Specially Designed Funnel-Control in Mechatronics

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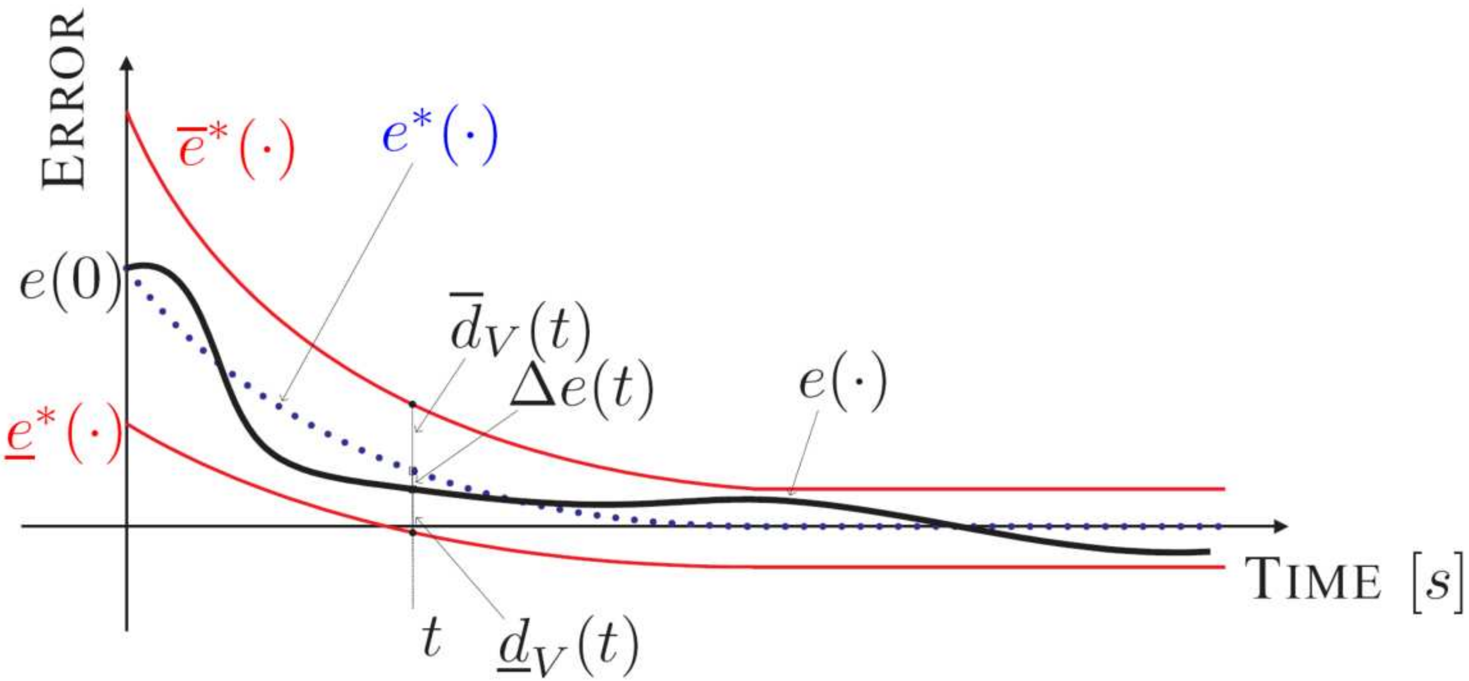

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Goals:

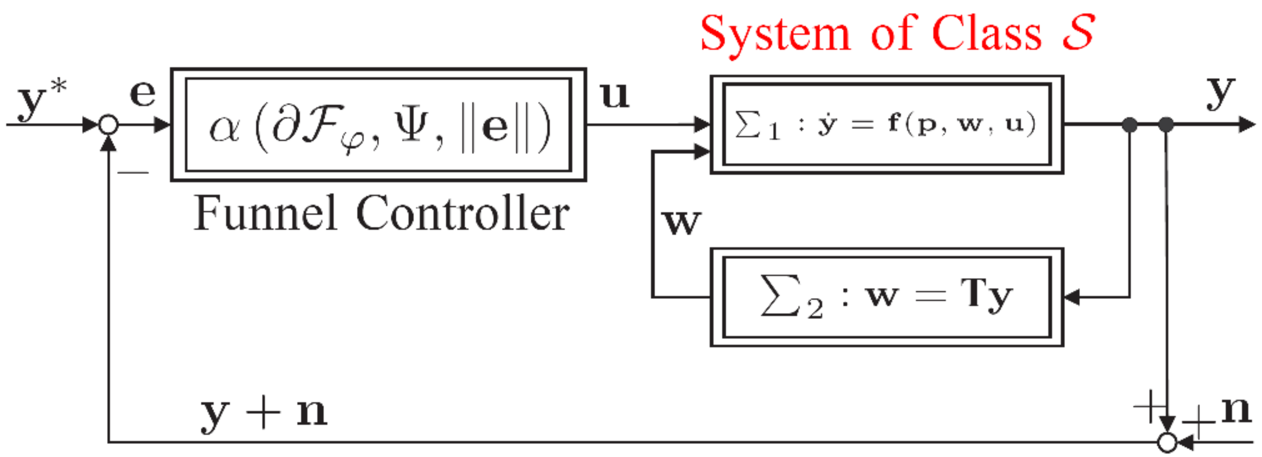
- stable, adaptive (time-varying) load speed control of nonlinear two-mass flexible servo system without knowledge of system parameters (only structural properties)
- precribed transient behavior (FC & ERC)
- accurate tracking within prescribed limits (FC & ERC)
- increase system damping & reduce overshoots (ERC)
- error guidance along desired error evolution (ERC)

Overview

1. Funnel-Control (FC)
2. Error Reference Control (ERC)
3. Nonlinear Two-Mass Flexible Servo System (2MS)
4. Measurement Results (Comparison SF, FC, ERC)
5. Conclusion & Outlook

Funnel-Control (FC) – Prerequisites

(developed by *Ilchmann et al.*)



Controller properties:

- Proportional Controller (simple and robust)
- Adaptive Time-Varying Gain
- Control Law:

$$\mathbf{u}(t) = \alpha\left(\partial\mathcal{F}_\varphi(t), \Psi(t), \|\mathbf{e}(t)\|\right) \cdot \mathbf{e}(t)$$

System of Class S - prerequisites:

- Relative Degree $r = 1$
- Minimum-Phase (stable zero-dynamics)
- Known High-Frequency Gain

