Applicability of Funnel-Control in Robotics

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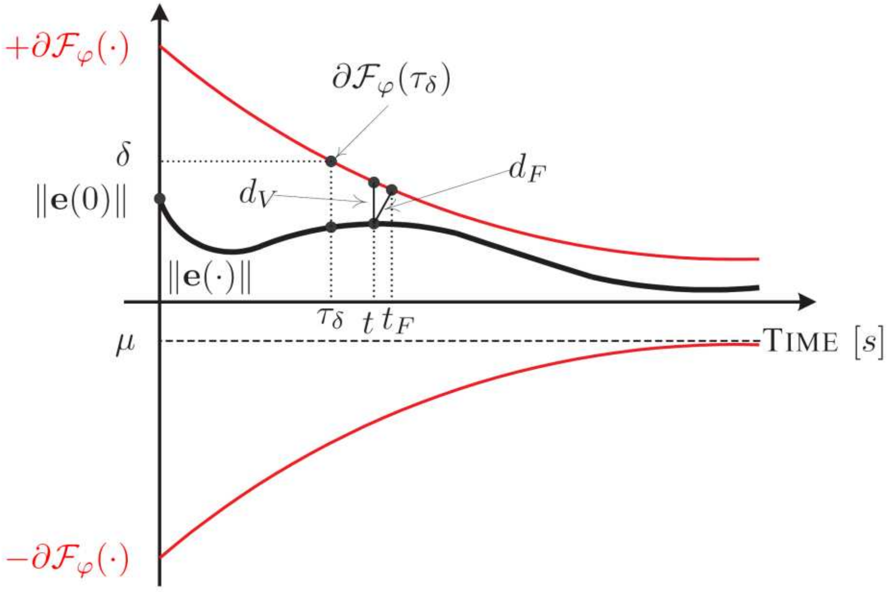

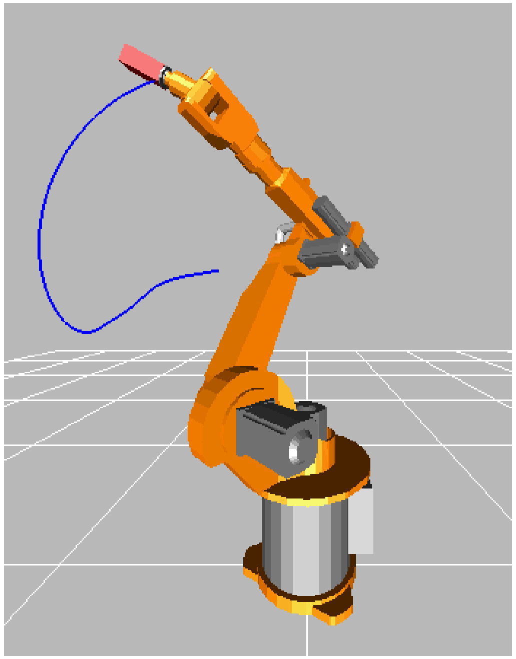

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Goals:

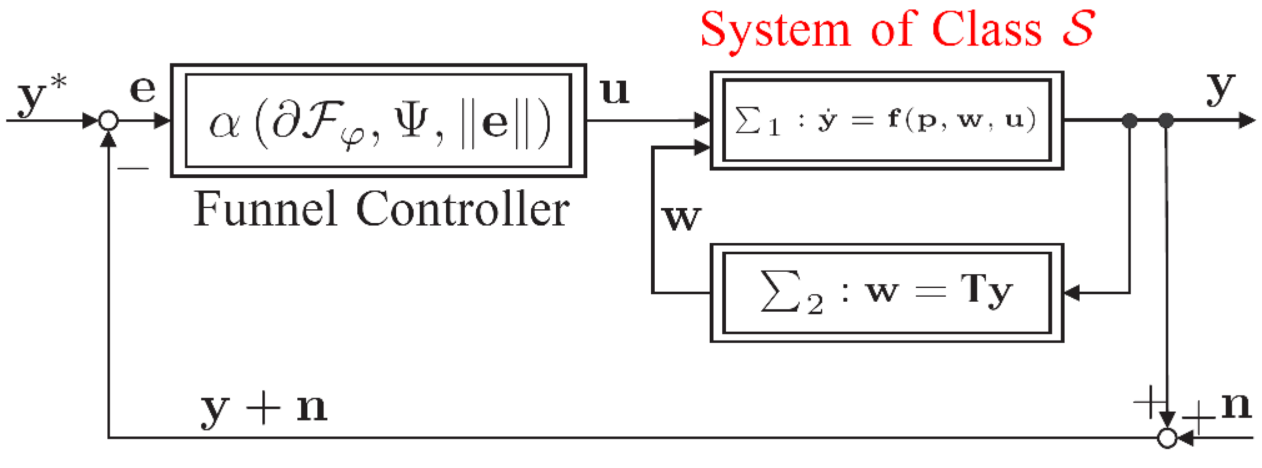
- show applicability of Funnel-Control in robotics
- check for prerequisits of auxiliary system
  (relative degree one, minimum-phase, positive sign of high-frequency gain)
- result of these first steps:
  stable, adaptive (time-varying) motor position control of nonlinear (rigid) robotic systems *without* knowledge of system like e.g. inertia matrix, nonlinearities, ... (only structural properties)
- tracking with prescribed transient behavior of auxiliary output

