Position funnel control for rigid revolute joint robotic manipulators with known inertia matrix

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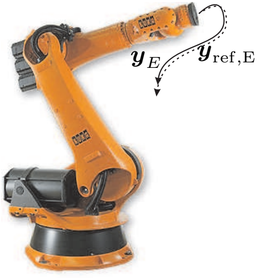
Outline

Position control of industrial robots



Kuka KR 150-2 (Serie 2000)
(http://www.kuka-robotics.com)

Applications

- (spot-)welding
- painting/enameling
- mounting/assembling
- laser-beam cutting
- ...

Goals:

- precise position control of end effector, i.e. for given $\lambda > 0$:

$$\boxed{\forall\, t \geq t_0: \qquad \|\boldsymbol{e}_E(t)\| = \left\|\boldsymbol{y}_{\mathrm{ref,E}}(t) - \boldsymbol{y}_E(t)\right\| \leq \lambda.}$$

- controller: simple and easy to tune

Rigid robotic manipulator with n revolute joints

Robot model (in joint space)

$$\left.\begin{array}{c} \boldsymbol{M}(\boldsymbol{y}(t))\ddot{\boldsymbol{y}}(t) + \boldsymbol{C}(\boldsymbol{y}(t),\dot{\boldsymbol{y}}(t))\dot{\boldsymbol{y}}(t) + (\mathfrak{F}\dot{\boldsymbol{y}})(t) + \boldsymbol{g}(\boldsymbol{y}(t)) + \boldsymbol{d}(t) = \boldsymbol{u}(t), \\ (\boldsymbol{y}(0),\, \dot{\boldsymbol{y}}(0))^\top = (\boldsymbol{y}_0,\, \boldsymbol{y}_1)^\top \in \mathbb{R}^{2n} \end{array}\right\} \tag{ROBOT}$$

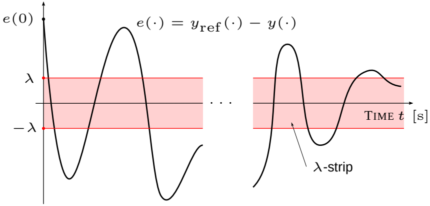
- standard assumptions
  - $\exists\, \overline{c}_{\boldsymbol{M}}, \underline{c}_{\boldsymbol{M}} > 0\ \forall\, \boldsymbol{y} \in \mathbb{R}^n: \quad 0 < \underline{c}_{\boldsymbol{M}} \boldsymbol{I}_n \le \boldsymbol{M}(\boldsymbol{y}) = \boldsymbol{M}(\boldsymbol{y})^\top \le \overline{c}_{\boldsymbol{M}} \boldsymbol{I}_n$
  - $\exists\, c_{\boldsymbol{C}} > 0\ \forall\, \boldsymbol{y}, \boldsymbol{v} \in \mathbb{R}^n: \quad \|\boldsymbol{C}(\boldsymbol{y},\boldsymbol{v})\boldsymbol{v}\| \le c_{\boldsymbol{C}} \|\boldsymbol{v}\|^2$
  - $\mathfrak{F}\colon \mathcal{C}(\mathbb{R}_{\ge 0};\mathbb{R}) \to \mathcal{L}^\infty_{\text{loc}}(\mathbb{R}_{\ge 0};\mathbb{R})$ (causal friction operator)
  - $\exists\, c_{\boldsymbol{g}} > 0\ \forall\, \boldsymbol{y} \in \mathbb{R}^n: \quad \|\boldsymbol{g}(\boldsymbol{y})\| \le c_{\boldsymbol{g}}$
  - $\boldsymbol{d}(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\ge 0};\mathbb{R}^n)$ (disturbance)
  - $\boldsymbol{u}(\cdot) \in \mathcal{C}(\mathbb{R}_{\ge 0};\mathbb{R}^n)$ (control input)
  - feedback signals: $\boldsymbol{y}(\cdot)$ *and* $\dot{\boldsymbol{y}}(\cdot)$
- 'structural properties'
  - (strict) relative degree $r = 2$
  - positive (definite) high-frequency gain
  - no internal dynamics (minimum-phase)
  - derivative feedback admissible