Funnel-Control (FC) - Basic Idea

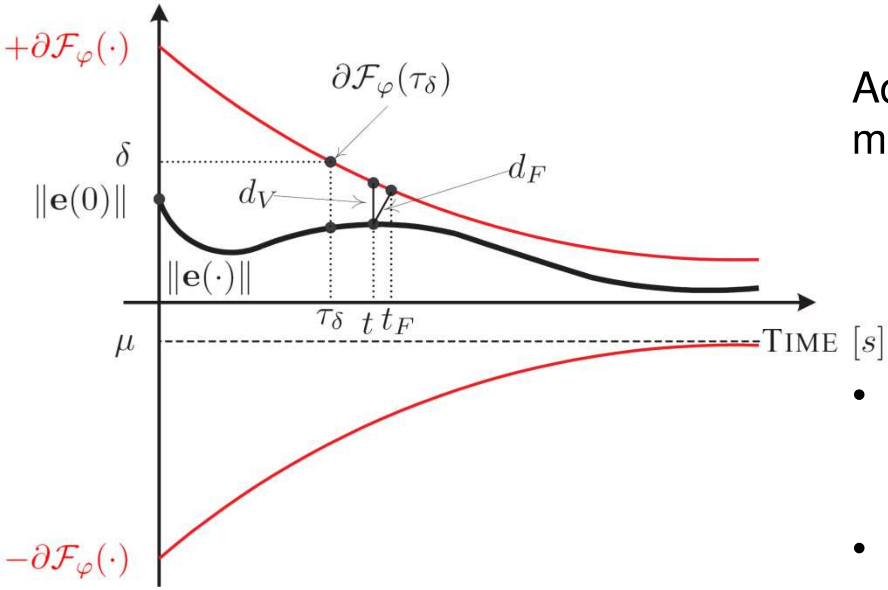


Fig.: FC - Error evolution within Funnel Boundary
[Predefinition of Transient Behavior possible!]

Adaption of gain depending on (vertical or minimal) distance:

$$d_V(t) = \partial\mathcal{F}_\varphi(t) - \|\mathbf{e}(t)\|$$

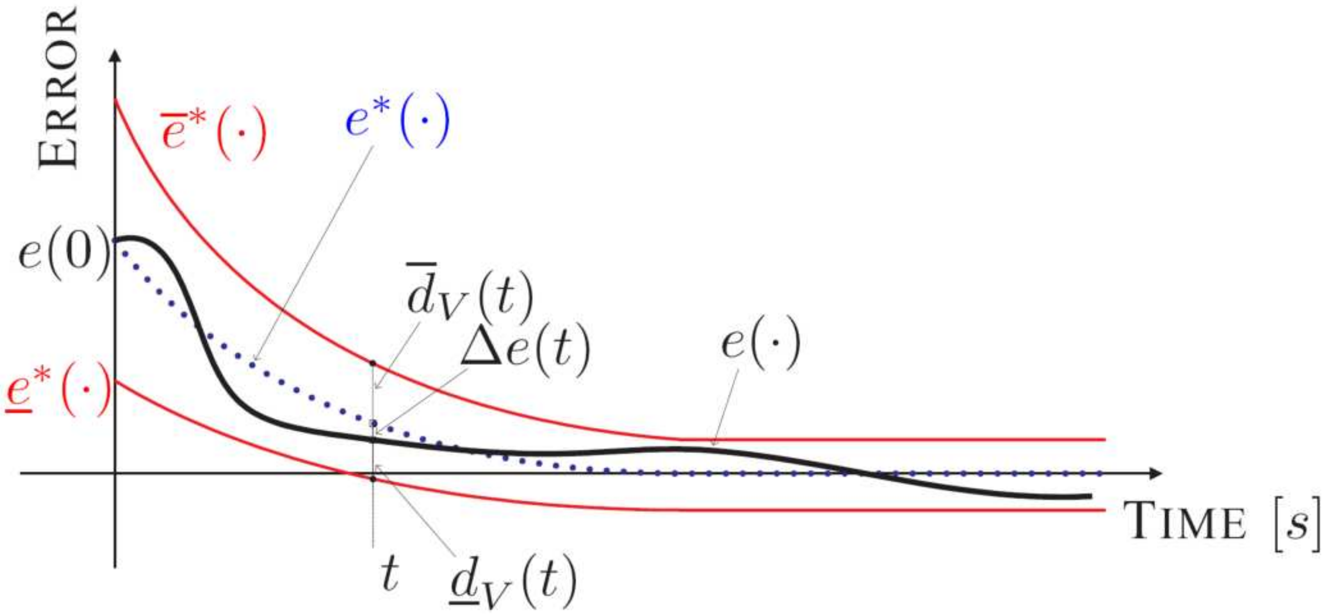
- Short distance => high gain (more aggressive control)
- Great distance => small gain (more relaxed control)

Time-varying (adaptive) gain $\Rightarrow$ Boundedness of control error (for sufficiently large but finite control inputs!)

$$\alpha\left(\partial\mathcal{F}_\varphi(t), \Psi(t), \|\mathbf{e}(t)\|\right) = \frac{\Psi(t)}{d_V(t)} = \frac{\Psi(t)}{\partial\mathcal{F}_\varphi(t) - \|\mathbf{e}(t)\|}$$

Error Reference Control (ERC) - Basic Idea

(direct derivative of Funnel-Control with specially designed boundaries and reference)



Predefinition of desired error evolution („reference error“) with $T_{exp} > 0$

$$e^*(t) = \underbrace{[y^*(t_0) - y(t_0)]}_{e(t_0)} \cdot \exp\left(-\frac{t - t_0}{T_{exp}}\right)$$

Design of Virtual Tube (upper & lower Boundary) with some $\lim_{t! 1} \cdot(t) = 0$

$$\overline{e}^{¤}(t) = e^{¤}(t) + {}^{1}(1 + \cdot(t))$$
$$\underline{e}^{¤}(t) = e^{¤}(t) \;¡\; {}^{1}(1 + \cdot(t))$$

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Funnel Control

Error Reference Control

Nonlinear 2MS

Measured Results

Conclusion & Outlook

Error Reference Control (ERC) - Basic Idea

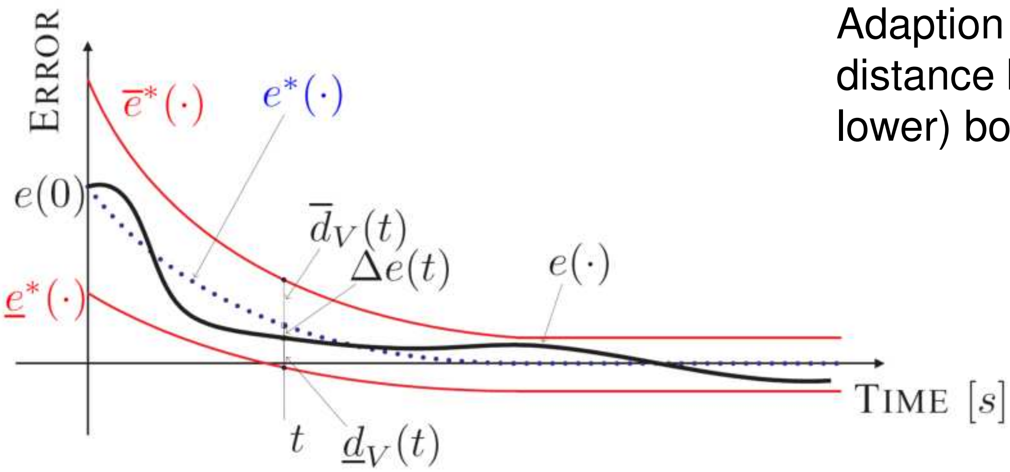


Fig.: ERC - Error evolution within Virtual Tube
[specially designed Funnel Boundary!]