Overview

1. Funnel-Control (FC)
   - Prerequisites, Basic Idea –
2. Rigid (Revolute) Robotics
3. Auxiliary System
4. Manutec r3 Simulation (SISO & MIMO)
5. Conclusion & Outlook

Funnel-Control (FC) – Prerequisites

(developed by *Ilchmann et al.*)



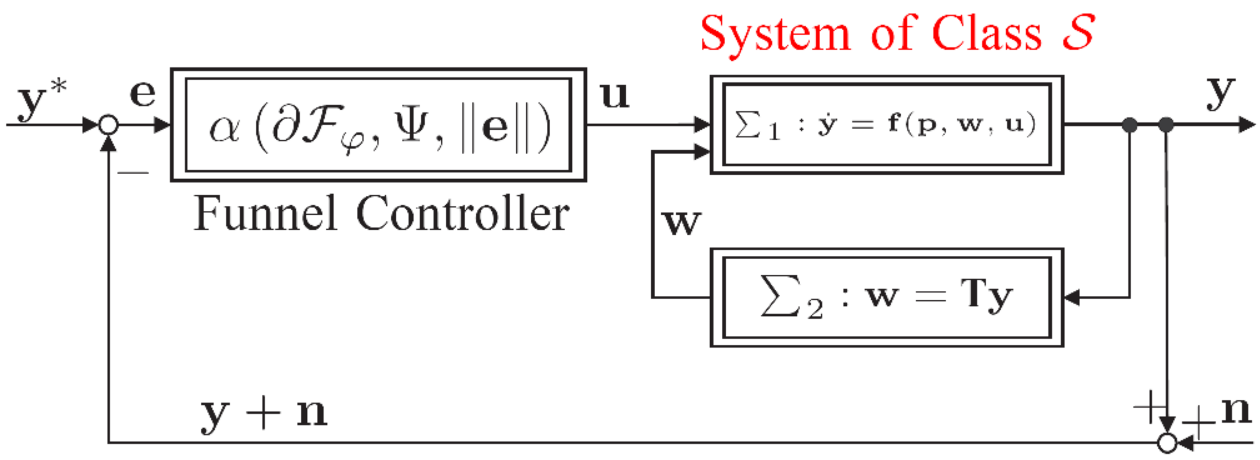
Controller properties:

- proportional controller (simple and robust)
- adaptive time-varying gain
- control law:

$$\boxed{\mathbf{u}(t) = \alpha\left(\partial\mathcal{F}_\varphi(t), \Psi(t), \|\mathbf{e}(t)\|\right) \cdot \mathbf{e}(t)}$$

Funnel-Control (FC) – Prerequisites

(developed by *Ilchmann et al.*)



System of class S - *three* prerequisites:

1. strict relative degree $\boxed{r = 1}$
2. minimum-phase, e.g.: (asymptotically stable zero-dynamics) $\boxed{\sigma(Q_{ZD}) \in \mathbb{C}_-}$
3. positive sign of high-frequency gain

$$\boxed{\|u_n\| \to \infty \;\; for \;\; n \to \infty \Rightarrow \min_{(p,w)\in C} \frac{\langle u_n, f(p,w,u_n)\rangle}{\|u_n\|} \to \infty}$$

