Funnel control for speed and position control
— An overview —

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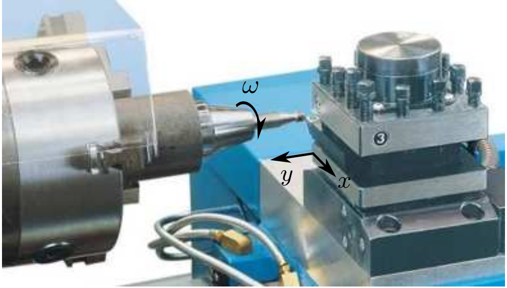
Outline

Reference tracking under load (CNC turning machine)



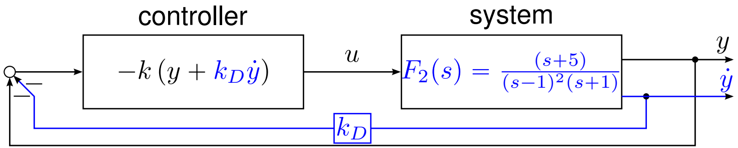
(http://www.knuth.de)

- Problem statement
  - reference $(\omega_{\text{ref}}, x_{\text{ref}}, y_{\text{ref}}) : \mathbb{R}_{\geqslant 0} \to \mathbb{R}^3$
  - precise position and speed control, e.g. for prescribed $\lambda > 0$

$$\boxed{\forall\, t \geqslant t_0 : \qquad |e(t)| = |y_{\text{ref}}(t) - y(t)| \leqslant \lambda.}$$

- Challenges
  - nonlinear effects (e.g. actuator saturation, friction)
  - friction and loads (disturbances) unknown and varying
  - system parameters (solely) roughly known

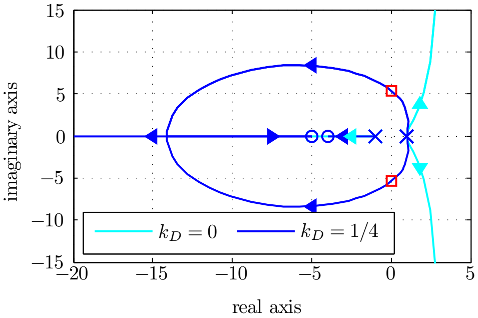
High-gain control - an intuition



‘structural properties’ of $F_2(s)$

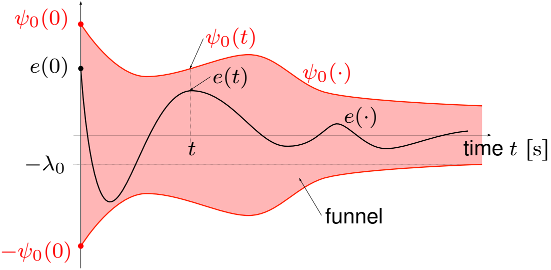
- relative degree (pole excess): $r = 2$
- positive high-frequency gain ($\lim_{s\to\infty} s^2 F_1(s) = 1$)
- minimum-phase (numerator is Hurwitz)

Root-locus ($\times$ poles, $\circ$ zeros)



Control objective and funnel controller ($r = 1$)

- objective: 'tracking with prescribed transient accuracy' of reference $y_{\mathrm{ref}}(\cdot)$



where
$e(t) = y_{\mathrm{ref}}(t) - y(t)$,
$y_{\mathrm{ref}}(\cdot) \in \mathcal{C}(\mathbb{R}_{\geqslant 0}; \mathbb{R})$,
$\psi_0(\cdot) \in \mathcal{C}(\mathbb{R}_{\geqslant 0}; [\lambda_0, \infty))$
with $\lambda_0 > 0$

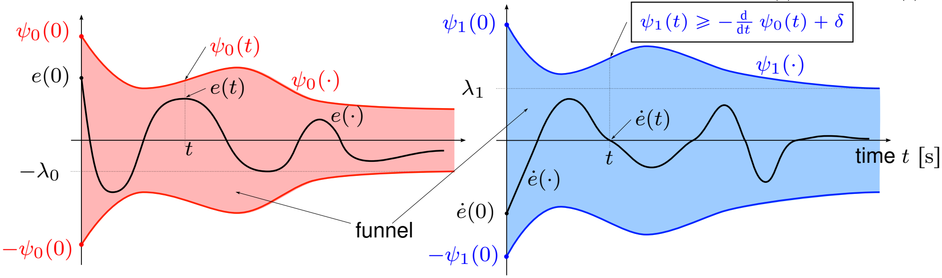
- funnel controller:

$$\boxed{u(t) = k_0(t)e(t) \quad \text{where} \quad k_0(t) = \frac{s_0(t)}{\psi_0(t) - |e(t)|}} \qquad (\mathrm{FC}_1)$$