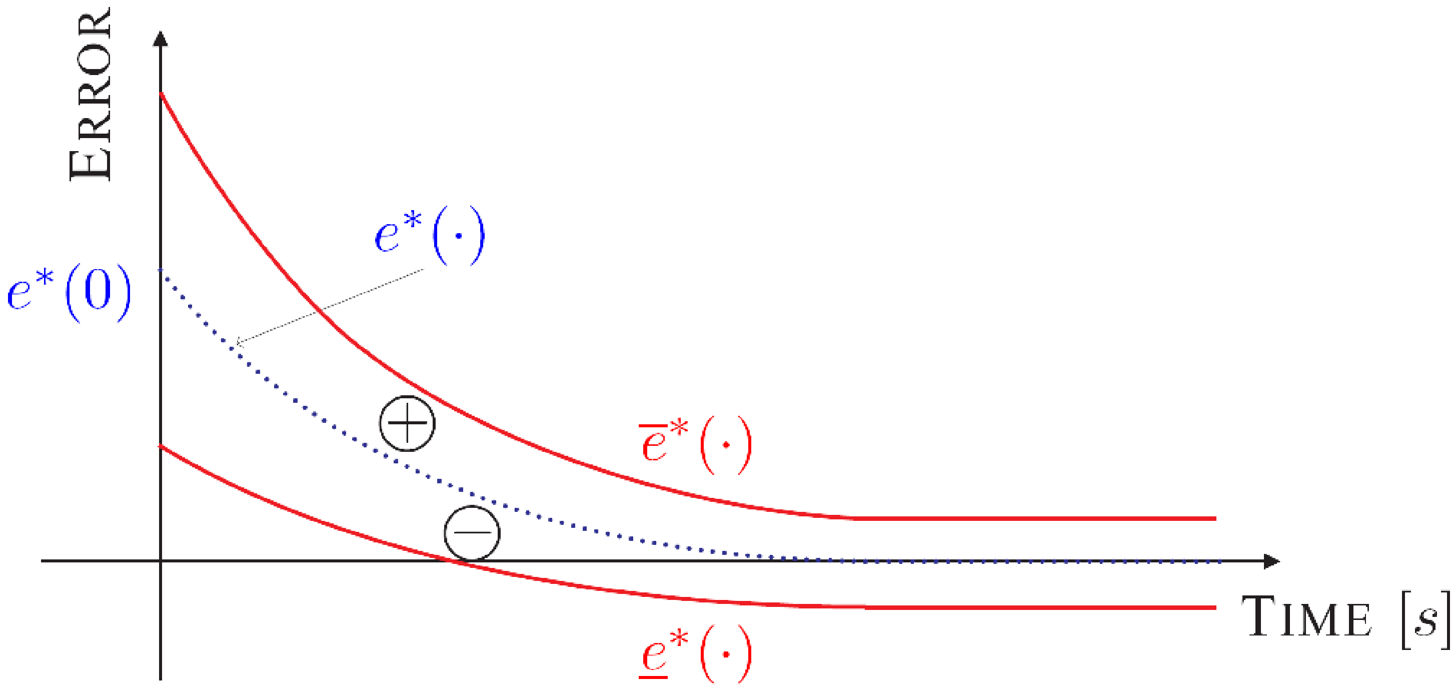
Adaption of gain depending on (vertical) distance between error and upper (or lower) boundary:

$$\overline{d}_V(t) = \overline{e}^*(t) - e(t)$$
$$\underline{d}_V(t) = e(t) - \underline{e}^*(t)$$

Time-varying (adaptive) gain $\Rightarrow$ Boundedness of control error (for sufficiently large but finite control inputs!)

$$\alpha_{ERC}(t; \gamma(t); e(t)) = \frac{\gamma(t)}{\min(\overline{d}_V(t); \underline{d}_V(t))}$$

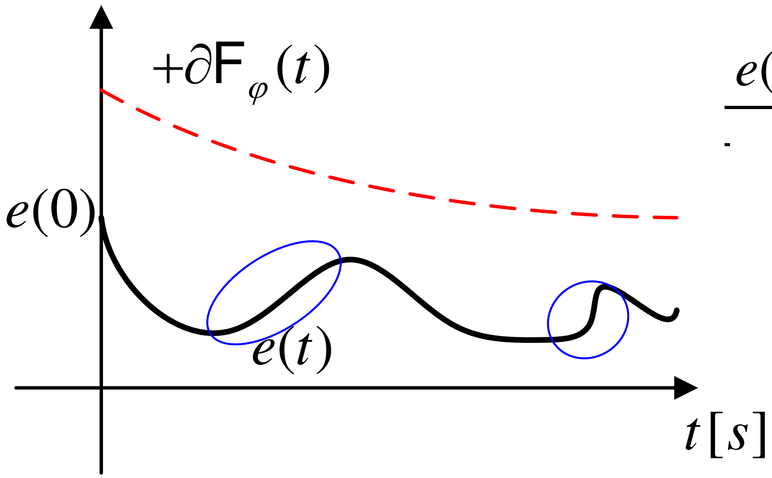
Error Reference Control (ERC) – Sign of Control Input



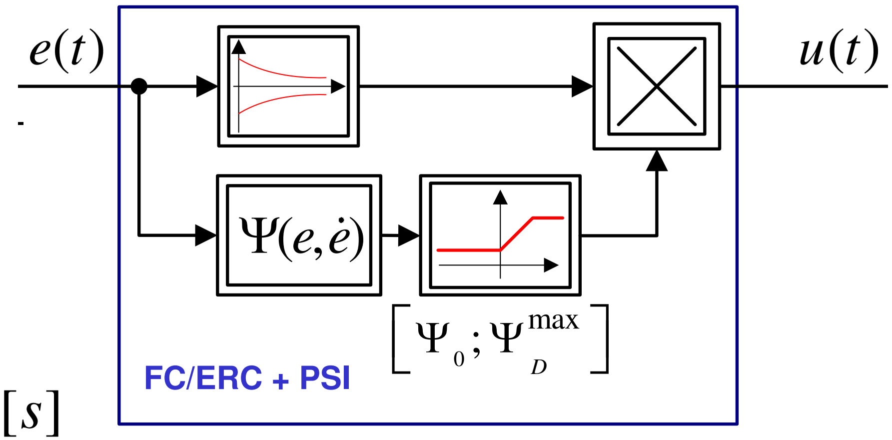
Control input changes its sign according to $\mathrm{sign}(u_{ERC}(\cdot)) = \mathrm{sign}(\cdot\, e(\cdot))$

$$
\boxed{
\begin{aligned}
u_{ERC}(t) &= \alpha_{ERC}(t) \cdot e(t) \\
&= \alpha_{ERC}(t)\,(e(t) - e^*(t))
\end{aligned}
}
$$

Active Damping by adequate Gain Scaling



Idea of Active Damping
(appropriate increase of Gain)

