

Funnel control in mechatronics: An introduction

Christoph Hackl

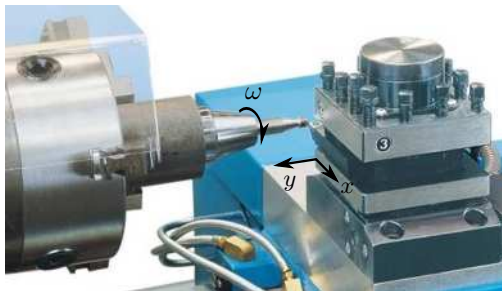
Technische Universität München
Lehrstuhl für Elektrische Antriebssysteme und Leistungselektronik

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Outline

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 - High-gain control
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 - Steady-state accuracy
- 3** Application: Motion control of stiffly coupled servo-systems
 - Laboratory setup
 - Speed control (relative-degree-one case)
 - Position control (relative-degree-two case)
- 4** Conclusion

Reference tracking under load (CNC turning machine)



(<http://www.knuth.de>)

■ Problem statement

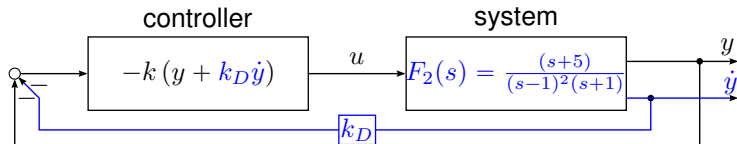
- reference $(\omega_{\text{ref}}, x_{\text{ref}}, y_{\text{ref}}): \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^3$
- precise position and speed control, e.g. for prescribed $\lambda > 0$

$$\forall t \geq t_0: \quad |e(t)| = |y_{\text{ref}}(t) - y(t)| \leq \lambda.$$

■ Challenges

- nonlinear effects (e.g. actuator saturation, friction)
- friction and loads (disturbances) unknown and varying
- system parameters (solely) roughly known

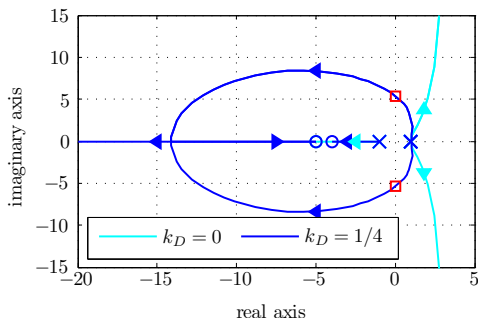
High-gain control - an intuition



‘structural properties’ of $F_2(s)$

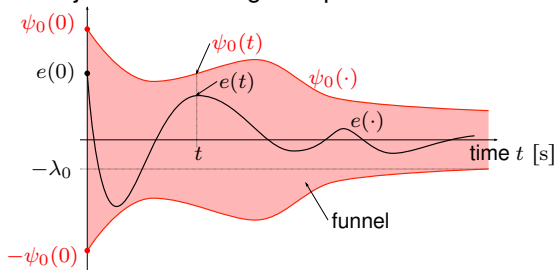
- relative degree (pole excess):
 $r = 2$
- positive high-frequency gain
 $(\lim_{s \rightarrow \infty} s^2 F_1(s) = 1)$
- minimum-phase
(numerator is Hurwitz)

Root-locus ($\times$ poles, $\circ$ zeros)



Control objective and funnel controller ($r = 1$)

- objective: 'tracking with prescribed transient accuracy' of reference $y_{\text{ref}}(\cdot)$



where

$$\begin{aligned} e(t) &= y_{\text{ref}}(t) - y(t), \\ y_{\text{ref}}(\cdot) &\in \mathcal{C}(\mathbb{R}_{\geq 0}; \mathbb{R}), \\ \psi_0(\cdot) &\in \mathcal{C}(\mathbb{R}_{\geq 0}; [\lambda_0, \infty)) \\ \text{with } \lambda_0 &> 0 \end{aligned}$$

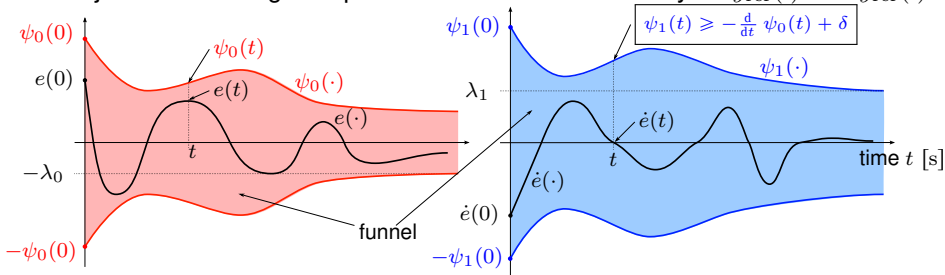
- funnel controller:

$$u(t) = k_0(t)e(t) \quad \text{where} \quad k_0(t) = \frac{s_0(t)}{\psi_0(t) - |e(t)|} \quad (\text{FC}_1)$$

