Funnel control in mechatronics: An overview

— Position funnel control of stiff industrial servo-systems —

C.M. Hackl[1], A.G. Hofmann[2] and R.M. Kennel[1]

[1]Institute for Electrical Drive Systems and Power Electronics (EAL),
Technische Universität München (TUM), Germany

[2]Institute for Power Electronics and Electrical Drives (ISEA),
RTWH Aachen University, Germany

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Outline

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   - Control law
3. Application: Position control of stiff industrial servo-systems
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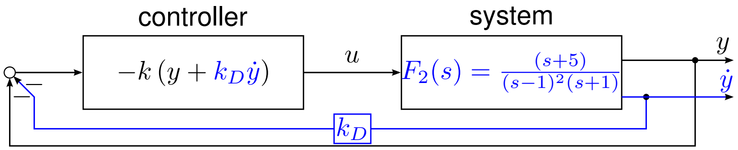
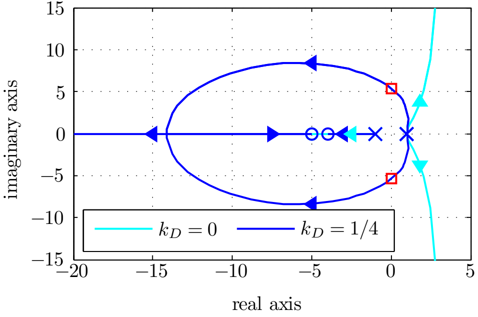
Motivation: High-gain control



'structural properties' of $F_2(s)$

- relative degree (pole excess): $r = 2$
- positive high-frequency gain ($\lim_{s\to\infty} s^2 F_1(s) = 1$)
- minimum-phase (numerator is Hurwitz)

Root-locus ($\times$ poles, $\circ$ zeros)



Funnel control ($r = 2$): Admissible system class

Only 'structural system knowledge' necessary:

- relative degree two
- positive (or known sign of) high-frequency gain
- stable zero-dynamics (minimum-phase in the LTI case)
- (derivative feedback admissible)

Example: LTI SISO system of form

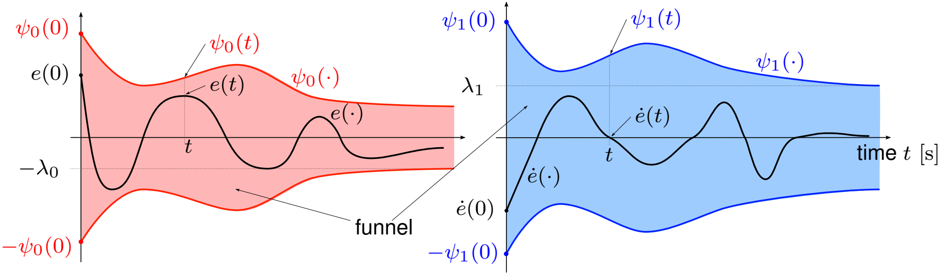
$$\begin{aligned} \tfrac{\mathrm{d}}{\mathrm{d}t}\,\boldsymbol{x}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}\,u(t), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \in \mathbb{R}^n \\ y(t) &= \boldsymbol{c}^\top \boldsymbol{x}(t) \end{aligned}$$

with

- $\boldsymbol{c}^\top \boldsymbol{b} = 0$ and $\boldsymbol{c}^\top \boldsymbol{A}\boldsymbol{b} \neq 0$ (relative degree two)
- $\boldsymbol{c}^\top \boldsymbol{A}\boldsymbol{b} > 0$ (positive high-frequency gain)
- $\det\left(\begin{bmatrix} s\boldsymbol{I}_n - \boldsymbol{A} & \boldsymbol{b} \\ \boldsymbol{c}^\top & 0 \end{bmatrix}\right) \neq 0$ for all $s \in \mathbb{C}$ with $\Re(s) \geq 0$ (minimum-phase)

Funnel control ($r = 2$): Control objective

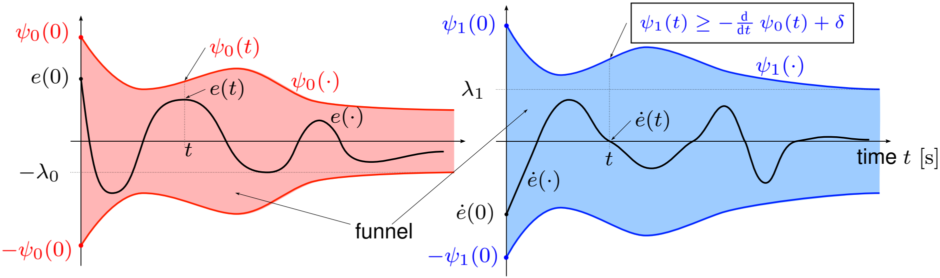
- 'tracking with prescribed transient accuracy' of $y_{\mathrm{ref}}(\cdot) \in \mathcal{C}^1 \cap \mathcal{L}^\infty(\mathbb{R}_{\geq 0}; \mathbb{R})$