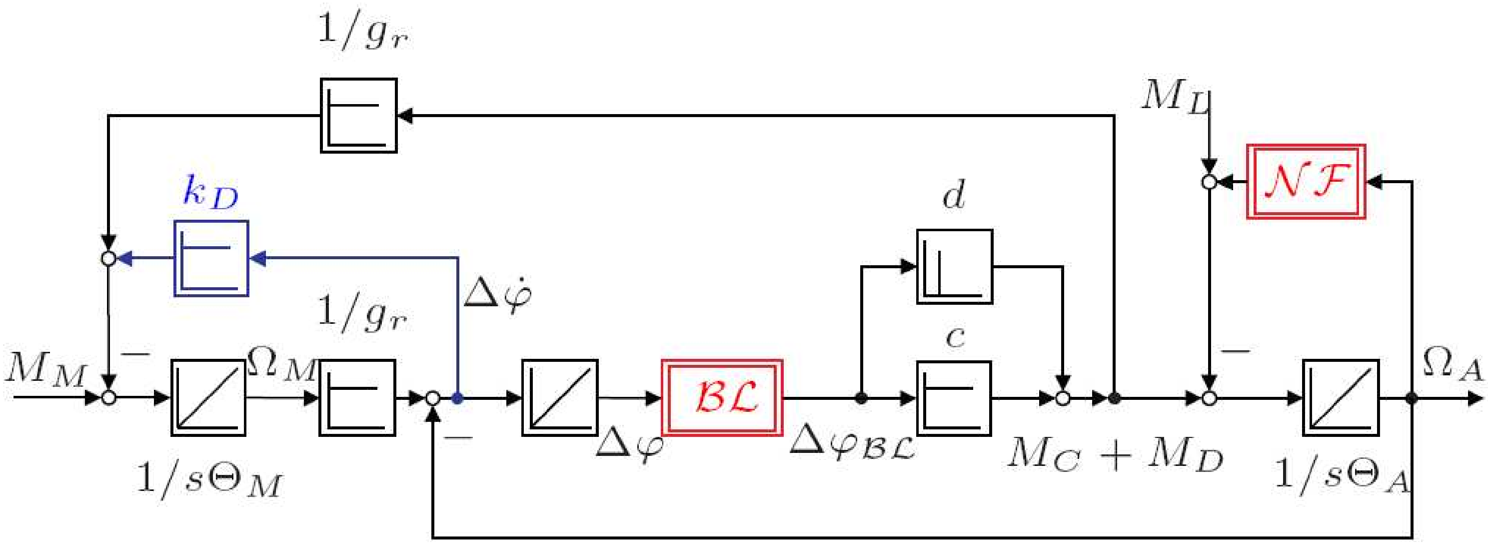


FC/ERC Extension for Gain Scaling

Active damping by online adjustment of scaling function
with $0 \le \Psi_D ; \Psi_D^{max} < 1 \; ; \Psi_0 \ge 1$

$$\boxed{\Psi(t; e(t); \dot{e}(t)) = \mathrm{sat}\left[\Psi_D\,(e(t) - e^*(t))(\dot{e}(t) - \dot{e}^*(t))\right]_0^{\Psi_D^{max}} + \Psi_0}$$

Nonlinear Two-Mass Flexible Servo System (2MS) – Model



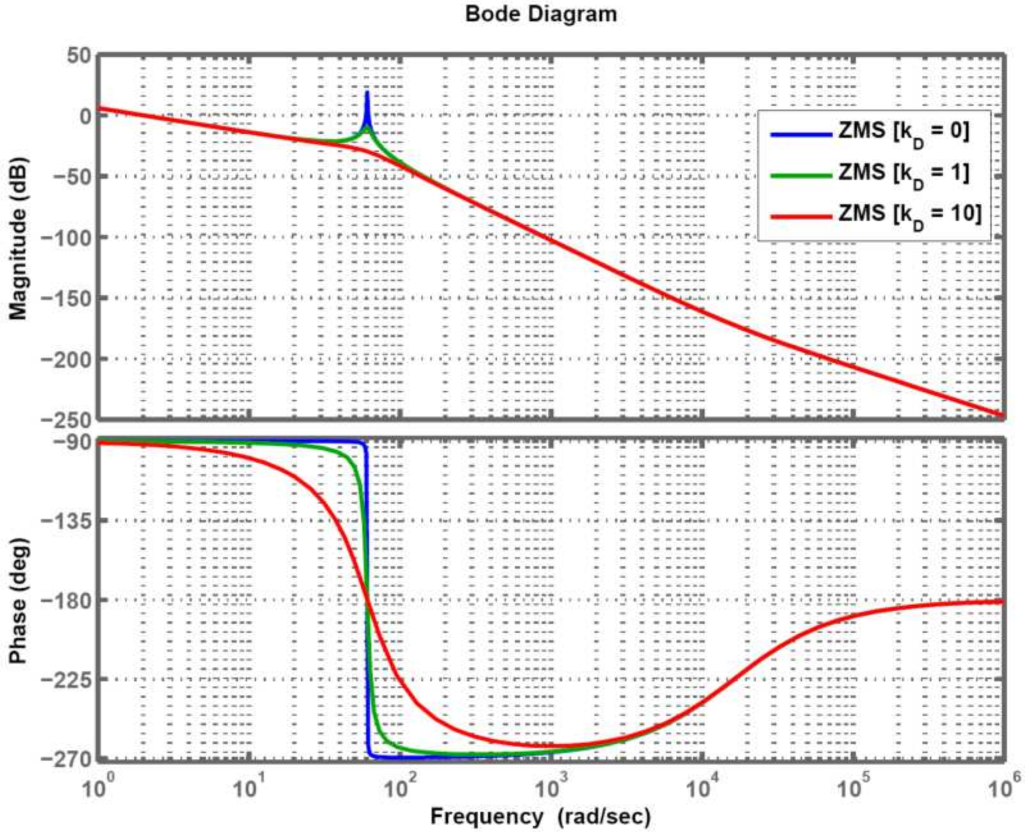
Nonlinear ordinary differential equation of 2MS:

$$\dot{\mathbf{x}} = \begin{bmatrix} \frac{1}{\Theta_M}\Big(M_M - k_D\big(\frac{1}{g_r}\Omega_M - \Omega_A\big) - \frac{d}{g_r}\frac{\partial \mathcal{BL}(\Delta\varphi)}{\partial t} - \frac{c}{g_r}\mathcal{BL}(\Delta\varphi)\Big) \\ \frac{1}{g_r}\Omega_M - \Omega_A \\ \frac{1}{\Theta_A}\Big(-M_L - \mathcal{NF}(\Omega_A) + d\frac{\partial \mathcal{BL}(\Delta\varphi)}{\partial t} + c\mathcal{BL}(\Delta\varphi)\Big) \end{bmatrix}$$

System states: $\mathbf{x}(t)^T = [\Omega_M(t)\ \Delta\varphi(t)\ \Omega_A(t)]$

System output: $y(t) = \Omega_A(t)$

Nonlinear 2MS – Static Feedback of time-derivative of Angle (Damper)