- with gain scaling $s_0(\cdot) \in \mathcal{C}(\mathbb{R}_{\geqslant 0}; \mathbb{R}_{>0})$
(e.g. to fix minimal gain: $k_0(t) \geqslant s_0(t)/\psi_0(t)$ for all $t \geqslant 0$)

Control objective and funnel controller ($r = 2$)

- objective: 'tracking with prescribed transient accuracy' of $y_{\mathrm{ref}}(\cdot)$ and $\dot{y}_{\mathrm{ref}}(\cdot)$



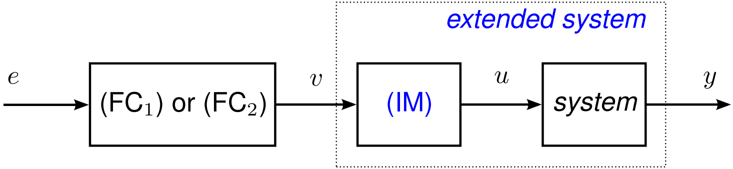
- funnel controller (with derivative feedback)

$$\boxed{u(t) = k_0(t)^2 \left(e(t) + \frac{k_1(t)}{k_0(t)} \dot{e}(t) \right), \qquad e(t) = y_{\mathrm{ref}}(t) - y(t)} \qquad (\mathrm{FC}_2)$$

- where $\quad k_0(t) = \dfrac{s_0(t)}{\psi_0(t) - |e(t)|} \quad$ and $\quad k_1(t) = \dfrac{s_1(t)}{\psi_1(t) - |\dot{e}(t)|}$
- scaling functions $s_i(\cdot) \in \mathcal{C}(\mathbb{R}_{\geqslant 0}; \mathbb{R}_{>0})$, $i = 0, 1$

Funnel control and steady-state accuracy

- asymptotic accuracy (i.e. $\lim_{t\to\infty} e(t) = 0$), cannot be guaranteed, since $\psi_0(t) \geqslant \lambda_0 > 0$ for all $t \geqslant 0$
- typical solution: use of internal model (IM) to achieve asymptotic tracking

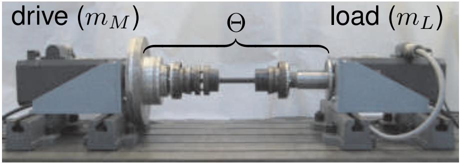


- if (IM) has
  - relative degree zero
  - positive high-frequency gain
  - and is minimum-phase

  then *extended system* has identical 'structural properties' as *system*
- standard internal model in industry (for constant signals): PI controller

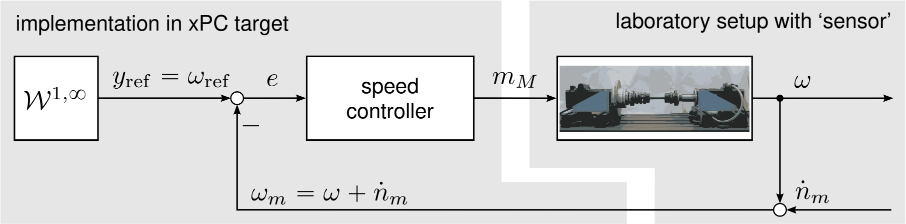
$$v(t) = u(t) + k_I \int_0^t u(\tau)\,\mathrm{d}\tau \quad \circ\!\!-\!\!\bullet \quad v(s) = \frac{s + k_I}{s}\,u(s), \quad k_I > 0 \qquad \text{(PI)}$$

Stiffly coupled servo-systems



- drive torque $m_M(\cdot) \in \mathcal{C}(\mathbb{R}_{\geqslant 0}; \mathbb{R})$ $[\mathrm{Nm}]$ (control input)
- load torque $m_L(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geqslant 0}; \mathbb{R})$ $[\mathrm{Nm}]$ (disturbance)
- inertia $\Theta > 0$ $\left[\mathrm{kg\,m^2}\right]$
- state $\boldsymbol{x} = (\phi, \omega)^\top$: position ϕ $[\mathrm{rad}]$, speed ω $[\mathrm{rad/s}]$
- friction (on drive & load side)
- gear with ratio $g_r \in \mathbb{R}\backslash\{0\}$ $[1]$ (neglecting dynamics & backlash)
- signals available for feedback: position ϕ and/or speed $\omega = \dot{\phi}$ (deteriorated by $n_m(\cdot)$ and/or $\dot{n}_m(\cdot)$, resp.)