High-gain adaptive control

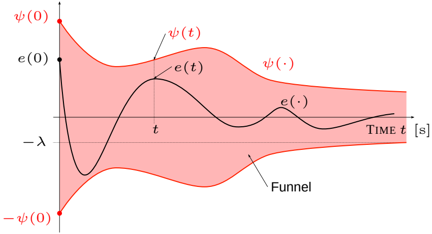
Promising advantages

- 'structural system knowledge' sufficient (robustness)
    - known relative degree $r \geq 1$
    - positive high-frequency gain (or known sign)
    - minimum-phase
- high gain in feedback reduces 'stick-slip'
- reference tracking with 'prescribed asymptotic accuracy'

Adaptive λ-tracking control



Funnel control



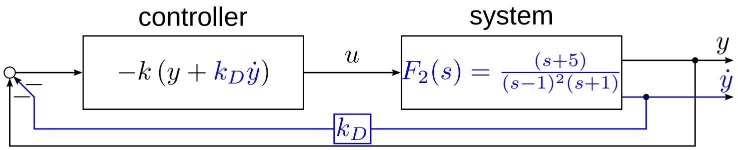
Available high-gain adaptive controllers

... in the Literature (for $r > 1$):

- adaptive λ-tracking control
  - with compensator (Ye [1999])
  - with high-gain observer (Bullinger & Allgöwer [2005])
- funnel control
  - with compensator (Ilchmann et al. [2007])
  - with state feedback (e.g. Ilchmann & Schuster [2009] or Hackl et al. [2009])
- drawbacks of these controllers
  - complex (e.g. recursive design due to backstepping)
  - very ‘noise sensitive’ (e.g. $k(t)^7$ for $r = 2$)
  - tracking with prescribed transient accuracy only for linear combination of states

$\Longrightarrow$ controllers **not** reasonable for application

Relative degree two and derivative feedback

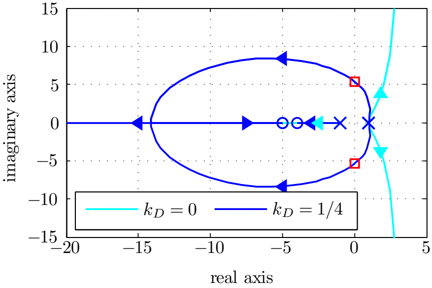


'Structural properties' of $F_2(s)$

- relative degree (pole excess): $r = 2$
- positive high-frequency gain ($\lim_{s\to\infty} s^2 F_2(s) = 1$)
- minimum-phase (Hurwitz numerator)

$\exists\, k^\star > 0$, such that closed-loop stable for $k_D > 0$ and $k > k^\star$

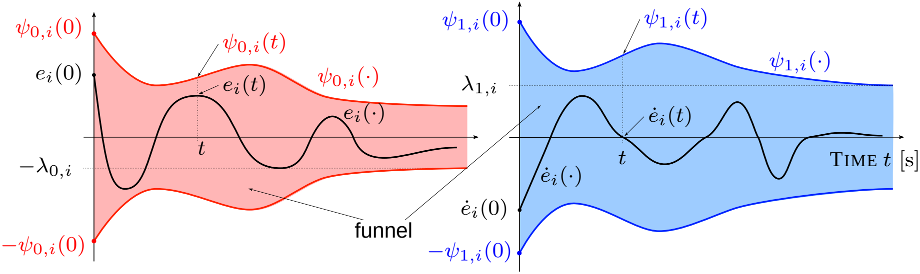
Root locus ($\times$ pole, $\circ$ zero)