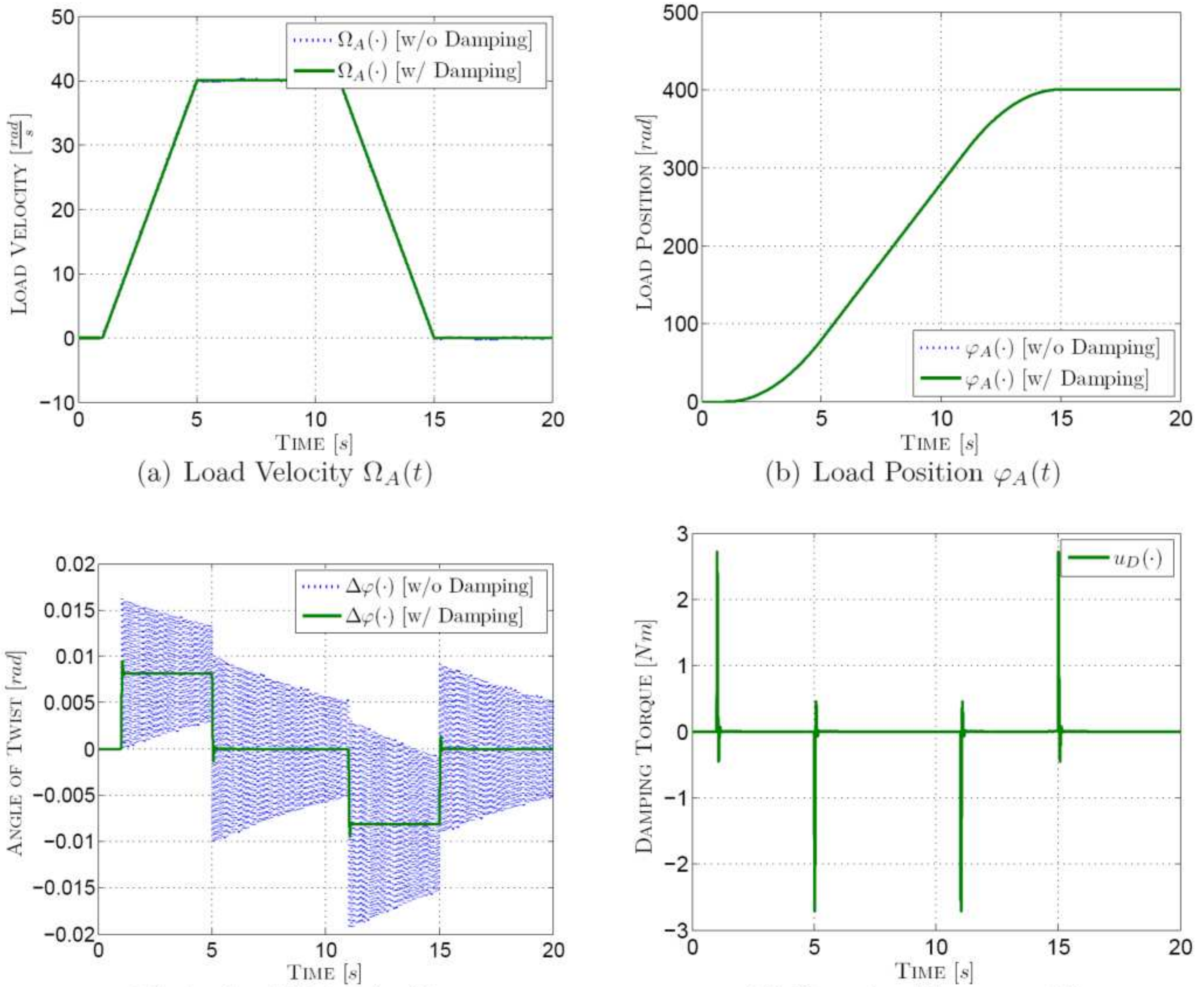
Angular frequency:

$$\omega_0 = \sqrt{\frac{(\Theta_M + \frac{\Theta_A}{g_r^2})c}{\Theta_M \Theta_A}}$$

Damping coefficient:

$$D = \frac{\omega_0}{2c}\left(d + \frac{\Theta_A}{g_r(\Theta_M + \frac{\Theta_A}{g_r^2})}k_D\right)$$

Nonlinear 2MS – Static Feedback of time-derivative of Angle (Damper)



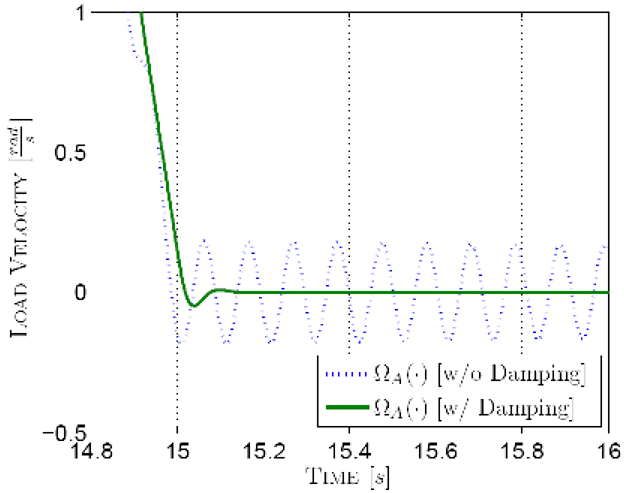
(a) Load Velocity $\Omega_A(t)$

(b) Load Position $\varphi_A(t)$

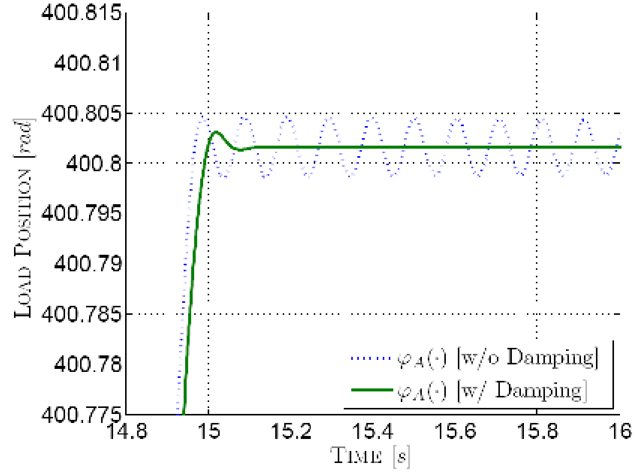
(c) Angle of Twist $\Delta\varphi(t)$

(d) Damping Torque $u_D(t)$

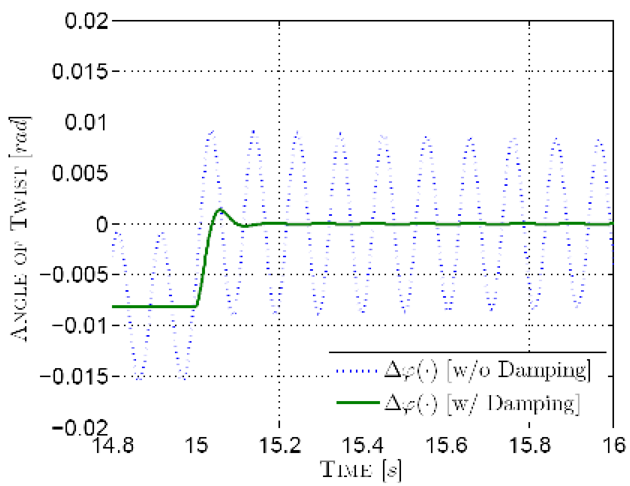
Nonlinear 2MS – Static Feedback of time-derivative of Angle (Damper)