Funnel-Control (FC) - Basic Idea

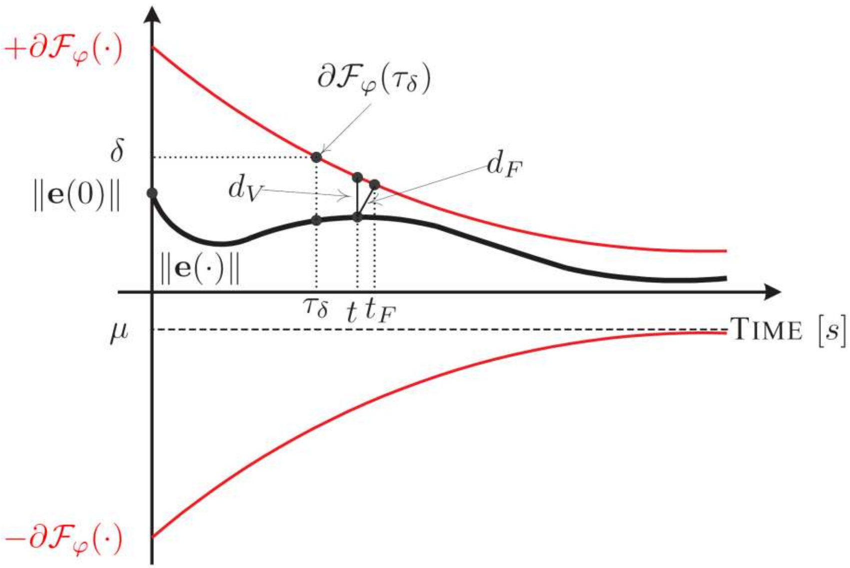


Fig. 2: Error evolution within Funnel Boundary
[Predefinition of transient behavior possible!]

Adaption of gain depending on vertical (or minimal) distance:

$$\boxed{d_V(t) = \partial\mathcal{F}_\varphi(t) - \|\mathbf{e}(t)\|}$$

- small distance => high gain (more aggressive control)
- great distance => small gain (more relaxed control)

Time-varying (adaptive) gain:
Boundedness of control error (for sufficiently large but finite control inputs!)

$$\boxed{\alpha\left(\partial\mathcal{F}_\varphi(t), \Psi(t), \|\mathbf{e}(t)\|\right) = \frac{\Psi(t)}{d_V(t)} = \frac{\Psi(t)}{\partial\mathcal{F}_\varphi(t) - \|\mathbf{e}(t)\|}}$$

Robotics (Rigid Case) - Dynamics

Rigid robotic dynamics from Lagrange's method

$$\boxed{\tau = M(q)\ddot{q} + n(q;\dot{q})}$$

τ	torque $[Nm]$
q	position $[rad]$
$\dot{q}$	velocity $[\frac{rad}{s}]$
$\ddot{q}$	acceleration $[\frac{rad}{s^2}]$
$M(q)$	inertia matrix $[\frac{Nms^2}{rad}]$

where

$$\boxed{n(q;\dot{q}) = V_m(q;\dot{q})\dot{q} + g(q) + F_d\dot{q} + f_S(\dot{q}) + t_d}$$

includes all nonlinearities, e.g.:

- Coriolis & centripedal terms $V_m(q;\dot{q})\dot{q}$
- gravitational forces $g(q)$
- friction effects (viscous & stiction) $F_d\dot{q} + f_S(\dot{q})$
- disturbances t_d

Robotics (Rigid Case) – State Space Representation

State space with states $\mathbf{x}(t) := \left[\mathbf{x}_{\mathbf{q}}(t)\ \ \mathbf{x}_{\dot{\mathbf{q}}}(t)\right]^T = \left[\mathbf{q}(t)\ \ \dot{\mathbf{q}}(t)\right]^T \in \mathbb{R}^{2M}$

$$\dot{\mathbf{x}} = \underbrace{\begin{pmatrix} \mathbf{x}_{\dot{\mathbf{q}}} \\ -\mathbf{M}(\mathbf{x}_{\mathbf{q}})^{-1}\mathbf{n}(\mathbf{x}_{\mathbf{q}}; \mathbf{x}_{\dot{\mathbf{q}}}) \end{pmatrix}}_{\mathbf{f}(\mathbf{x}) \in \mathbb{R}^N} + \underbrace{\begin{pmatrix} 0 \\ \mathbf{M}(\mathbf{x}_{\mathbf{q}})^{-1} \end{pmatrix}}_{\mathbf{G}(\mathbf{x}) \in \mathbb{R}^{N \times M}} \cdot \boldsymbol{\tau}$$

System output (goal position control):

$$\mathbf{y}(t) = \mathbf{q}(t) \in \mathbb{R}^M$$

Properties of the inertia matrix:

- symmetric & positive definite $\quad \mathbf{M}(\mathbf{q}) = \mathbf{M}^{\mathrm{T}}(\mathbf{q}) > 0$
- non-singular $\quad \det(\mathbf{M}(\mathbf{q})) \neq 0$
- uniformerly bounded away from zero with $\varepsilon_m, \varepsilon_M > 0$

$$0 < \varepsilon_m \cdot \mathbf{I} < \lambda_m(\mathbf{M}(\mathbf{q})) \cdot \mathbf{I} \leq \mathbf{M}(\mathbf{q}) \leq \lambda_M(\mathbf{M}(\mathbf{q})) \cdot \mathbf{I} < \varepsilon_M \cdot \mathbf{I} < \infty$$