with tracking error $e(t) = y_{\mathrm{ref}}(t) - y(t)$

- funnel boundary design:
  - $\psi_i(\cdot)$ absolutely continuous and $\psi_i(t) \geq \lambda_i > 0$ for all $t \geq 0$ and $i \in \{0, 1\}$;
  - $|e(0)| < \psi_0(0)$ and $|\dot{e}(0)| < \psi_1(0)$
  - $\psi_1(t) \geq -\frac{\mathrm{d}}{\mathrm{d}t}\,\psi_0(t) + \delta$ for all $t \geq 0$

Funnel control ($r = 2$): Control law



- funnel controller (with derivative feedback)

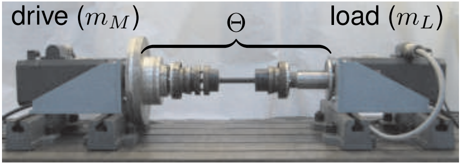
$$u(t) = k_0(t)^2 \left(e(t) + \frac{k_1(t)}{k_0(t)} \dot{e}(t) \right), \qquad e(t) = y_{\mathrm{ref}}(t) - y(t)$$

- with 'time-varying' gains

$$k_0(t) = \frac{s_0(t)}{\psi_0(t) - |e(t)|} \qquad \text{and} \qquad k_1(t) = \frac{s_1(t)}{\psi_1(t) - |\dot{e}(t)|}$$

- scaling functions $s_i(\cdot) \in \mathcal{C} \cap \mathcal{L}^\infty(\mathbb{R}_{\geq 0}; [m_i, \infty))$ where $m_i > 0$, $i \in \{0, 1\}$

Application: Stiff industrial servo-systems



- drive torque $m_M(\cdot) \in \mathcal{C}(\mathbb{R}_{\geq 0}; \mathbb{R})$ $[\mathrm{Nm}]$ (control input)
- load torque $m_L(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0}; \mathbb{R})$ $[\mathrm{Nm}]$ (disturbance)
- inertia $\Theta > 0$ $\left[\mathrm{kg\,m}^2\right]$
- state $\boldsymbol{x} = (\phi, \omega)^\top$: position ϕ $[\mathrm{rad}]$, speed ω $[\mathrm{rad/s}]$
- friction (on drive & load side)
  - unbounded viscous friction with coefficients $\nu_1, \nu_2 \geq 0$ $[\mathrm{Nms/rad}]$
  - bounded dynamic friction $\mathfrak{F}_1, \mathfrak{F}_2 \colon \mathcal{C}(\mathbb{R}_{\geq 0}; \mathbb{R}) \to \mathcal{L}^\infty(\mathbb{R}_{\geq 0}; \mathbb{R})$ $[\mathrm{Nm}]$
- gear with ratio $g_r \in \mathbb{R} \setminus \{0\}$ $[1]$ (neglecting dynamics & backlash)
- signals available for feedback: position ϕ and speed $\omega = \dot{\phi}$ (deteriorated by $n_m(\cdot) \in \mathcal{W}^{2,\infty}(\mathbb{R}_{\geq 0}; \mathbb{R})$ & $\dot{n}_m(\cdot)$, resp.)

Application: Model of stiff industrial servo-systems

LTI SISO system with input disturbance and nonlinear perturbation

$$\frac{\mathrm{d}}{\mathrm{d}t}\overbrace{\begin{pmatrix}\phi(t)\\ \omega(t)\end{pmatrix}}^{=:\boldsymbol{x}(t)} = \overbrace{\begin{bmatrix}0 & 1\\ 0 & -\frac{\nu_1+\nu_2/g_r^2}{\Theta}\end{bmatrix}}^{=:\boldsymbol{A}}\begin{pmatrix}\phi(t)\\ \omega(t)\end{pmatrix} + \overbrace{\begin{pmatrix}0\\ \frac{1}{\Theta}\end{pmatrix}}^{=:\boldsymbol{b}}\big(\overbrace{m_M(t)}^{=:u(t)} - (\mathfrak{F}_1\omega)(t)\big)$$