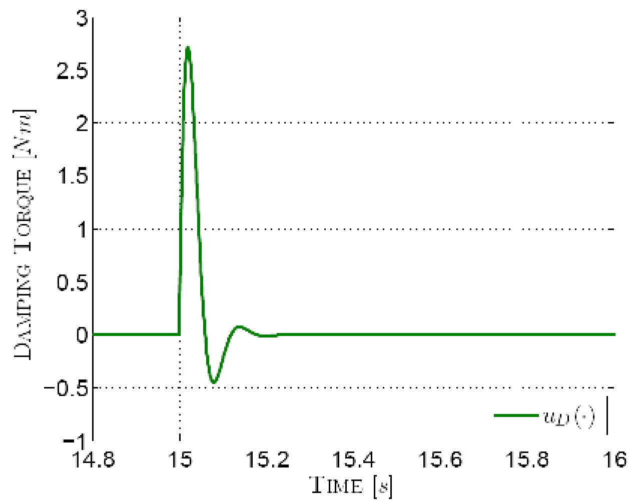
(a) Load Velocity $\Omega_A(t)$



(b) Load Position $\varphi_A(t)$

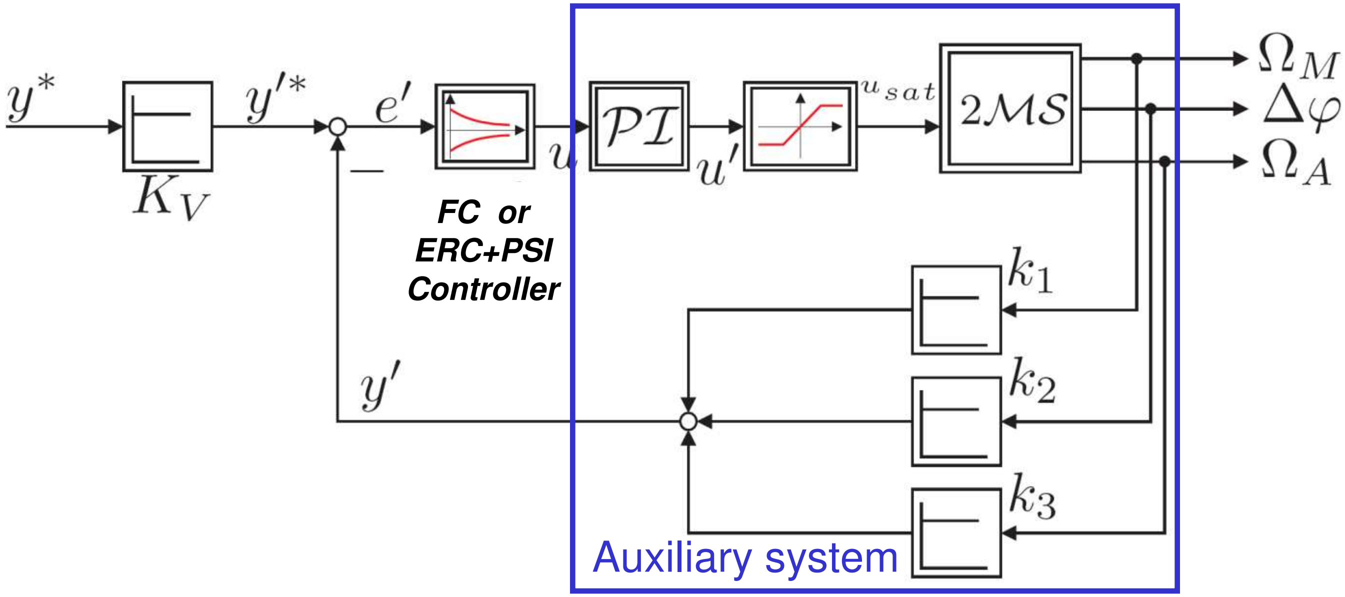


(c) Angle of Twist $\Delta\varphi(t)$



(d) Damping Torque $u_D(t)$

Nonlinear 2MS – Control Loop for FC & ERC



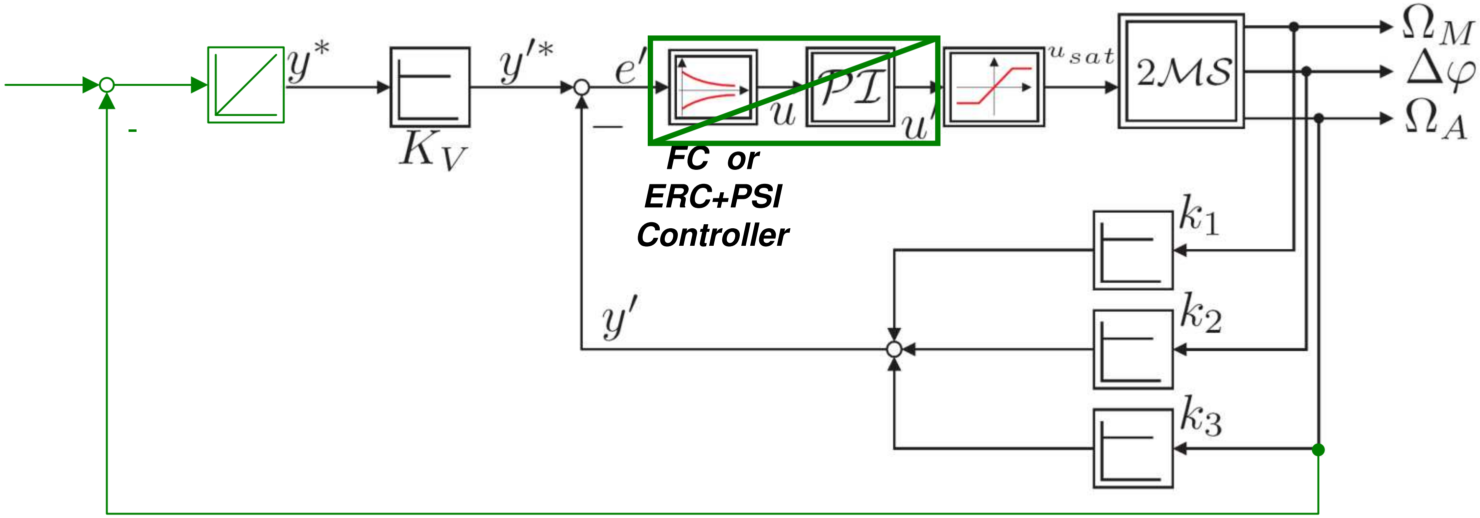
Auxiliary system (element of class S) output:

$$\boxed{y'(t) = k_1 \Omega_M(t) + k_2 \Delta\varphi(t) + k_3 \Omega_A(t)}$$

Allowed range of constant feedback coefficients (independent of system parameters!):