Speed control implementation



- Standard PI controller

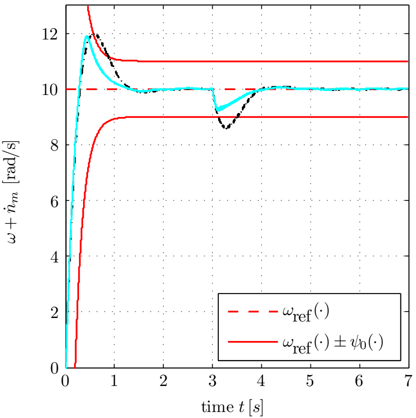
$$m_M(t) = k_P\, e(t) + k_I \int_0^t e(\tau)\,\mathrm{d}\tau$$

- PI-funnel controller

$$m_M(t) = k_0(t)\, e(t) + k_I \int_0^t k_0(\tau) e(\tau)\,\mathrm{d}\tau \qquad \text{where} \qquad k_0(t) = \frac{s_0(t)}{\psi_0(t) - |e(t)|}$$

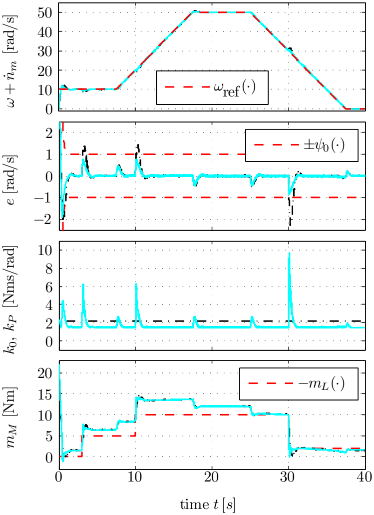
Speed control measurement results

Set-point tracking

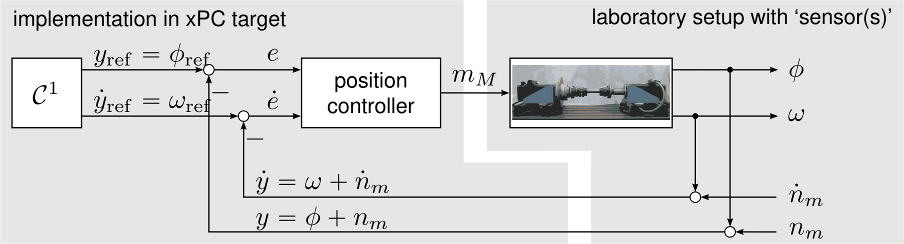


PI-funnel controller
PI controller

Reference tracking



Position control implementation



- Standard PID controller (with feedforward)

$$m_M(t) = k_P\, e(t) + k_I \int_0^t e(\tau)\,\mathrm{d}\tau \; + k_D \dot{e}(t) + u_F(t)$$

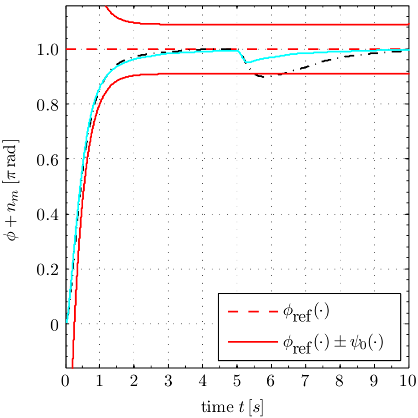
- PID-funnel controller

$$m_M(t) = k_0(t)^2 \left(e(t) + \frac{k_1(t)}{k_0(t)} \dot{e}(t) \right) + k_I \int_0^t k_0(\tau)^2 \left(e(\tau) + \frac{k_1(\tau)}{k_0(\tau)} \dot{e}(\tau) \right) \mathrm{d}\tau$$

where $k_i(t) = \frac{s_i(t)}{\psi_i(t) - |e^{(i)}(t)|}$ for $i = 0, 1$.