Auxiliary System – Introduction

We introduce the new auxiliary output

$$\mathbf{y}' = \mathbf{h}'(\mathbf{x}) = \mathbf{K}_P \mathbf{x}_q + \mathbf{K}_D \mathbf{x}_{\dot{q}}$$

and the (linear!) coordinate transformation $[\boldsymbol{\xi}, \boldsymbol{\eta}]^T = \boldsymbol{\Phi}\mathbf{x}$

with transformation matrix
$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\Phi}_\xi \\ \boldsymbol{\Phi}_\eta \end{bmatrix} = \begin{bmatrix} \mathbf{K}_P & \mathbf{K}_D \\ \mathbf{1} & \mathbf{0} \end{bmatrix}$$

and deduce the new system represenation

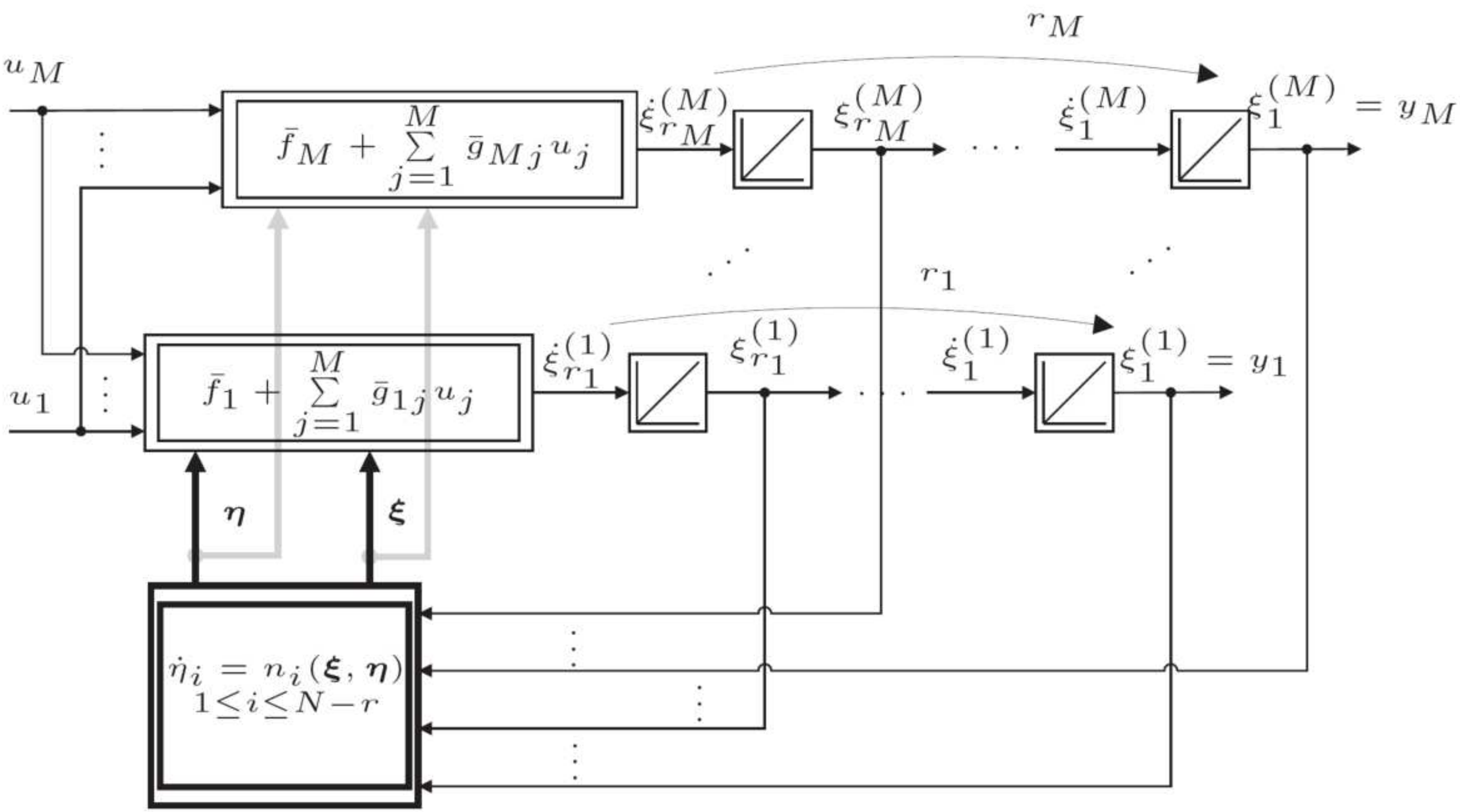
Forward dynamics:
$$\dot{\xi} = \underbrace{K_P \; K_D^{-1}\xi - K_D^{-1}K_P\eta - K_D M(\eta)^{-1} n(\Phi^{-1}(\xi,\eta))}_{=:f(\xi,\eta)} + \underbrace{K_D M(\eta)^{-1}}_{=:G(\xi,\eta)}\tau$$

Internal dynamics:
$$\dot{\eta} = \underbrace{K_D^{-1}}_{=:P}\xi + \underbrace{(-K_D^{-1}K_P)}_{=:Q}\eta$$