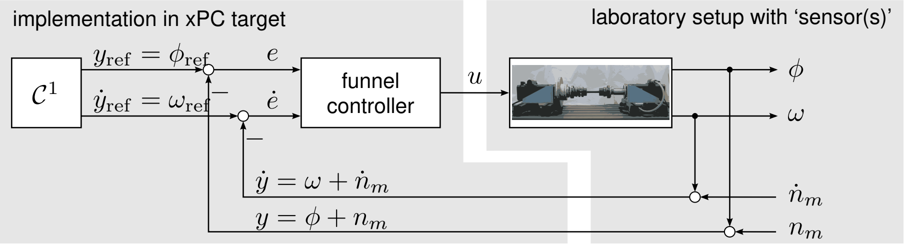
$$- \begin{pmatrix}0\\ \frac{1}{g_r\Theta}\end{pmatrix}\big(m_L(t) + (\mathfrak{F}_2\tfrac{\omega}{g_r})(t)\big)$$

$$y(t) = \underbrace{\begin{pmatrix}1 & 0\end{pmatrix}}_{=:\boldsymbol{c}^\top}\boldsymbol{x}(t), \qquad (\phi(0),\,\omega(0))^\top = (\phi^0,\,\omega^0)^\top$$

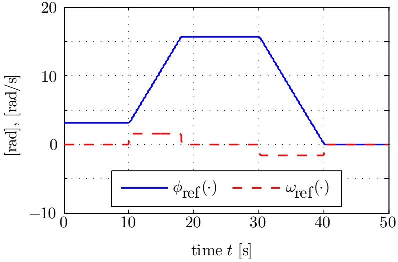
'Structural properties' of model:

- $\boldsymbol{c}^\top\boldsymbol{b} = 0$ and $\boldsymbol{c}^\top\boldsymbol{A}\boldsymbol{b} = 1/\Theta > 0$
  $\Rightarrow$ relative degree two and positive high-frequency gain
- $\det\left(\begin{bmatrix}s\boldsymbol{I}_n - \boldsymbol{A} & \boldsymbol{b}\\ \boldsymbol{c}^\top & 0\end{bmatrix}\right) \neq 0$ for all $s \in \mathbb{C}$ with $\Re(s) \geq 0$ (unperturbed system is minimum-phase)
- and $m_L(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0};\mathbb{R})$, $\mathfrak{F}_1, \mathfrak{F}_2\colon\ \mathcal{C}(\mathbb{R}_{\geq 0};\mathbb{R}) \to \mathcal{L}^\infty(\mathbb{R}_{\geq 0};\mathbb{R})$ (bounded disturbance & perturbations)
  $\Rightarrow$ stable zero-dynamics

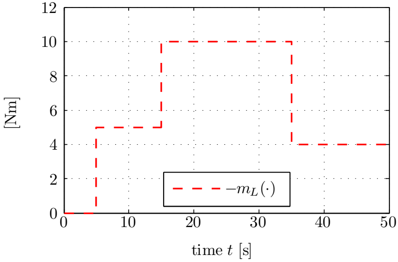
Application: Position control implementation



Reference

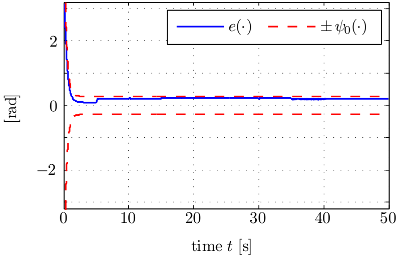


Load

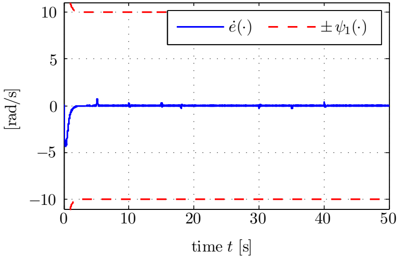


Application: Position control measurement results

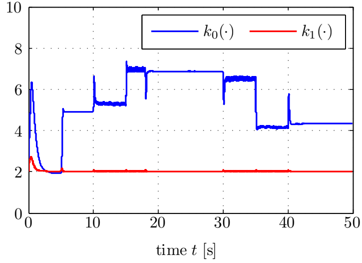
Position error



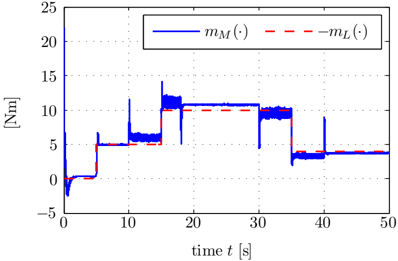
Speed error



Gains



Torque



Conclusion

- funnel control applicable for position control
- for application only structural proporties must be checked
- system identification or parameter estimation is not required
- ‘time-varying’ gains (not monotone)
- ‘tracking with prescribed transient accuracy’ independent of system parameters (robustness)

Some more results (not presented)

- funnel control applicable for speed control
- steady state accuracy is feasible in conjunction with PI-like extension
- funnel control is also applicable for speed and position control of elastic servo-systems (two mass-systems)
- position funnel control of rigid revolute joint robotic manipulators is feasible (if inertia matrix is known)
- funnel control in presence of actuator saturation is feasible (conservative feasibility condition must be satisfied)

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