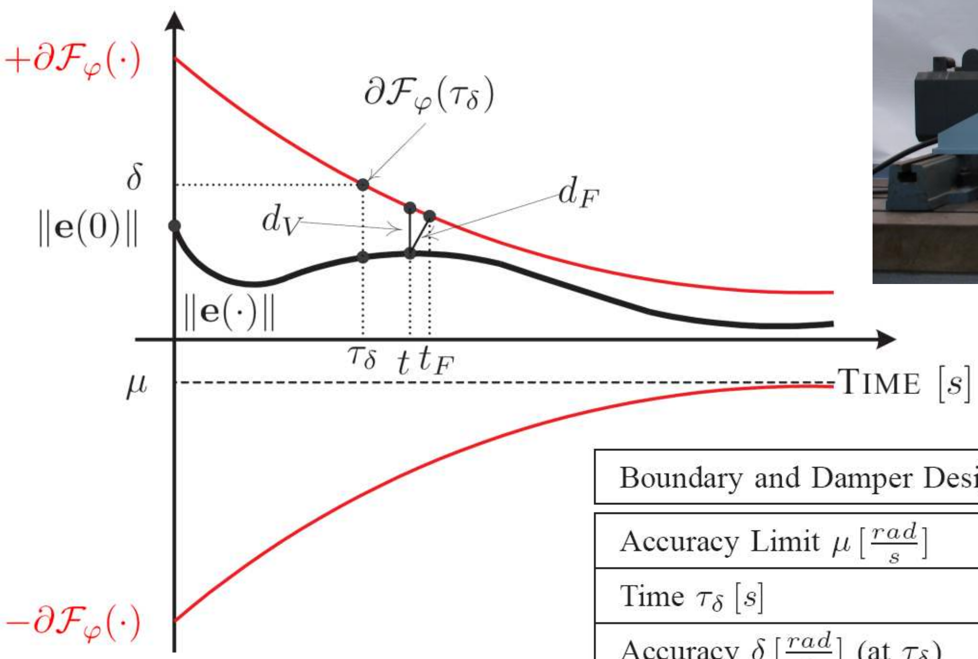
$$\boxed{\begin{array}{rcl} k_1 & > & 0 \\ k_2 & \geq & 0 \\ 0 > k_3 & > & -k_1 \end{array}}$$

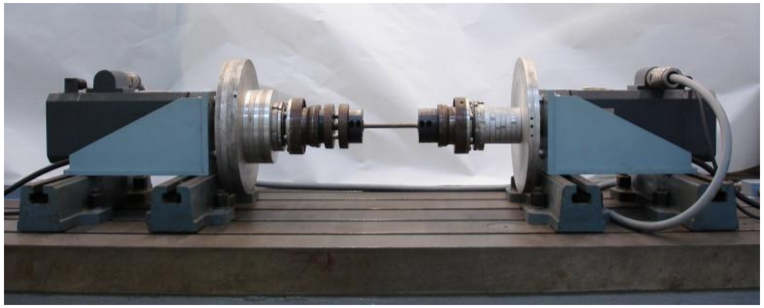
Nonlinear 2MS – Control Loop Design (SF, FC, ERC)



Control Parameters	SF	FC	ERC
$k_1\ [\frac{Nms}{rad}]$ (for Ω_M)	3.95	1	1
$k_2\ [\frac{Nm}{rad}]$ (for $\Delta\varphi$)	35.45	0	0
$k_3\ [\frac{Nms}{rad}]$ (for Ω_A)	2.25	-0.1	-0.1
$\mathcal{PI}$–Integrator $T_I\,[s]$	–	0.0667	0.0667
Prefilter K_V [1]	18.97	0.9	0.9
Damper $k_D\ [\frac{Nms}{rad}]$	–	0	0

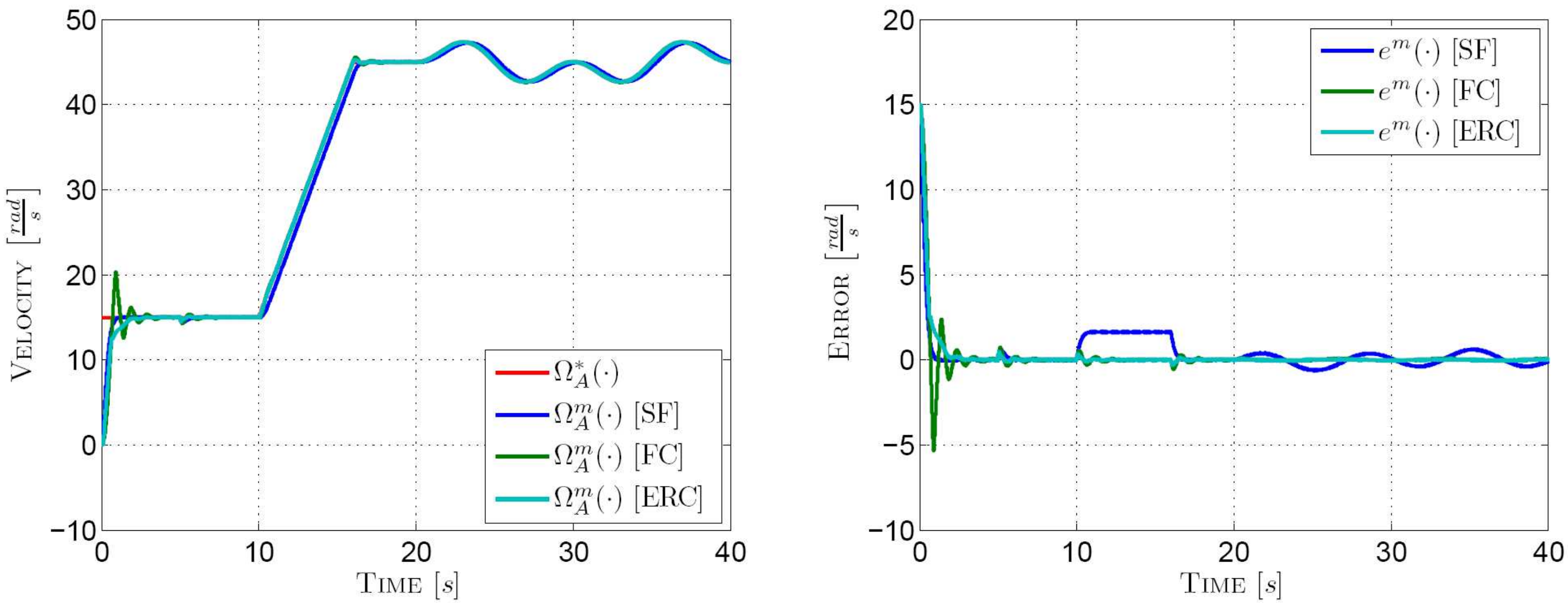
Measurement Results – Controller & Design Parameters





Boundary and Damper Design	FC	ERC
Accuracy Limit $\mu\,[\frac{rad}{s}]$	0.8	0.8
Time $\tau_\delta\,[s]$	1	1
Accuracy $\delta\,[\frac{rad}{s}]$ (at τ_δ)	2	2
$\partial\mathcal{F}_{exp}(0)$ and $\overline{e}^*(0)\,[\frac{rad}{s}]$ $(> K_V \cdot \Omega_A^*(0))$	25.1	25.1
Time constant $T_{exp}\,[s]$	0.329	0.329
Design Parameter $\kappa(t)$	–	$e^*(t)$
Scaling $\Psi(t)$ [1]	1	$\Psi(t, e(t), \dot{e}(t))$
Damper Gain Ψ_D [1]	–	2
Damper Upper Limit Ψ_D^{max} [1]	–	19
Damper Offset Ψ_0 [1]	–	1