Control objective

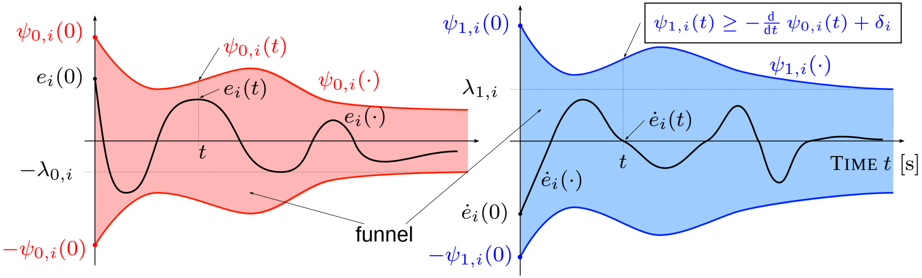
- tracking with '**prescribed transient behavior**' for **each joint** $i \in \{1, \dots, n\}$:

$$\boxed{\forall\, t \geq 0: \quad |e_i(t)| = |y_{\mathrm{ref,i}}(t) - y_i(t)| < \psi_{0,i}(t) \quad \text{and} \quad |\dot{e}_i(t)| < \psi_{1,i}(t)}$$



- joint reference $y_{\mathrm{ref,i}}(\cdot) \in \mathcal{W}^{2,\infty}(\mathbb{R}_{\geq 0}; \mathbb{R})$
- joint funnel boundary $(\psi_{0,i}(\cdot),\ \psi_{1,i}(\cdot))$:
  - $|e_i(0)| < \psi_{0,i}(0)$ and $|\dot{e}_i(0)| < \psi_{1,i}(0)$
  - $\psi_{0,i}(\cdot),\ \psi_{1,i}(\cdot) \in \mathcal{L}^{\infty}(\mathbb{R}_{\geq 0}; \mathbb{R}_{>0})$ and globally Lipschitz continuous
  - $\exists \lambda_{0,i},\ \lambda_{1,i} > 0\ \forall t \geq 0:\ \psi_{0,i}(t) \geq \lambda_{0,i}$ and $\psi_{1,i}(t) \geq \lambda_{1,i}$
  - $\exists\, \delta_i > 0\ \forall t \geq 0:\ \psi_{1,i}(t) \geq -\frac{\mathrm{d}}{\mathrm{d}t}\, \psi_{0,i}(t) + \delta_i$

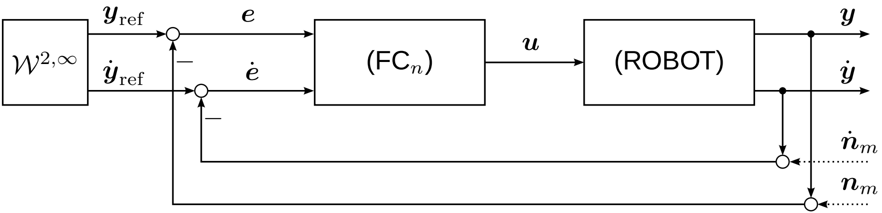
MIMO funnel controller



$$\boxed{\boldsymbol{u}(t) = \boldsymbol{M}(\boldsymbol{y}(t))\Big(\underbrace{\boldsymbol{K}_0(t)^2\boldsymbol{e}(t)}_{\textbf{P}} + \underbrace{\boldsymbol{K}_0(t)\boldsymbol{K}_1(t)\dot{\boldsymbol{e}}(t)}_{\textbf{D}}\Big)} \qquad (\mathrm{FC}_n)$$

