Position funnel control with linear internal model

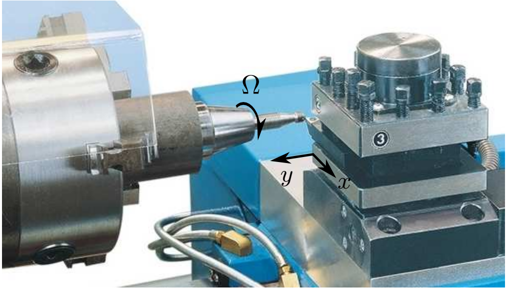
Christoph Hackl

Technische Universität München
Lehrstuhl für Elektrische Antriebssysteme und Leistungselektronik

2012 IEEE Multi-Conference on Systems and Control
October 3-5, 2012, Dubrovnik, Croatia

Outline

Reference tracking under load (CNC turning machine)



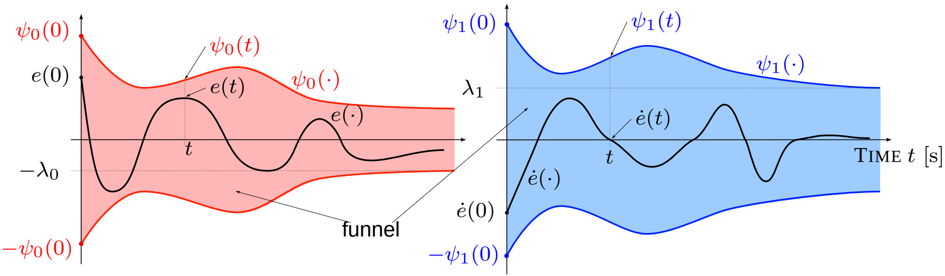
(http://www.knuth.de)

- Problem statement
  - reference $(\Omega_{\text{ref}}, x_{\text{ref}}, y_{\text{ref}}) : \mathbb{R}_{\geq 0} \to \mathbb{R}^3$
  - precise position and speed control, e.g. for prescribed $\lambda > 0$

$$\boxed{\forall\, t \geq t_0 : \qquad |e(t)| = |y_{\text{ref}}(t) - y(t)| \leq \lambda.}$$

- Challenges
  - nonlinear effects (e.g. actuator saturation, friction)
  - friction and loads (disturbances) unknown and varying
  - system parameters (solely) roughly known

Funnel control for relative-degree-two systems (SISO)



- control objective: 'prescribed transient accuracy' via funnel boundary design

$$\boxed{\forall\, t \geq 0: \quad |e(t)| = |y_{\text{ref}}(t) - y(t)| < \psi_0(t) \quad \text{and} \quad |\dot{e}(t)| < \psi_1(t)}$$

- adaptive PD controller (see [Hackl et al. (2010)] and [Hackl (2011)]):

$$\boxed{u(t) = \underbrace{k_0(t)^2\, e(t)}_{\text{P-part}} + \underbrace{k_0(t) k_1(t) \dot{e}(t)}_{\text{D-part}}} \tag{FC}$$

where $k_0(t) = \dfrac{s_0(t)}{\psi_0(t) - |e(t)|}$ and $k_1(t) = \dfrac{s_1(t)}{\psi_1(t) - |\dot{e}(t)|}$.

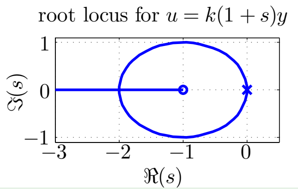
Why funnel control for position control problem?

- prescribed control performance guaranteed
- high gains reduce/eliminate ‘stick-slip’
- admissible system class $\mathcal{S}_2$ ($\Longrightarrow$ robustness)
  - known relative degree $r = 2$
  - positive high-frequency gain (or known sign)
  - BIBO stable zero dynamics (minimum-phase)
  - and derivative feedback admissible ($\Longrightarrow$ PD controller)

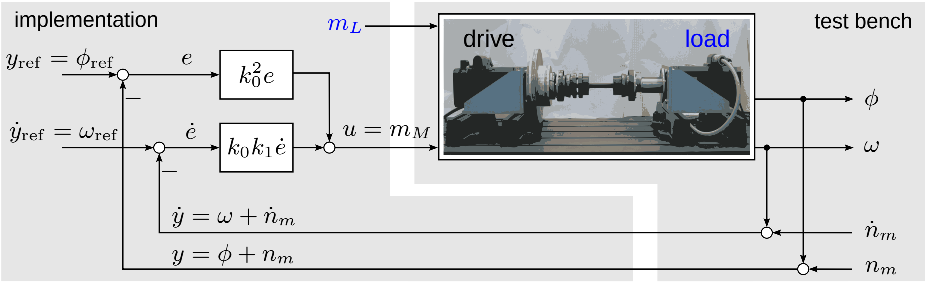
Example (LTI SISO system in frequency domain)