Output:
$$y^0 = \left(\xi_1^{(1)}, \ldots, \xi_1^{(M)}\right)^T$$

Auxiliary System - Decomposition

The new system is decomposed in forward and internal dynamics



where each forward branch has relative degree $r_i \geq 1 \ \forall i = 1, \ldots, M$ (see later)

Auxiliary System – Relative degree

1. Check for relative degree one:

The auxiliary system has the auxiliary output

$$\mathbf{y}' = \mathbf{h}'(\mathbf{x}) = \mathbf{K}_P \mathbf{x}_q + \mathbf{K}_D \mathbf{x}_{\dot{q}}$$

Differentiating this the auxiliary output gives

$$\begin{aligned} \dot{y}' &= \frac{\partial h'(x)}{\partial x}\dot{x} = K_P \dot{x}_q + K_D \ddot{x}_q \\ &= K_P \dot{x}_q + K_D \left(-M(x_q)^{-1} n(x_q; \dot{x}_q) + M(x_q)^{-1} \cdot \tau \right) \end{aligned}$$

Hence, for any positive definite matrix

$$\boxed{K_D M(x_q)^{-1} > 0}$$

the relative degree is **one** (reduction of the relative degree by linear combination).

Simplistic, $K_D = \beta \cdot 1$ with $\beta > 0$ assures positive definitiness.

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Funnel Control

Rigid Robotics

Auxiliary System

Manutec r3 Simulation

Conclusion & Outlook

Auxiliary System - Zero-Dynamics

2. Check for stable Zero-Dynamics:

Setting

$$\mathbf{y}(t) = \mathbf{0}$$

gives the internal dynamics (zero-dynamics) of the auxiliary system

$$\dot{\eta} = -\mathbf{K}_D^{-1}\mathbf{K}_P\eta = \mathbf{Q}\eta$$

which are asymptotically stable for all

$$\boxed{\sigma(\mathbf{Q}) = \sigma\left(-\mathbf{K}_D^{-1}\mathbf{K}_P\right) \in \mathbb{C}_-}$$

achievable due to free choice of the gain matrices (design matrices).

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Auxiliary System - High-Frequency Gain

3. Check for positve sign of HFG:

For any sequence

$$\|\mathbf{u}_n\| \to \infty \text{ as } n \to \infty$$

the generalization of the sign of the high-frequency gain yields

$$\min_{[\xi;\eta]\in\mathcal{C}} \frac{\langle u_n;\dot{\xi}\rangle}{\|u_n\|} = \min_{[\xi;\eta]\in\mathcal{C}} \frac{\langle u_n; \mathbf{f}(\xi;\eta) + \mathbf{G}(\xi;\eta)u_n\rangle}{\|u_n\|}$$

$$= \min_{[\xi;\eta]\in\mathcal{C}} \left(\frac{u_n^T \mathbf{f}(\xi;\eta)}{\|u_n\|} + \frac{u_n^T \mathbf{G}(\xi;\eta) u_n}{\|u_n\|} \right)$$

$$\geq -\max_{[\xi;\eta]\in\mathcal{C}} \frac{\cancel{\|u_n\|}}{\cancel{\|u_n\|}} \underbrace{\|\mathbf{f}(\xi;\eta)\|}_{\text{bounded in } \mathcal{C}} + \underbrace{\frac{\beta}{\varepsilon}}_{=:M>0} \frac{\|u_n\|^{\cancel{2}}}{\cancel{\|u_n\|}}$$

