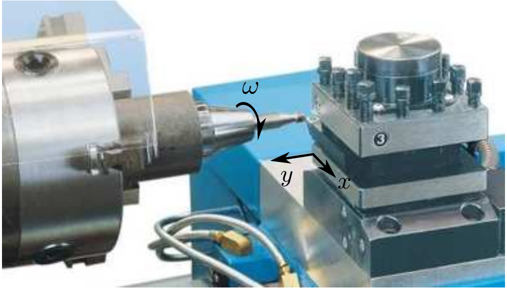
Funnel control for motion control of mechatronic systems

Christoph Hackl

Hochschule Augsburg, University of Applied Sciences
December 6^{th}, 2013

Outline

Reference tracking under load (CNC turning machine)



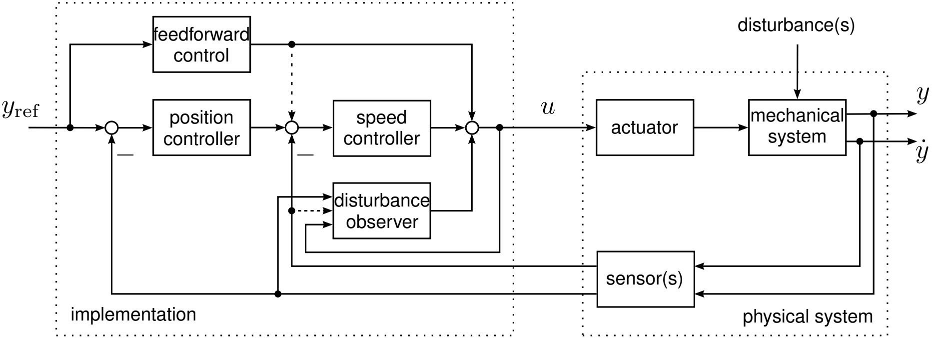
(http://www.knuth.de)

- Problem statement
  - reference $(\omega_{\mathrm{ref}}, x_{\mathrm{ref}}, y_{\mathrm{ref}}) : \mathbb{R}_{\geqslant 0} \to \mathbb{R}^3$
  - precise position and speed control, e.g. for prescribed $\lambda > 0$

$$\boxed{\forall\, t \geqslant t_0: \qquad |e(t)| = |y_{\mathrm{ref}}(t) - y(t)| \leqslant \lambda.}$$

- Challenges
  - nonlinear effects (e.g. actuator saturation, friction)
  - friction and loads (disturbances) unknown and varying
  - uncertain system parameters

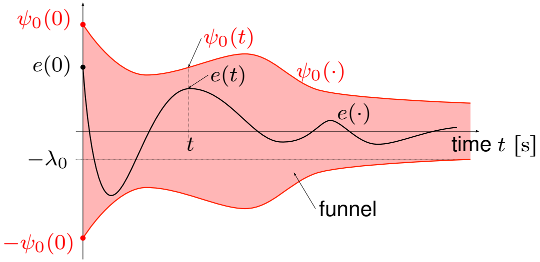
Standard solution (one axis)



- *heuristic or empirical* controller design ⇒ 'trial-and-error'
- *model-based* controller design ⇒ system identification
- *general challenge* ⇒ desired tracking accuracy *not* included in design

$$\cancel{\forall\, t \geqslant t_0: \qquad |e(t)| = |y_{\mathrm{ref}}(t) - y(t)| \leqslant \lambda} \quad ?$$

Speed funnel control (relative-degree-one case)