- with gain scaling $s_0(\cdot) \in \mathcal{C}(\mathbb{R}_{\geq 0}; \mathbb{R}_{>0})$
(e.g. to fix minimal gain: $k_0(t) \geq s_0(t)/\psi_0(t)$ for all $t \geq 0$)

Control objective and funnel controller ($r = 2$)

- objective: 'tracking with prescribed transient accuracy' of $y_{\text{ref}}(\cdot)$ and $\dot{y}_{\text{ref}}(\cdot)$



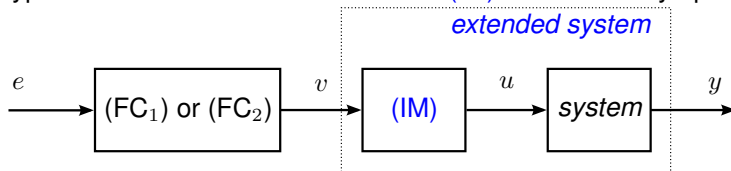
- funnel controller (with derivative feedback)

$$u(t) = k_0(t)^2 \left(e(t) + \frac{k_1(t)}{k_0(t)} \dot{e}(t) \right), \quad e(t) = y_{\text{ref}}(t) - y(t) \quad (\text{FC}_2)$$

- where $k_0(t) = \frac{s_0(t)}{\psi_0(t) - |e(t)|}$ and $k_1(t) = \frac{s_1(t)}{\psi_1(t) - |\dot{e}(t)|}$
- scaling functions $s_i(\cdot) \in \mathcal{C}(\mathbb{R}_{\geq 0}; \mathbb{R}_{> 0})$, $i = 0, 1$

Funnel control and steady-state accuracy

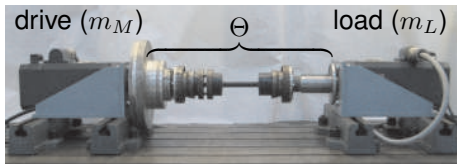
- asymptotic accuracy (i.e. $\lim_{t \rightarrow \infty} e(t) = 0$), cannot be guaranteed, since $\psi_0(t) \geq \lambda_0 > 0$ for all $t \geq 0$
- typical solution: use of internal model (IM) to achieve asymptotic tracking



- if (IM) has
 - relative degree zero
 - positive high-frequency gain
 - and is minimum-phase
 then *extended system* has identical 'structural properties' as *system*
- standard internal model in industry (for constant signals):
PI controller

$$v(t) = u(t) + k_I \int_0^t u(\tau) d\tau \quad \circ \longrightarrow \bullet \quad v(s) = \frac{s + k_I}{s} u(s), \quad k_I > 0 \quad (\text{PI})$$

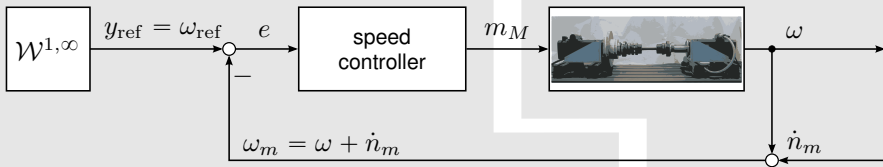
Stiffly coupled servo-systems



- drive torque $m_M(\cdot) \in \mathcal{C}(\mathbb{R}_{\geq 0}; \mathbb{R})$ [Nm] (control input)
- load torque $m_L(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0}; \mathbb{R})$ [Nm] (disturbance)
- inertia $\Theta > 0$ [kg m²]
- state $x = (\phi, \omega)^\top$: position ϕ [rad], speed ω [rad/s]
- friction (on drive & load side)
- gear with ratio $g_r \in \mathbb{R} \setminus \{0\}$ [1] (neglecting dynamics & backlash)
- signals available for feedback: position ϕ and/or speed $\omega = \dot{\phi}$ (deteriorated by $n_m(\cdot)$ and/or $\dot{n}_m(\cdot)$, resp.)