$$\to \infty \text{ as } n \to \infty$$

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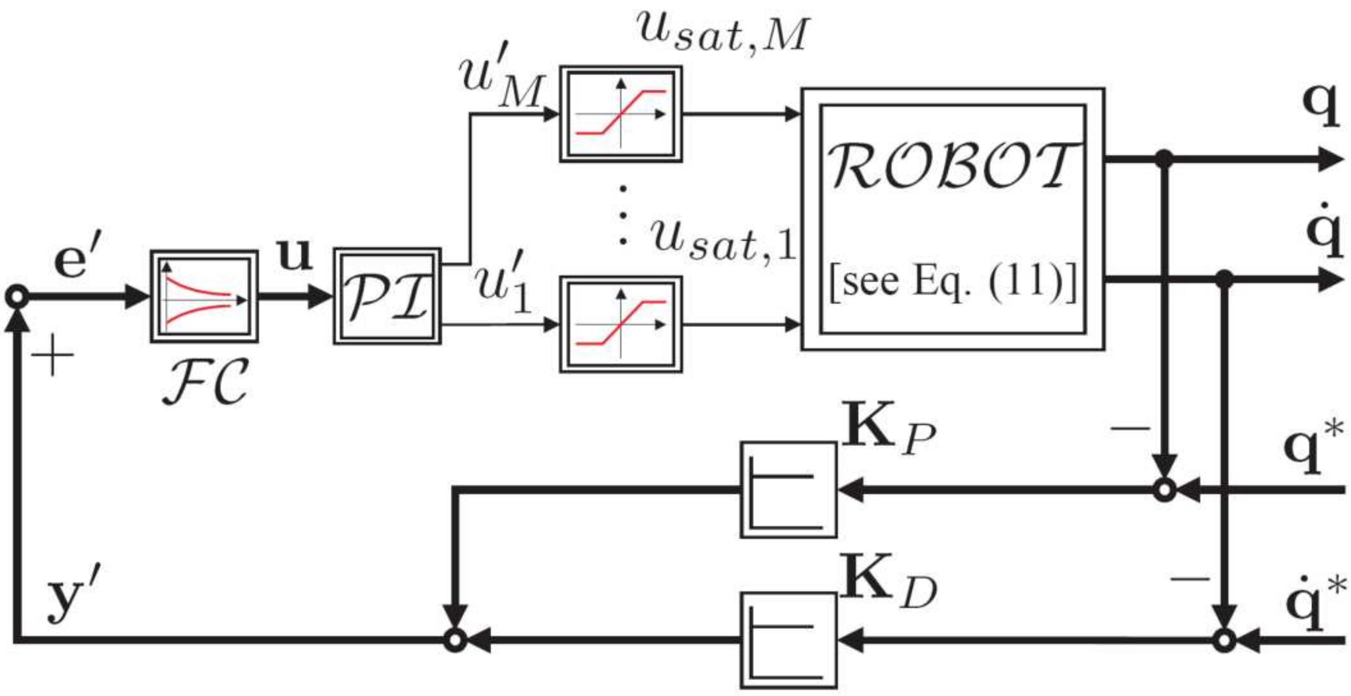
Auxiliary System (Control-Loop: MIMO)



(a) *One* MIMO Controller for All Joints

MIMO PI controller:

$$\boxed{\mathbf{u}'(t) = \mathbf{u}(t) + \frac{1}{T_I}\int \mathbf{u}(\tau)d\tau}$$

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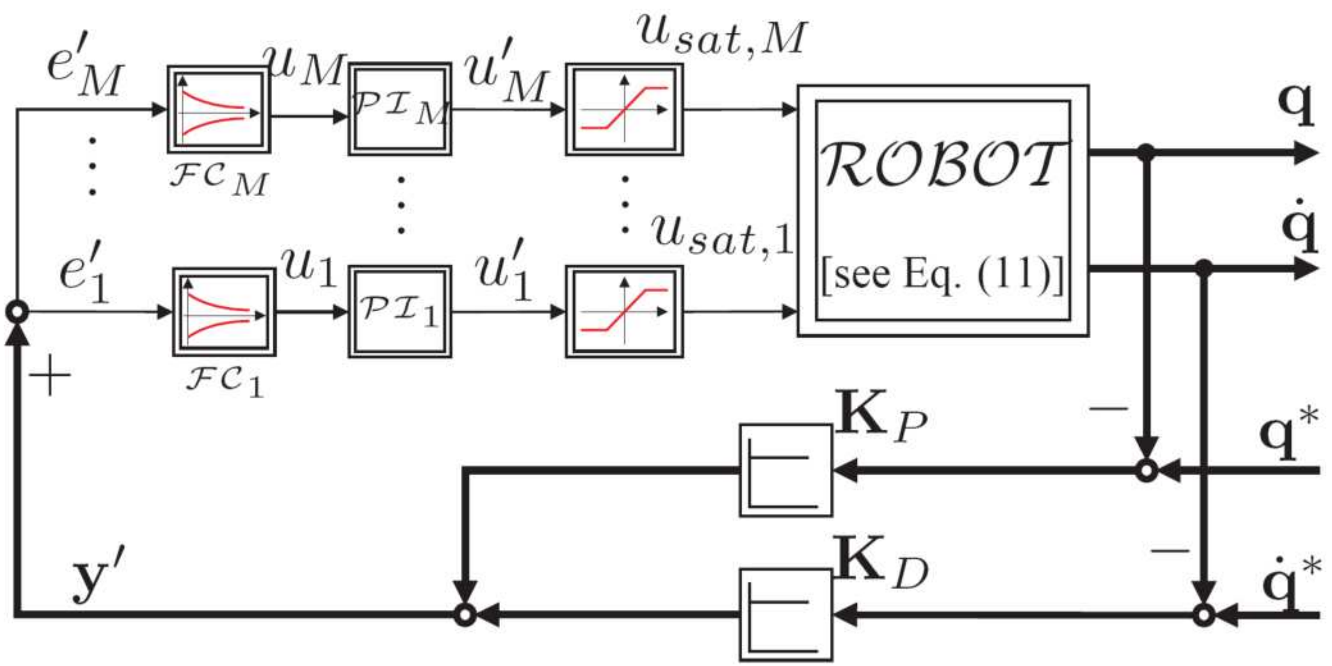
Auxiliary System (Control-Loop: SISO)