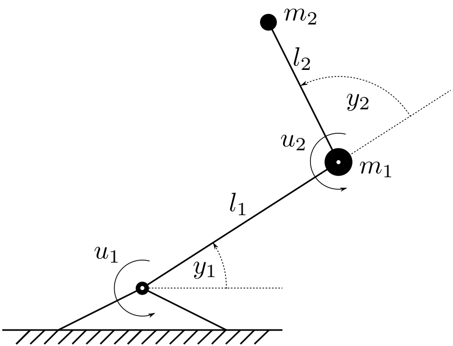
- $\boldsymbol{e}(t) = \begin{pmatrix} e_1(t) \\ \vdots \\ e_n(t) \end{pmatrix} = \begin{pmatrix} y_{\mathrm{ref},1}(t) \\ \vdots \\ y_{\mathrm{ref,n}}(t) \end{pmatrix} - \begin{pmatrix} y_1(t) \\ \vdots \\ y_n(t) \end{pmatrix}$
- $\boldsymbol{K}_0(t) = \mathrm{diag}\left\{k_{0,1}(t),\ \ldots,\ k_{0,n}(t)\right\}$ and $\boldsymbol{K}_1(t) = \mathrm{diag}\left\{k_{1,1}(t),\ \ldots,\ k_{1,n}(t)\right\}$
- $\forall\, i \in \{1, \ldots, n\}: \quad k_{0,i}(t) = \dfrac{1}{\psi_{0,i}(t) - |e_i(t)|}$ and $k_{1,i}(t) = \dfrac{1}{\psi_{1,i}(t) - |\dot{e}_i(t)|}$

Properties of closed-loop system [Theorem 3.1]



- (ROBOT): $\boxed{\boldsymbol{M}(\boldsymbol{y}(t))\ddot{\boldsymbol{y}}(t) + \boldsymbol{C}(\boldsymbol{y}(t), \dot{\boldsymbol{y}}(t))\dot{\boldsymbol{y}}(t) + (\mathfrak{F}\dot{\boldsymbol{y}})(t) + \boldsymbol{g}(\boldsymbol{y}(t)) + \boldsymbol{d}(t) = \boldsymbol{u}(t)}$
- (FC$_n$): $\boxed{\boldsymbol{u}(t) = \boldsymbol{M}(\boldsymbol{y}(t))\left(\boldsymbol{K}_0(t)^2\boldsymbol{e}(t) + \boldsymbol{K}_0(t)\boldsymbol{K}_1(t)\dot{\boldsymbol{e}}(t)\right)}$
- For **known** $\boldsymbol{M}(\boldsymbol{y})$:
  - $\forall i \in \{1, \ldots, n\}\ \exists \varepsilon_i > 0\ \forall t \geq 0\colon\ \psi_{0,i}(t) - |e_i(t)| \geq \varepsilon_i$ and $\psi_{1,i}(t) - |\dot{e}_i(t)| \geq \varepsilon_i$
  - $\boldsymbol{K}_0(\cdot), \boldsymbol{K}_1(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0}; \mathbb{R}^n_{>0})$
  - $\boldsymbol{u}(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0}; \mathbb{R}^n)$
  - $\boldsymbol{n}_m(\cdot) \in \mathcal{W}^{2,\infty}(\mathbb{R}_{\geq 0}; \mathbb{R}^n)$ (measurement errors admissible)