- control objective: 'prescribed transient accuracy' via funnel boundary design

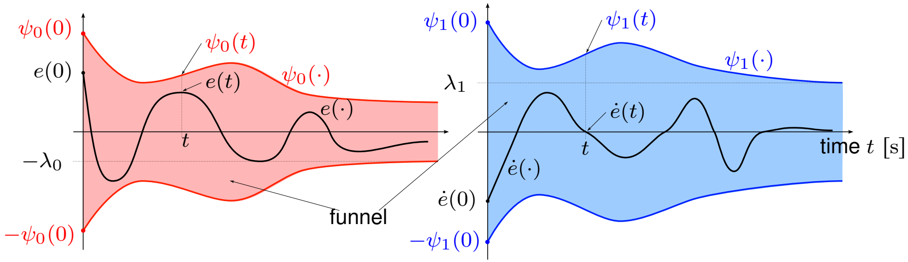
$$\boxed{\forall\, t \geqslant 0: \quad |e(t)| = |\omega_{\mathrm{ref}}(t) - \omega(t)| < \psi_0(t)}$$

- "adaptive" proportional controller (see [1] and [2]):

$$\boxed{u(t) = k_0(t)\, e(t)} \qquad (\mathrm{FC}_1)$$

where $\quad k_0(t) = \dfrac{1}{\psi_0(t) - |e(t)|}.$

Position funnel control (relative-degree-two case)



- control objective: 'prescribed transient accuracy' via funnel boundary design

$$\boxed{\forall\, t \geqslant 0\colon \quad |e(t)| = |y_{\text{ref}}(t) - y(t)| < \psi_0(t) \quad \text{and} \quad |\dot{e}(t)| < \psi_1(t)}$$

- "adaptive" proportional-derivative controller (see [3] and [4]):

$$\boxed{u(t) \;=\; \underbrace{k_0(t)^2\, e(t)}_{\text{P-part}} \;+\; \underbrace{k_0(t)k_1(t)\dot{e}(t)}_{\text{D-part}}} \qquad (\text{FC}_2)$$

where $k_0(t) = \dfrac{1}{\psi_0(t) - |e(t)|}$ and $k_1(t) = \dfrac{1}{\psi_1(t) - |\dot{e}(t)|}$.

Why funnel control for motion control problems?

- simple
- prescribed tracking accuracy
- high gains reduce/eliminate ‘stick-slip’
- robust: “structural” system properties sufficient (system classes $\mathcal{S}_1$ or $\mathcal{S}_2$)
  - known relative degree (pole excess): $r = 1$ or $r = 2$
  - positive high-frequency gain (or known sign)
  - minimum-phase (all zeros in left complex half-plane)
  - for $r = 2$: derivative feedback available ($\Longrightarrow$ PD controller)

Example (LTI SISO system in frequency domain)

$$F(s) = \frac{y(s)}{u(s)} = \gamma_0 \frac{N(s)}{D(s)} = 5 \frac{s+1}{s^2(s+2)}$$

- $\deg(D) - \deg(N) = 3 - 1$ (pole excess) $\Longrightarrow r = 2$
- $\gamma_0 = 5 > 0 \Longrightarrow$ positive high-frequency gain
- $N(s) = s + 1 \Longrightarrow$ zero: $z_1 = -1 \Longrightarrow$ minimum-phase
- measured signals: $y(\cdot)$ and $\dot{y}(\cdot) \Longrightarrow \boxed{F(s) \in \mathcal{S}_2}$

Funnel control and steady state accuracy

- $\lim_{t\to\infty} e(t) = 0$ not guaranteed (since $\psi_0(t) \geqslant \lambda_0 > 0$ for all $t \geqslant 0$)
- PI controller

$$v(s) = \underbrace{1 + \frac{k_I}{s}}_{=:F_{PI}(s)} = \frac{s + k_I}{s}\, u(s), \quad k_I > 0 \qquad \text{(PI)}$$

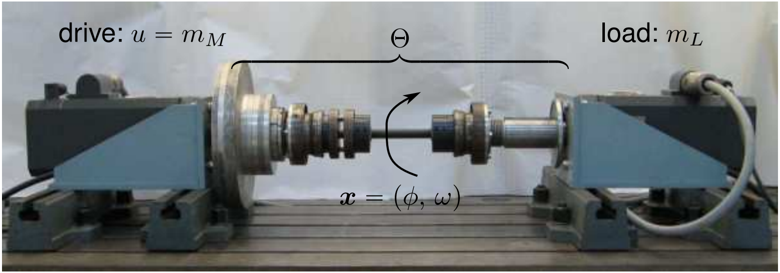
- (PI) does not change system affiliation!