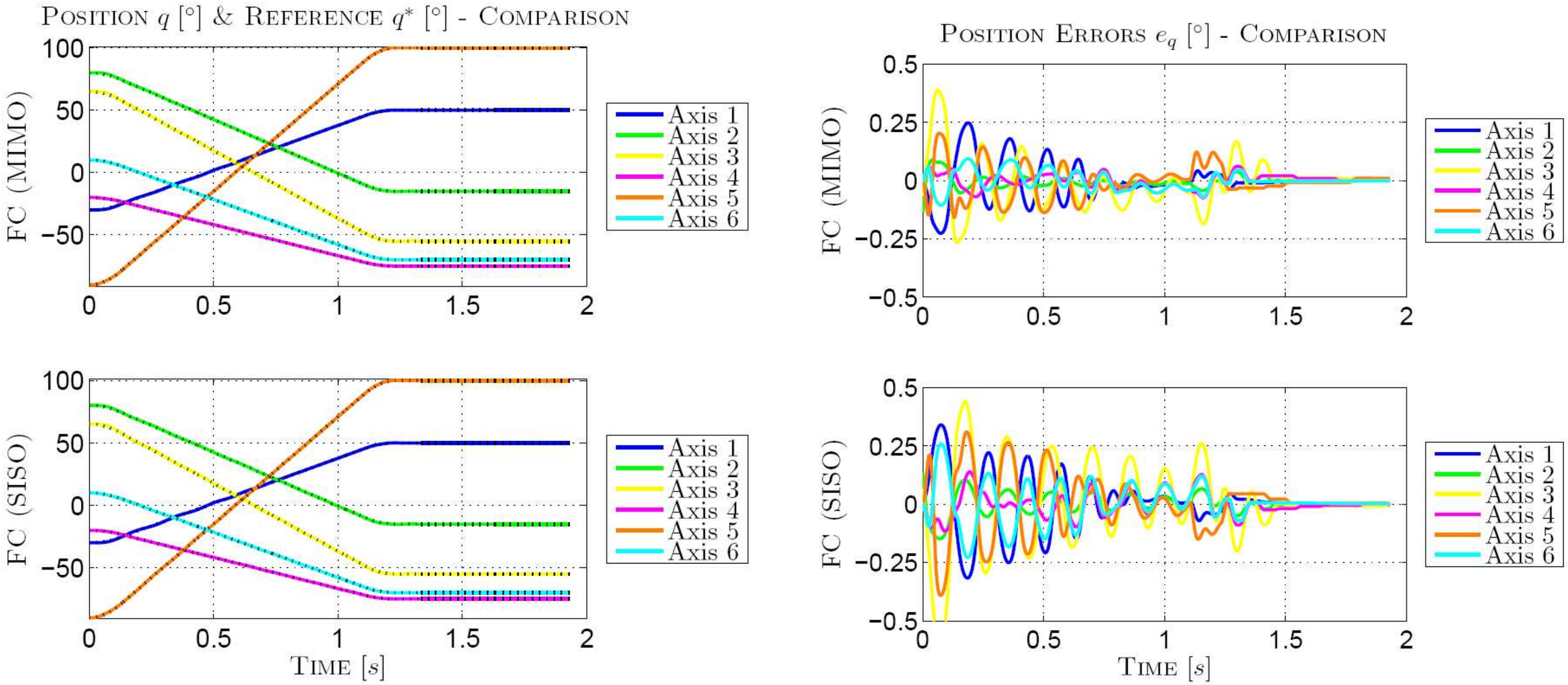


(b) *One* SISO Controller for Each Joint

SISO PI controllers:
(for each joint)

$$u_i'(t) = u_i(t) + \frac{1}{T_{I_i}} \int u_i(\tau)\,d\tau$$

Manutec r3 Simulation (Modelica/Simulink)