Measurements Results – Velocities and Errors



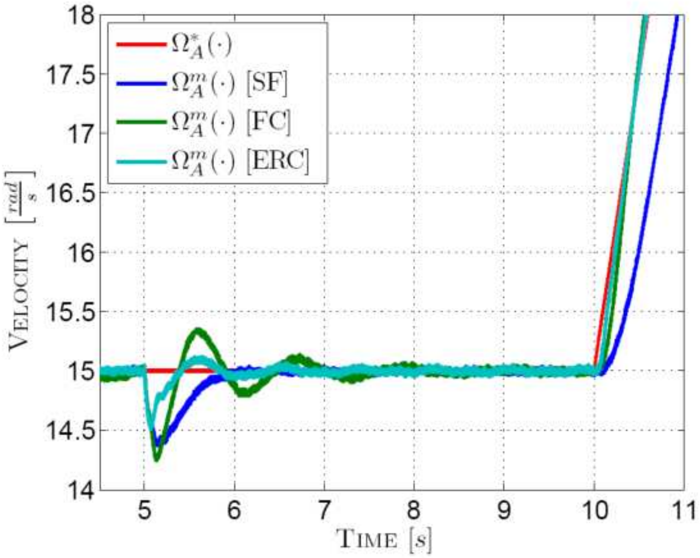
(a) Reference & Velocities

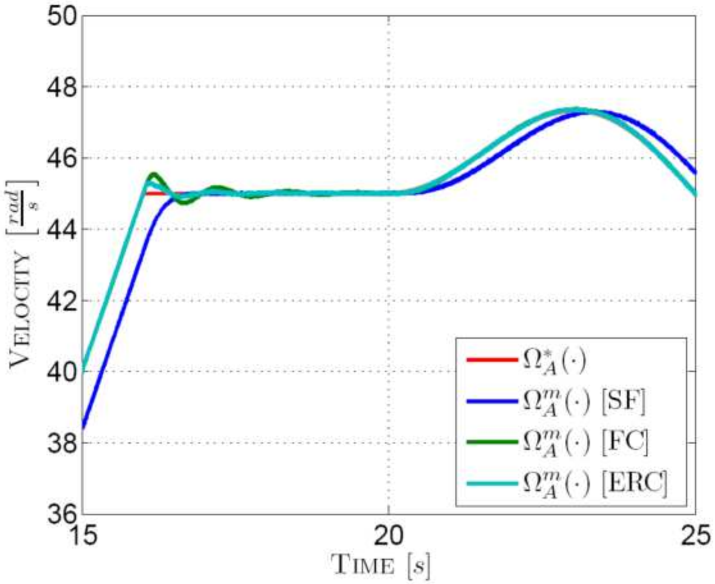
(b) Velocity Errors

Reference Trajectory:
(Step, Ramp, Sinusoidal)

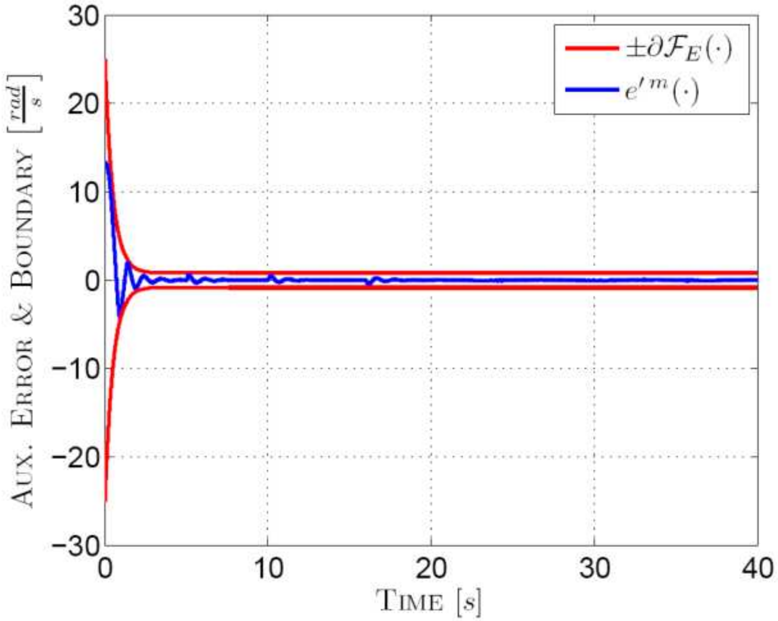
$$\Omega_A^*(t) = 15\frac{rad}{s} \cdot \sigma(t) \ldots$$
$$+ 5\frac{rad}{s^2}(t-10s) \cdot (\sigma(t-10s) - \sigma(t-16s)) \ldots$$
$$+ 3\frac{rad}{s} \sin(\frac{\pi}{10}t) \sin(\frac{\pi}{5}t) \cdot \sigma(t-20s)$$

Measurements Results – Velocities (Zoom)

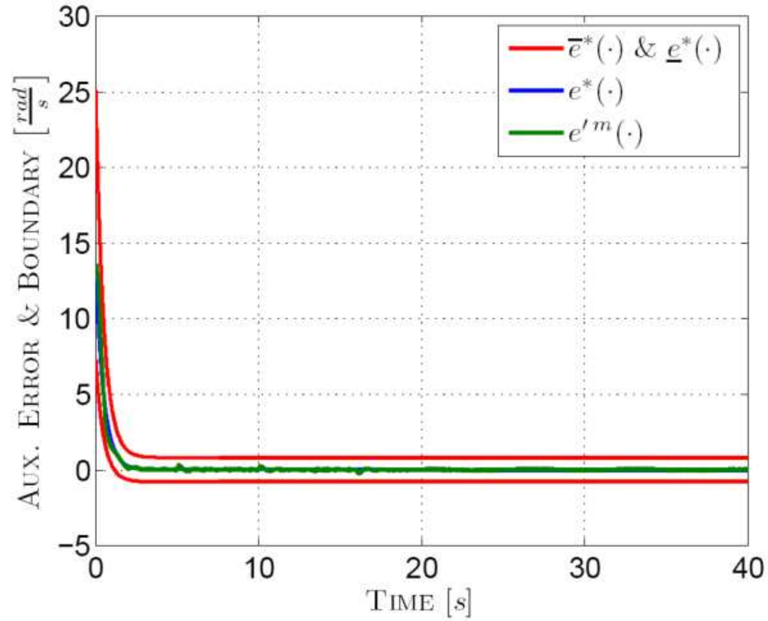




Measurements Results – Auxiliary Error Evolutions (FC & ERC)



(a) Funnel-Control



(b) Error Reference Control

Conclusion:

- Simple, robust and adaptive control structures (FC & ERC)
- No system identification is needed, only structural proporties must be assured (FC & ERC)
- Measurement noise is admissible (FC & ERC)
- Pre-definition of desired transient behavior possible within Funnel Boundary (FC) or Virtual Tube (ERC)
- Pre-definition of and guidance along desired error evolution (ERC)
- Acceleration and/or Deceleration possible due to appropriate change of control input sign (ERC)
- Gain scaling improves performance (especially for ERC)
- Static Feedback of time-derivative of torsional angle allows to (arbitrarilly) damp the system *without* exact system knowledge (parameters)

Outlook:

- Extension of ERC to MIMO case
- Translation of results into position control and application in robotics