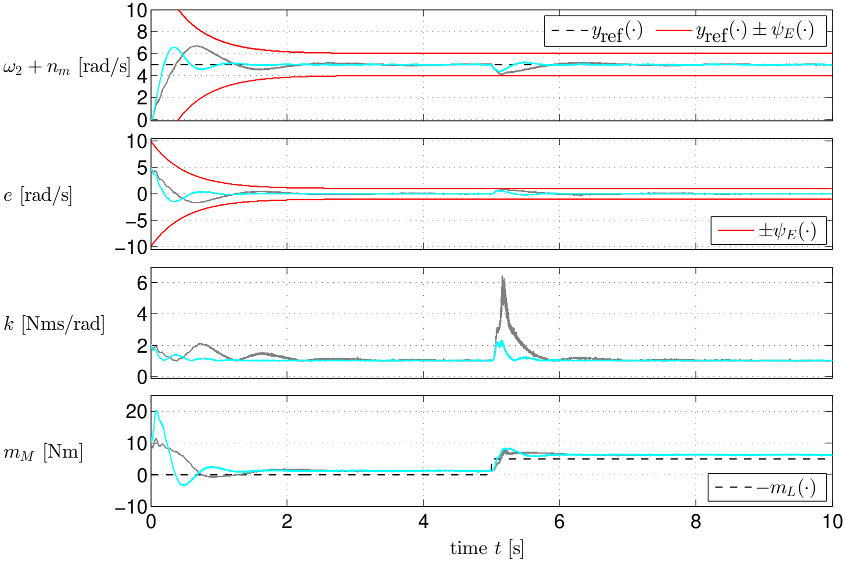
Implementation



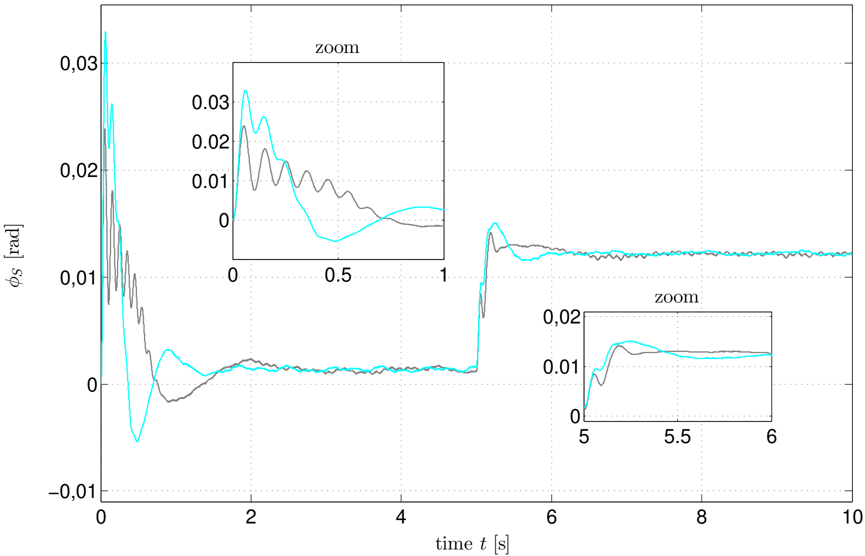
Disturbance observer design

$$k_{\mathrm{DO}} = 0 \qquad \wedge \qquad T_{\mathrm{DO}} = 0.03\ [\mathrm{s}] \qquad \left(\frac{1}{T_{\mathrm{DO}}} = 33.3\,[\mathrm{Hz}] > f_0 = 9.7\,[\mathrm{Hz}]\right)$$

Measurement results: —— PI+FC, —— FC+DO



Meas. results (zoom): —— PI+FC, —— FC+DO



Conclusions and future work

To take home:

- Funnel control with disturbance observer admissible for 2MS (mathematical and experimental validation)
- Active damping of oscillations
- Tracking with prescribed transient accuracy (on motor side)

Open questions:

- ... impact of actuator saturation?
- ... prescribed accuracy also for load side?
- ... other (disturbance) observer designs admissible and beneficial?

References

Y. Hori, H. Sawada, and Y. Chun. Slow resonance ratio control for vibration suppression and disturbance rejection in torsional system. *IEEE Transactions on Industrial Electronics*, 46(1): 162–168, 1999.

A. Ilchmann and H. Schuster. PI-funnel control for two mass systems. *IEEE Transactions on Automatic Control*, 54(4):918–923, 2009.

A. Ilchmann, E. P. Ryan, and C. J. Sangwin. Tracking with prescribed transient behaviour. *ESAIM: Control, Optimisation and Calculus of Variations*, 7:471–493, 2002.