Position control measurement results

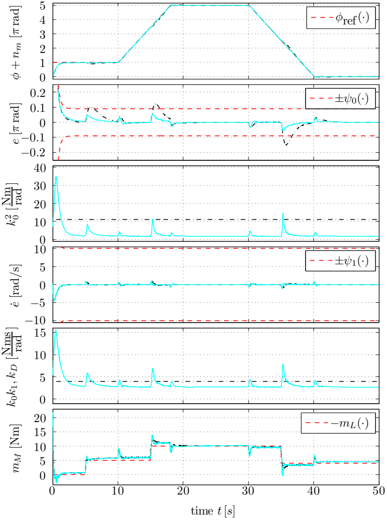
Set-point tracking



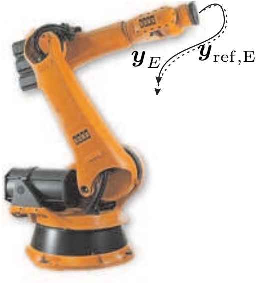
PID-funnel controller

PID controller (with feedforward)

Reference tracking



Position control of rigid revolute joint robots



Kuka KR 150-2 (Serie 2000)
(http://www.kuka-robotics.com)

Applications

- (spot-)welding
- painting/enameling
- mounting/assembling
- laser-beam cutting
- ...

Goals:

- Accurate position control of end effector , i.e. for given $\lambda > 0$:

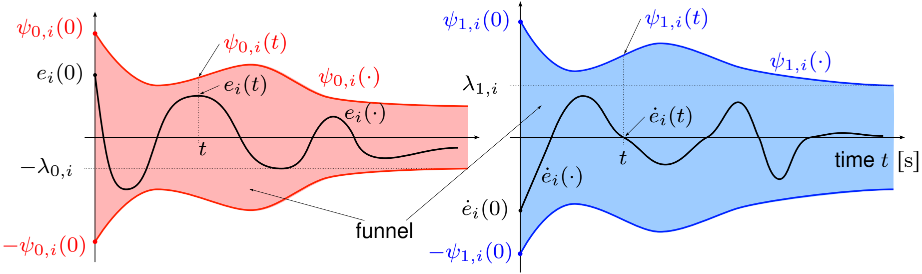
$$\boxed{\forall\, t \geqslant t_0: \qquad \|\boldsymbol{e}_E(t)\| = \left\|\boldsymbol{y}_{\mathrm{ref,E}}(t) - \boldsymbol{y}_E(t)\right\| \leqslant \lambda.}$$

- controller: simple and 'intuitive' to tune

Control objective and MIMO funnel controller

- Tracking with prescribed transient accuracy for **each joint** $i \in \{1, \ldots, n\}$:

$$\boxed{\forall\, t \geqslant 0: \quad |e_i(t)| = |y_{\mathrm{ref,i}}(t) - y_i(t)| < \psi_{0,i}(t) \quad \text{and} \quad |\dot{e}_i(t)| < \psi_{1,i}(t)}$$

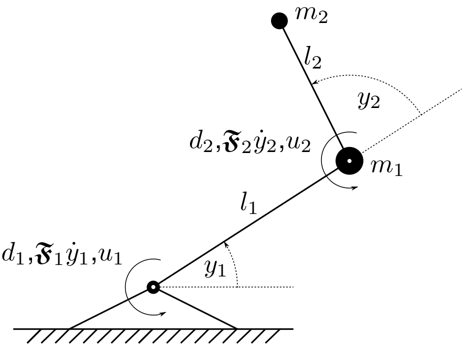


- MIMO funnel controller

$$\boxed{\boldsymbol{u}(t) = \widehat{\boldsymbol{M}}(\boldsymbol{y}(t))\Big(\boldsymbol{K}_0(t)^2\boldsymbol{e}(t) + \boldsymbol{K}_0(t)\boldsymbol{K}_1(t)\dot{\boldsymbol{e}}(t)\Big)} \qquad (\mathrm{FC}_n)$$

- $\boldsymbol{K}_0(t) = \mathrm{diag}\left\{k_{0,1}(t), \ldots, k_{0,n}(t)\right\}$ & $\boldsymbol{K}_1(t) = \mathrm{diag}\left\{k_{1,1}(t), \ldots, k_{1,n}(t)\right\}$

Example: Planar robot (2DOF)