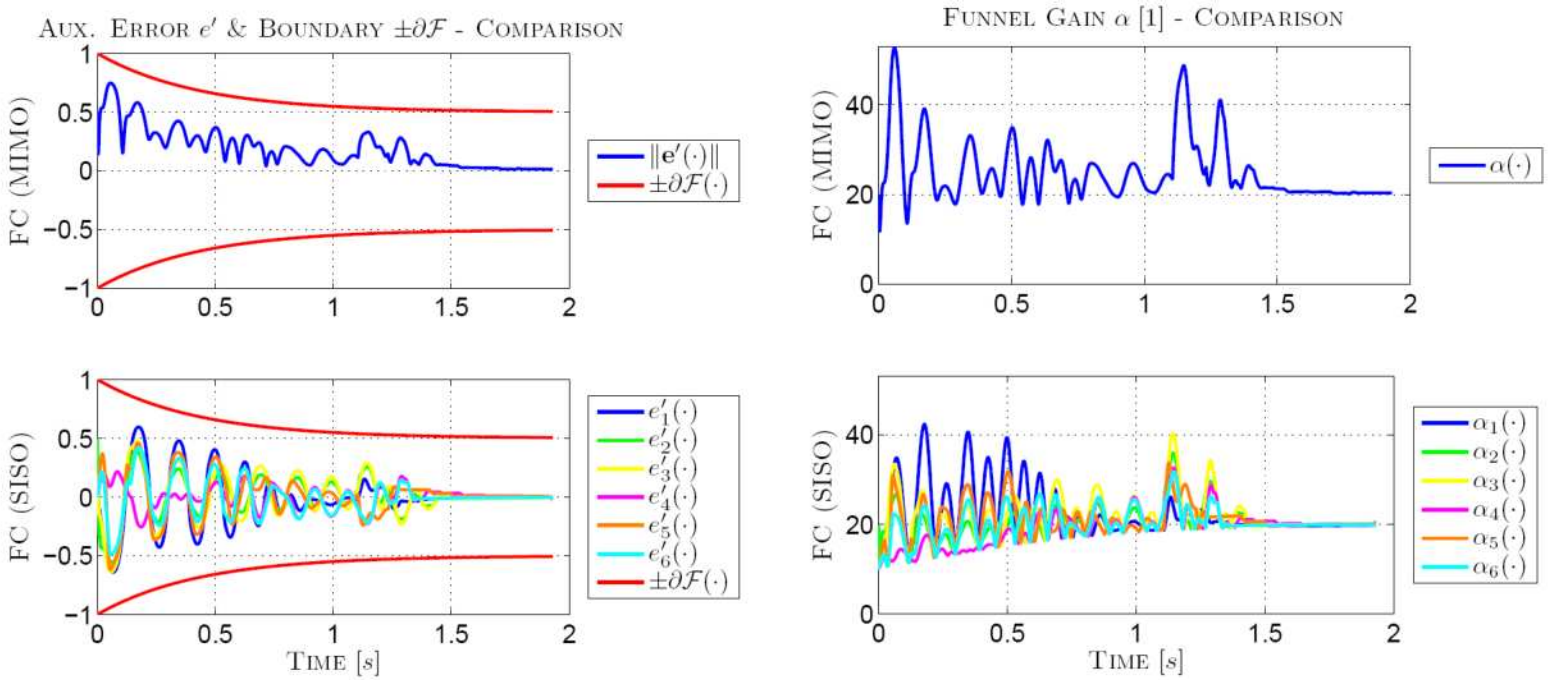


(a) Position (colored) & Reference (dotted)

(b) Position Errors (colored)

Reference for Joint Angles	Start	End
Axis 1 q_1^* [°]	-30	50
Axis 2 q_2^* [°]	80	-15
Axis 3 q_3^* [°]	65	-55
Axis 4 q_4^* [°]	-20	75
Axis 5 q_5^* [°]	-90	100
Axis 6 q_6^* [°]	10	-70

Manutec r3 Simulation (Modelica/Simulink)



(a) Auxiliary Error (colored) & Funnel Boundary (red) (b) Funnel Gain (colored)

Controller Design Parameters	Value
Accuracy Limit μ [°]	0.5
Time τ_δ [s]	0.3
Accuracy δ [$\frac{\circ}{s}$] (at τ_δ)	0.75
Initial Boundary $\partial\mathcal{F}_{exp}(0)$	1
$\mathcal{PI}$ Integrator Time Constant T_I [s]	0.1
Scaling Factor Ψ [1]	10
Proportional Matrix $\mathbf{K}_P$ [$\frac{Nm}{\circ}$]	$\mathbf{1} \in \mathbb{R}^{6\times 6}$
Derivative Matrix $\mathbf{K}_D$ [$\frac{Nms}{\circ}$]	$0.01 \cdot \mathbf{1} \in \mathbb{R}^{6\times 6}$

Conclusion:

- Application of Funnel-Control in Robotics is possible (yet only by introduction of the auxiliary system)
- Simple, robust and adaptive (time-varying) controller structure
- *One* controller for *all* joints (MIMO) **or** *one* controller for *each* joint (SISO)
- Pre-definition of desired transient behavior (indirectly) possible
- Measurement noise is admissible
- No system parameters need to be identified or estimated

Outlook:

- Application of Funnel-Control directly for Position Control (relative degree 2 case)
- Exploit non-diagonal entries of gain matrices to achieve better decoupling of joints
- Increase system damping in flexible case (flexible joints, rigid links)