Example (PI in series to LTI SISO system)

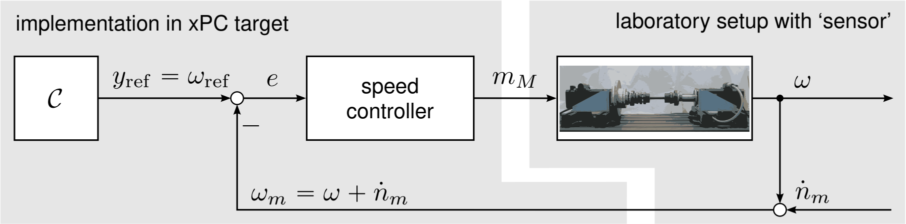
$$\hat{F}(s) = \frac{y(s)}{v(s)} = \hat{\gamma}_0 \frac{\hat{N}(s)}{\hat{D}(s)} = F_{PI}(s)\, F(s) = 5\, \frac{s + k_I}{s} \frac{s+1}{s^2(s+2)}$$

- $\deg(\hat{D}) - \deg(\hat{N}) = 4 - 2$ (pole excess) $\Longrightarrow \hat{r} = 2$
- $\widehat{\gamma_0} = 5 > 0 \Longrightarrow$ positive high-frequency gain
- $\hat{N}(s) = (s+1)(s+k_I) \Longrightarrow$ zeros: $z_1 = -1$ and $z_2 = -k_I \Longrightarrow$ minimum-phase
- measured signals: $y(\cdot)$ and $\dot{y}(\cdot) \Longrightarrow \boxed{\hat{F}(s) \in \mathcal{S}_2}$

Laboratory setup: Stiffly coupled system



Speed control implementation



- Standard PI controller

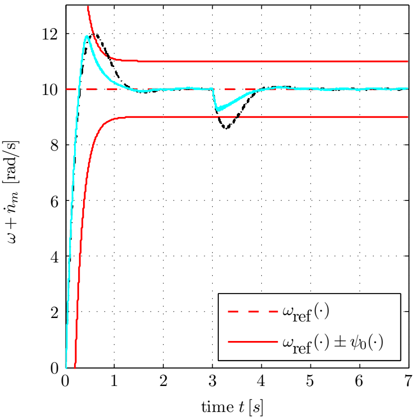
$$m_M(t) = k_P\, e(t) + k_I \int_0^t e(\tau)\,\mathrm{d}\tau$$

- PI-funnel controller

$$m_M(t) = k_0(t)\, e(t) + k_I \int_0^t k_0(\tau) e(\tau)\,\mathrm{d}\tau \quad \text{where} \quad k_0(t) = \frac{1}{\psi_0(t) - |e(t)|}$$

Speed control measurement results

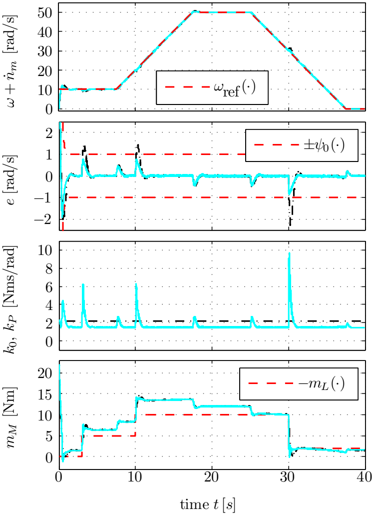
Set-point tracking



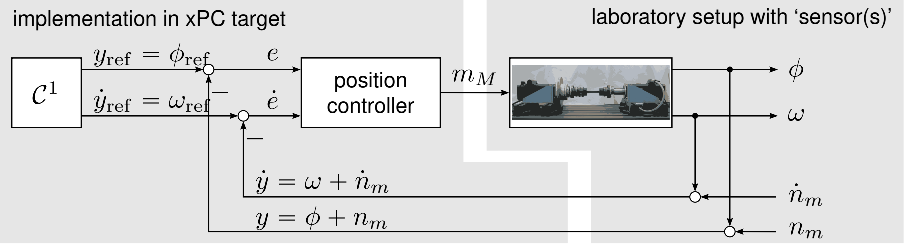
PI-funnel controller

PI controller

Reference tracking



Position control implementation



- Standard PID controller (with feedforward)