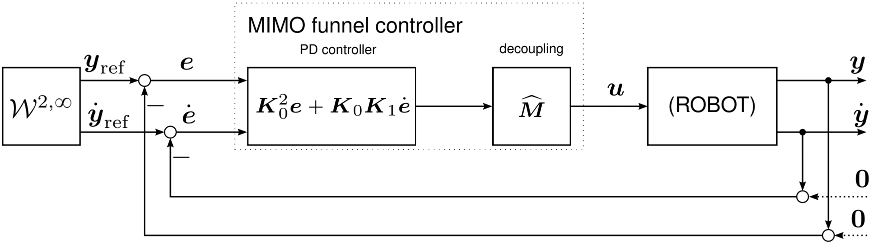


- (point-)masses m_1, m_2 $[\mathrm{kg}]$
- length of the links l_1, l_2 $[\mathrm{m}]$
- joint angle $\boldsymbol{y} := \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ $[\mathrm{rad}]^2$
- torque $\boldsymbol{u} := \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ $[\mathrm{Nm}]^2$
- friction $\mathfrak{F}\dot{\boldsymbol{y}} := \begin{pmatrix} \mathfrak{F}_1 \dot{y}_1 \\ \mathfrak{F}_2 \dot{y}_2 \end{pmatrix}$ $[\mathrm{Nm}]^2$
- disturbance $\boldsymbol{d} := \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$ $[\mathrm{Nm}]^2$

$$\boxed{\left.\begin{array}{c} \boldsymbol{M}(\boldsymbol{y}(t))\ddot{\boldsymbol{y}}(t) + \boldsymbol{C}(\boldsymbol{y}(t), \dot{\boldsymbol{y}}(t))\dot{\boldsymbol{y}}(t) + (\mathfrak{F}\dot{\boldsymbol{y}})(t) + \boldsymbol{g}(\boldsymbol{y}(t)) + \boldsymbol{d}(t) = \boldsymbol{u}(t), \\ (\boldsymbol{y}(0),\ \dot{\boldsymbol{y}}(0))^\top = (\boldsymbol{0},\ \boldsymbol{0})^\top \in \mathbb{R}^4 \end{array}\right\}}$$

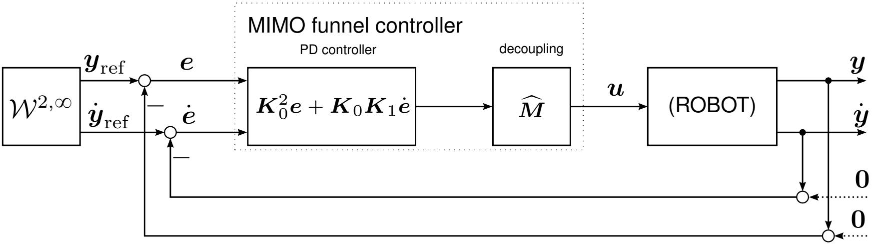
(ROBOT)

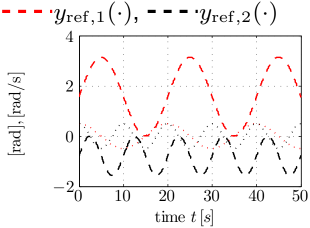
Implementation: MIMO funnel controller

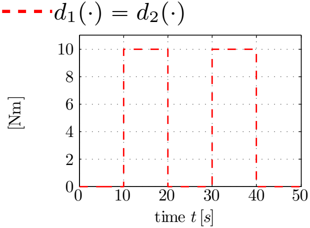


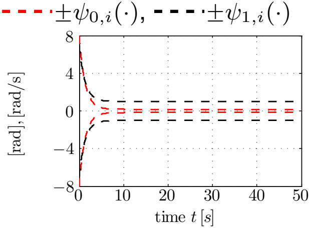
- $$\boldsymbol{u}(t) = \overbrace{\boldsymbol{M}(\boldsymbol{y}(t)) \cdot \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}}^{\widehat{\boldsymbol{M}}(\boldsymbol{y}(t))} \cdot \left(\begin{bmatrix} k_{0,1}(t)^2 & 0 \\ 0 & k_{0,2}(t)^2 \end{bmatrix} \boldsymbol{e}(t) + \begin{bmatrix} k_{0,1}(t)k_{1,1}(t) & 0 \\ 0 & k_{0,2}(t)k_{1,2}(t) \end{bmatrix} \dot{\boldsymbol{e}}(t) \right)$$
- $$\forall\, i \in \{1,2\}: \; k_{0,i}(t) = \frac{\psi_{0,i}(t)}{\psi_{0,i}(t) - |e_i(t)|} \geqslant 1 \;\wedge\; k_{1,i}(t) = \frac{5\psi_{1,i}(t)}{\psi_{1,i}(t) - |\dot{e}_i(t)|} \geqslant 5$$