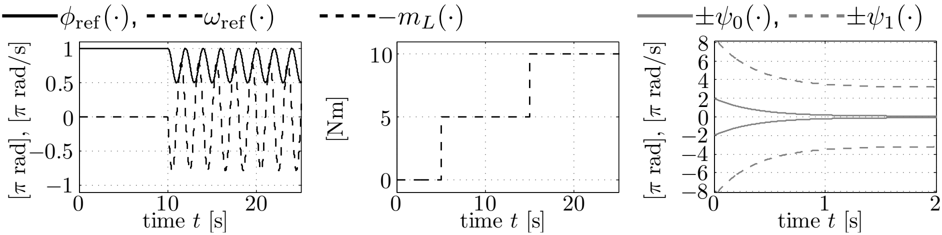
$$F(s) = \frac{y(s)}{u(s)} = \gamma_0 \frac{N(s)}{D(s)} = \frac{1}{s^2} \qquad \begin{cases} \gamma_0 \neq 0, \\ N, D \in \mathbb{R}[s] \text{ monic and coprime}, \\ N \text{ Hurwitz } \wedge \deg(N) = \deg(D) - 2 \end{cases}$$

- relative degree $r = 2$ (pole excess)
- $\mathrm{sign}(\gamma_0)$ known
- minimum-phase
- measured signals: $y(\cdot)$ and $\dot{y}(\cdot)$



Position funnel control of stiff one-mass system (1MS)





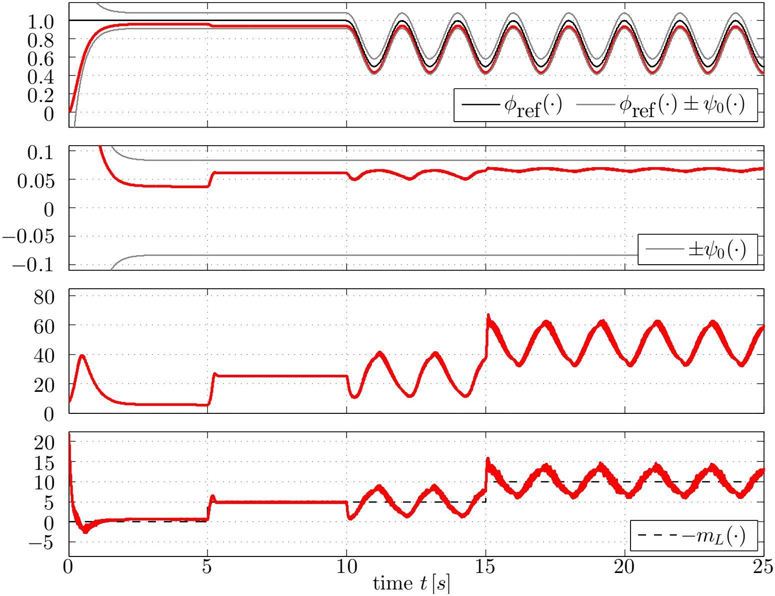
Measurements: ——(FC)

angle
$\phi + n_m$ [π rad]

error
e [π rad]

P-gain
k_0^2 [Nm/rad]

torque
m_M [Nm]



Funnel control and steady state accuracy

- $\lim_{t\to\infty} e(t) = 0$ not guaranteed (since $\psi_0(t) \geq \lambda_0 > 0$ for all $t \geq 0$)
- PI controller ('standard' internal model)

$$\left.\begin{aligned} \dot{x}_I(t) &= u(t), \quad x_I(0) = 0 \\ v(t) &= k_P u(t) + k_I x_I(t) \end{aligned}\right\} \quad \circ\!\!-\!\!\bullet \quad v(s) = \underbrace{\frac{k_P\, s + k_I}{s}}_{=:F_{PI}(s)} u(s) \qquad \text{(PI)}$$

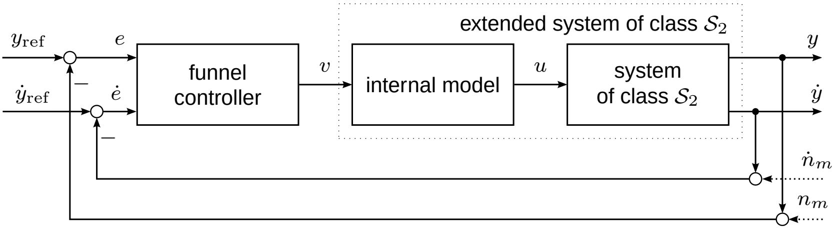
- Application of (linear) internal model admissible, if
  - relative degree is zero
  - high-frequency gain is positive
  - it is controllable and observable ($\Longrightarrow$ minimum-phase)

Example (PI in series to LTI SISO system)

$$\widehat{F}(s) = \frac{y(s)}{v(s)} = \underbrace{\gamma_0 k_P}_{=:\widehat{\gamma_0}} \frac{s + k_I/k_P}{s} \frac{N(s)}{D(s)} \qquad \begin{cases} \gamma_0 \neq 0,\ k_P, k_I > 0 \\ N, D \in \mathbb{R}[s] \text{ monic and coprime,} \\ N \text{ Hurwitz } \wedge\ \deg(N) = \deg(D) - 2 \end{cases}$$