$$m_M(t) = k_P\, e(t) + k_I \int_0^t e(\tau)\,\mathrm{d}\tau \; + k_D \dot{e}(t) + u_F(t)$$

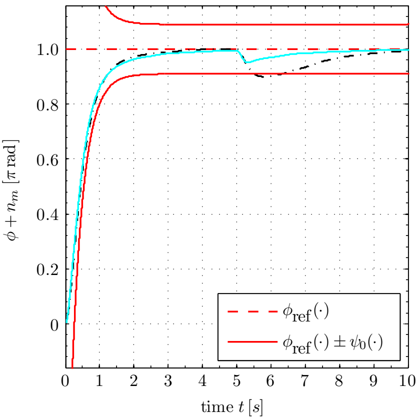
- PID-funnel controller

$$m_M(t) = k_0(t)^2 e(t) + k_0(t) k_1(t) \dot{e}(t) + k_I \int_0^t k_0(\tau)^2 e(\tau) + k_1(\tau) k_0(\tau) \dot{e}(\tau)\,\mathrm{d}\tau$$

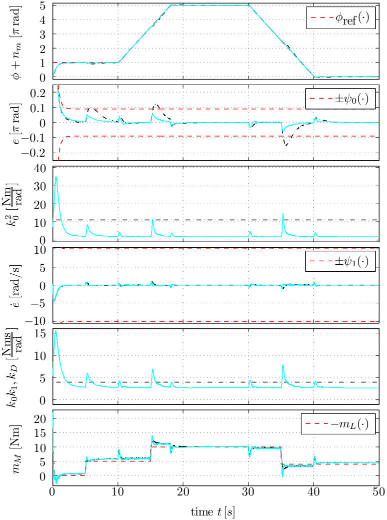
where $k_0(t) = \frac{1}{\psi_0(t) - |e(t)|}$ and $k_1(t) = \frac{1}{\psi_1(t) - |\dot{e}(t)|}$

Position control measurement results

Set-point tracking



Reference tracking



PID-funnel controller

PID controller (with feedforward)

References

[1] C. M. Hackl, A. G. Hofmann, R. W. De Doncker, and R. M. Kennel. Funnel control for speed & position control of electrical drives: A survey. In *Proceedings of the 19th Mediterranean Conference on Control and Automation*, pages 181–188, Corfu, Greece, 2011.

[2] C. M. Hackl, A. G. Hofmann, and R. M. Kennel. Funnel control in mechatronics: An overview. In *Proceedings of the 50th IEEE Conference on Decision and Control and European Control Conference*, pages 8000–8007, Orlando, USA, 2011.

[3] C. M. Hackl, N. Hopfe, A. Ilchmann, M. Mueller, and S. Trenn. Funnel control for systems with relative degree two. *SIAM Journal on Control and Optimization*, 51(2):965–995, 2013

[4] C. M. Hackl. High-gain adaptive position control. *International Journal of Control*, 84(10): 1695–1716, 2011.

[5] C. M. Hackl and R. M. Kennel. Position funnel control for rigid revolute joint robotic manipulators with known inertia matrix. *Proceedings of the 20th Mediterranean Conference on Control and Automation (Barcelona, Spain)*, p. 615–620, 2012.

[6] C. M. Hackl. Contributions to high-gain adaptive control in mechatronics. *PhD thesis, Lehrstuhl für Elektrische Antriebssyteme & Leistungselektronik, Technische Universität München, Deutschland*, 2012.