Implementation: Reference, disturbances & funnel

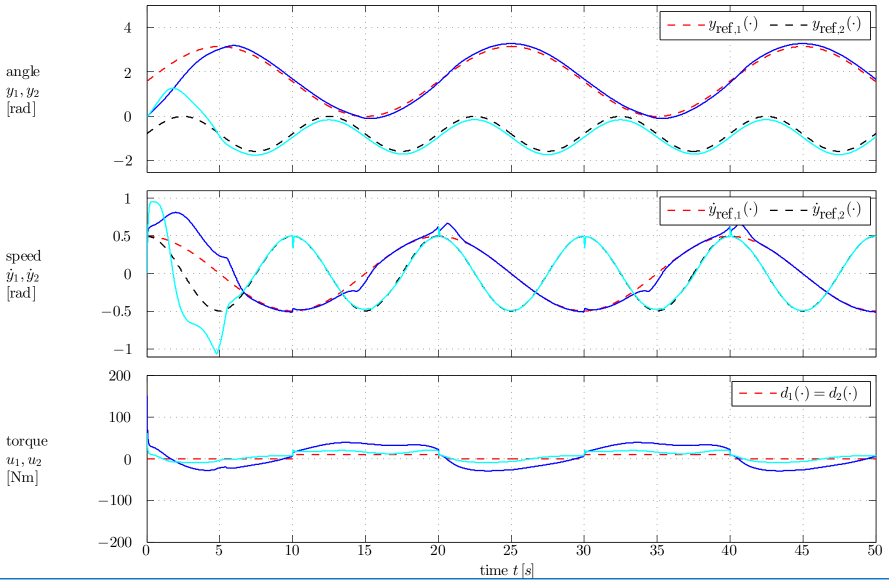








Simulation results (joints: —— #1, —— #2)



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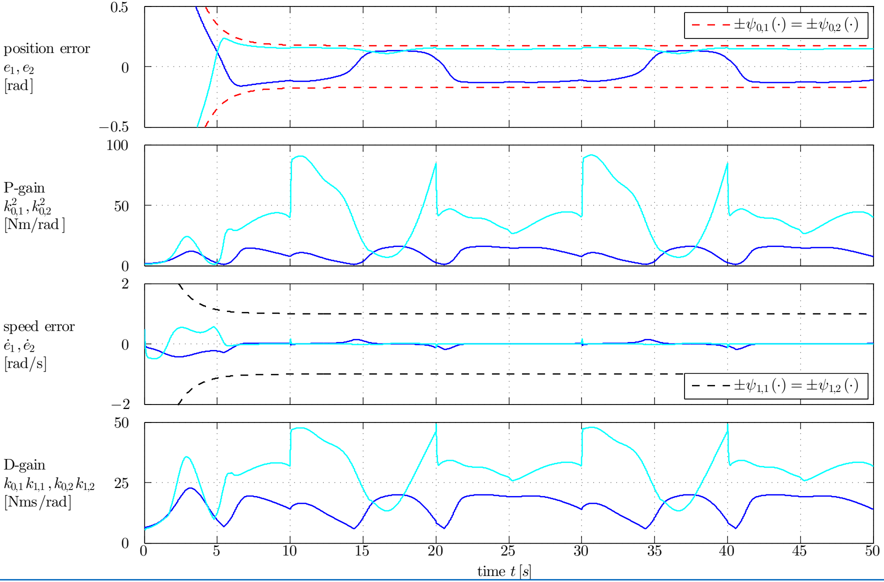

Conclusion

To take home

- **funnel control applicable** for speed and position control
- **only structural properties** must be checked (robustness)
- **no** system identification or parameter estimation necessary
- **time-varying gains** (also decrease possible)
- ‘**tracking with prescribed transient accuracy**’
- **steady state accuracy** in conjunction with PI controller
- position funnel control of **rigid revolute joint robotic manipulators** is feasible (if inertia matrix is roughly known)

Some more results (not presented)

- funnel control is also applicable for speed and position control of **elastically coupled servo-systems** (two-mass systems)
- funnel control in presence of **actuator saturation** is feasible (conservative feasibility condition must be satisfied)

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