Example: planar robot



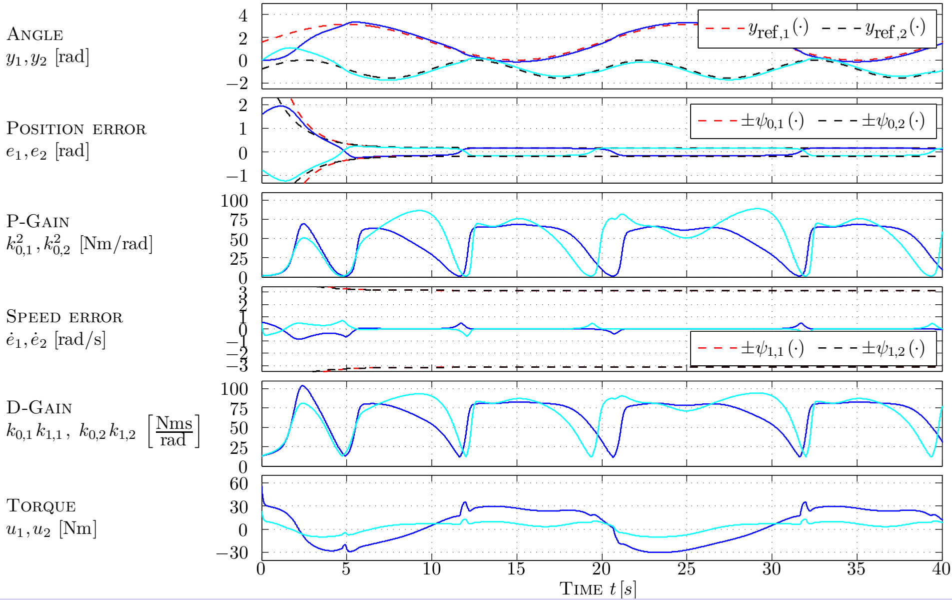
- (point-)masses m_1, m_2 $[\mathrm{kg}]$
- link lengths l_1, l_2 $[\mathrm{m}]$
- joint angle vector $\boldsymbol{y} := \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ $[\mathrm{rad}]^2$
- drive torque vector $\boldsymbol{u} := \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ $[\mathrm{Nm}]^2$
- friction & load neglected (simplicity)

- $\boldsymbol{u}(t) =$

$$\boldsymbol{M}(\boldsymbol{y}(t)) \left(\underbrace{\begin{bmatrix} k_{0,1}(t)^2 & 0 \\ 0 & k_{0,2}(t)^2 \end{bmatrix} \boldsymbol{e}(t)}_{\mathbf{P}} + \underbrace{\begin{bmatrix} k_{0,1}(t)k_{1,1}(t) & 0 \\ 0 & k_{0,2}(t)k_{1,2}(t) \end{bmatrix} \dot{\boldsymbol{e}}(t)}_{\mathbf{D}} \right)$$

- $\boldsymbol{e}(t) = \begin{pmatrix} y_{\mathrm{ref},1}(t) - y_1(t) \\ y_{\mathrm{ref},2}(t) - y_2(t) \end{pmatrix}$ and $\dot{\boldsymbol{e}}(t) = \begin{pmatrix} \dot{y}_{\mathrm{ref},1}(t) - \dot{y}_1(t) \\ \dot{y}_{\mathrm{ref},2}(t) - \dot{y}_2(t) \end{pmatrix}$

Simulation results (——joint 1, ——joint 2)



Summary and future work

To take home:

- simple but time-varying (adaptive) PD-controller
- tracking with prescribed transient accuracy (for **each** joint)
- boundary design very flexible
- measurement noise admissible
- **no** compensation of
  - friction
  - disturbances (unknown but bounded)
  - Coriolis and/or gravity effects
- inertia matrix must be **known**

Future work

- measurement results (for robotic manipulators)
- position funnel control
  - for rigid revolute joint robots **without** knowledge of inertia matrix?
  - for **elastic joint** robots (with/without knowledge of inertia matrix)?
- use of internal model(s)
- consideration of actuator saturation

Bibliography

X. Ye.
Universal λ-tracking for nonlinearly-perturbed systems without restrictions on the relative degree.
Automatica, 35(1):109–119, 1999.

E. Bullinger and F. Allgöwer.
Adaptive λ-tracking for nonlinear higher relative degree systems.
Automatica, 41:1191–1200, 2005.

A. Ilchmann, E. Ryan, and P. Townsend.
Tracking with prescribed transient behaviour for nonlinear systems of known relative degree.
SIAM Journal on Control and Optimization, 46(1):210–230, 2007.

A. Ilchmann and H. Schuster.
PI-Funnel Control for Two Mass Systems.
IEEE Transactions on Automatic Control, 54(4):918–923, 2009.

C. M. Hackl, C. Endisch and D. Schröder.
Contributions to non-identifier based adaptive control in mechatronics.
Robotics and Autonomous Systems, 57(10):996–1005, 2009.