Speed control implementation

implementation in xPC target



- Standard PI controller

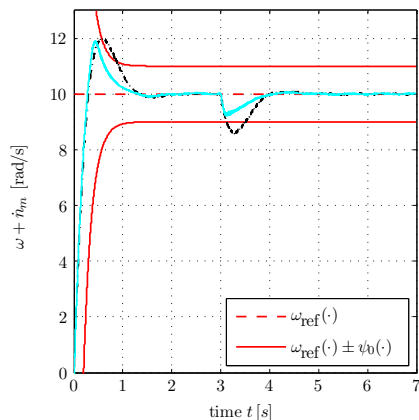
$$m_M(t) = k_P e(t) + k_I \int_0^t e(\tau) d\tau$$

- PI-funnel controller

$$m_M(t) = k_0(t) e(t) + k_I \int_0^t k_0(\tau) e(\tau) d\tau \quad \text{where} \quad k_0(t) = \frac{s_0(t)}{\psi_0(t) - |e(t)|}$$

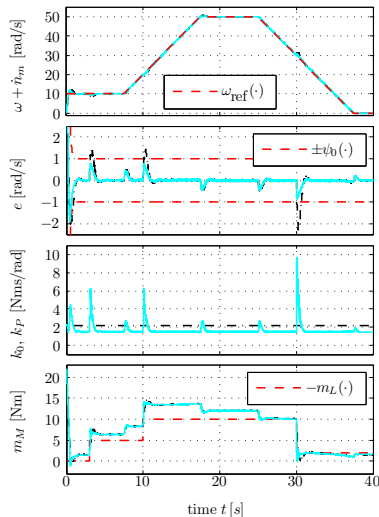
Speed control measurement results

Set-point tracking



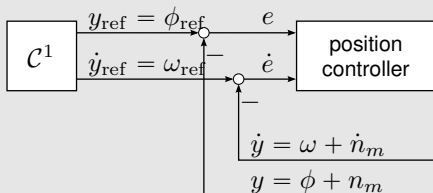
— PI-funnel controller
 - - - PI controller

Reference tracking

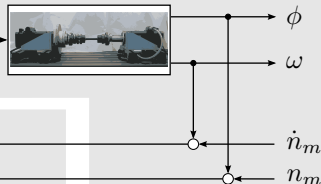


Position control implementation

implementation in xPC target



laboratory setup with 'sensor(s)'



- Standard PID controller (with feedforward)

$$m_M(t) = k_P e(t) + k_I \int_0^t e(\tau) d\tau + k_D \dot{e}(t) + u_F(t)$$

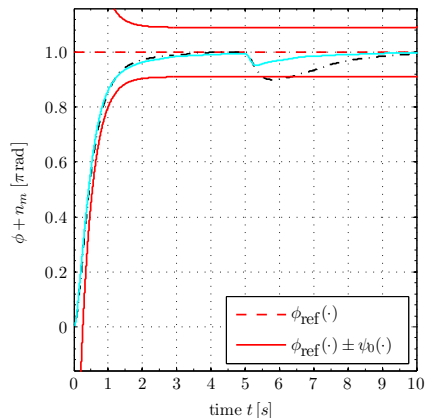
- PID-funnel controller

$$m_M(t) = k_0(t)^2 \left(e(t) + \frac{k_1(t)}{k_0(t)} \dot{e}(t) \right) + k_I \int_0^t k_0(\tau)^2 \left(e(\tau) + \frac{k_1(\tau)}{k_0(\tau)} \dot{e}(\tau) \right) d\tau$$

where $k_i(t) = \frac{s_i(t)}{\psi_i(t) - |e^{(i)}(t)|}$ for $i = 0, 1$.

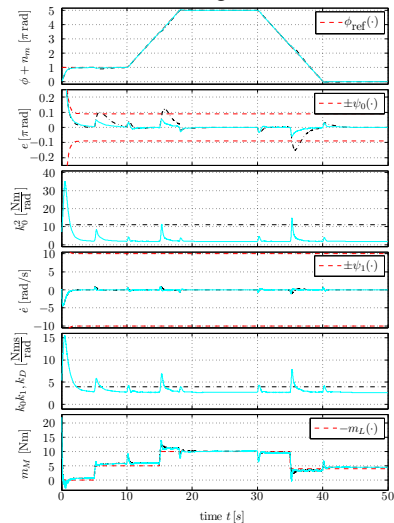
Position control measurement results

Set-point tracking



— PID-funnel controller
 - - - PID controller (with feedforward)

Reference tracking



Conclusion

To take home

- **funnel control applicable** for speed and position control
- **only structural properties** must be checked (robustness)
- **no** system identification or parameter estimation necessary
- **time-varying gains** (also decrease possible)
- **'tracking with prescribed transient accuracy'**
- **steady state accuracy** in conjunction with PI controller

Some more results (not presented)

- funnel control is also applicable for speed and position control of **elastically coupled servo-systems** (two-mass systems)
- position funnel control of **rigid revolute joint robotic manipulators** is feasible (if inertia matrix is roughly known)
- funnel control in presence of **actuator saturation** is feasible (conservative feasibility condition must be satisfied)

References



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