- relative degree $\widehat{r} = 2$ (pole excess)
- $\mathrm{sign}(\widehat{\gamma_0})=\mathrm{sign}(\gamma_0)$ known
- minimum-phase
- measured signals: $y(\cdot)$ and $\dot{y}(\cdot) \Longrightarrow \widehat{F}(s) \in \mathcal{S}_2$

Design and combination of internal models



- reference

$$y_{\mathrm{ref}}(t) := \begin{cases} y_0 & , 0 \leq t < t_0 \\ \frac{y_0}{4}\cos(2\pi f_0(t - t_0)) + \frac{3y_0}{4} & , t_0 \leq t \leq t_{end} \end{cases}, \qquad \begin{array}{l} y_0 \in \mathbb{R}\ [\mathrm{rad}] \\ f_0 \geq 0\ [1/\mathrm{s}] \end{array}$$

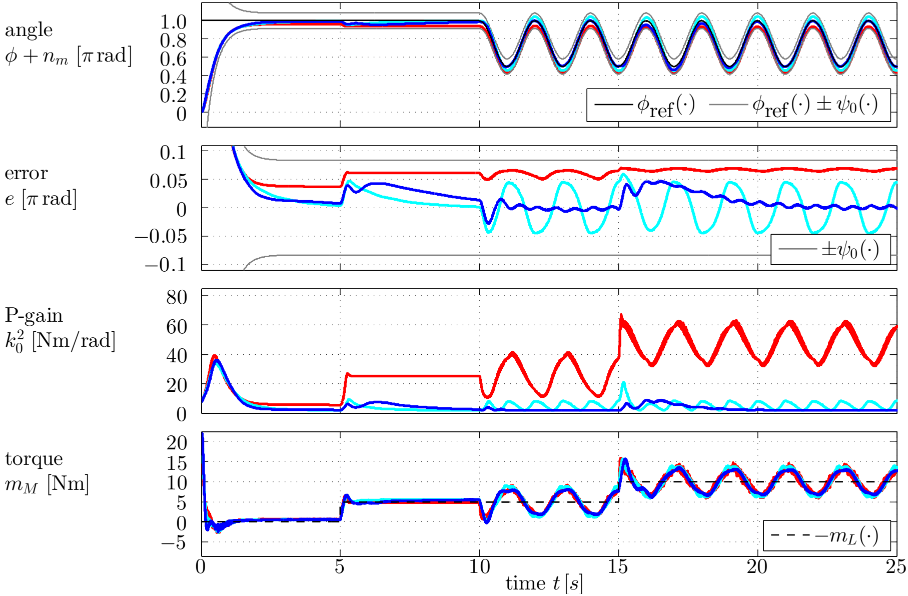
- design: (i) $y_0 \circ\!\!-\!\!\bullet\ \frac{y_0}{s} \implies F_{PI}(s) = \frac{k_P\, s + k_I}{s}$ and (ii)

$$\cos(2\pi f_0 t) \circ\!\!-\!\!\bullet\ \frac{1}{s^2 + (2\pi f_0)^2} \implies F_{COS}(s) = \frac{(s + z_0)^2}{s^2 + (2\pi f_0)^2}, \quad z_0, f_0 > 0.$$

- combination

$$F_{IM}(s) = \frac{u(s)}{v(s)} = k_P \cdot \frac{s + k_I/k_P}{s} \cdot \frac{(s + z_0)^2}{s^2 + (2\pi f_0)^2}, \quad k_P, k_I, z_0, f_0 > 0. \qquad \text{(IM)}$$

Measurem.: —(FC), —(FC)+(PI), —(FC)+(IM)



Conclusion

To take home:

- funnel control (with ‘derivative feedback’)
  - robust (only ‘structural’ assumptions)
  - prescribed transient accuracy guaranteed
  - applicable for e.g. position control problems
  - *but:* steady state error might remain
- funnel control with linear internal model(s) admissible
  - internal model design easy using Laplace transforms of (known) reference
  - serial interconnection of internal models admissible
  - adequate internal model design improves control performance significantly (asymptotic tracking possible)

Open questions

- funnel control with linear internal model in presence of actuator saturation
  - integral windup effects?
  - anti-windup strategies applicable?
- switching between internal models ?
- generalization to nonlinear internal models?

References

C. M. Hackl, N. Hopfe, A. Ilchmann, M. Mueller, and S. Trenn.
Funnel control for systems with relative degree two.
accepted for publication in SIAM Journal on Control and Optimization, 2010.

C. M. Hackl.
High-gain adaptive position control.
International Journal of Control, 84(10):1695–1716, 2011.

C. M. Hackl.
Contributions to high-gain adaptive control in mechatronics.
PhD thesis, Lehrstuhl für Elektrische Antriebssyteme & Leistungselektronik, Technische Universität München, Deutschland, 2012.