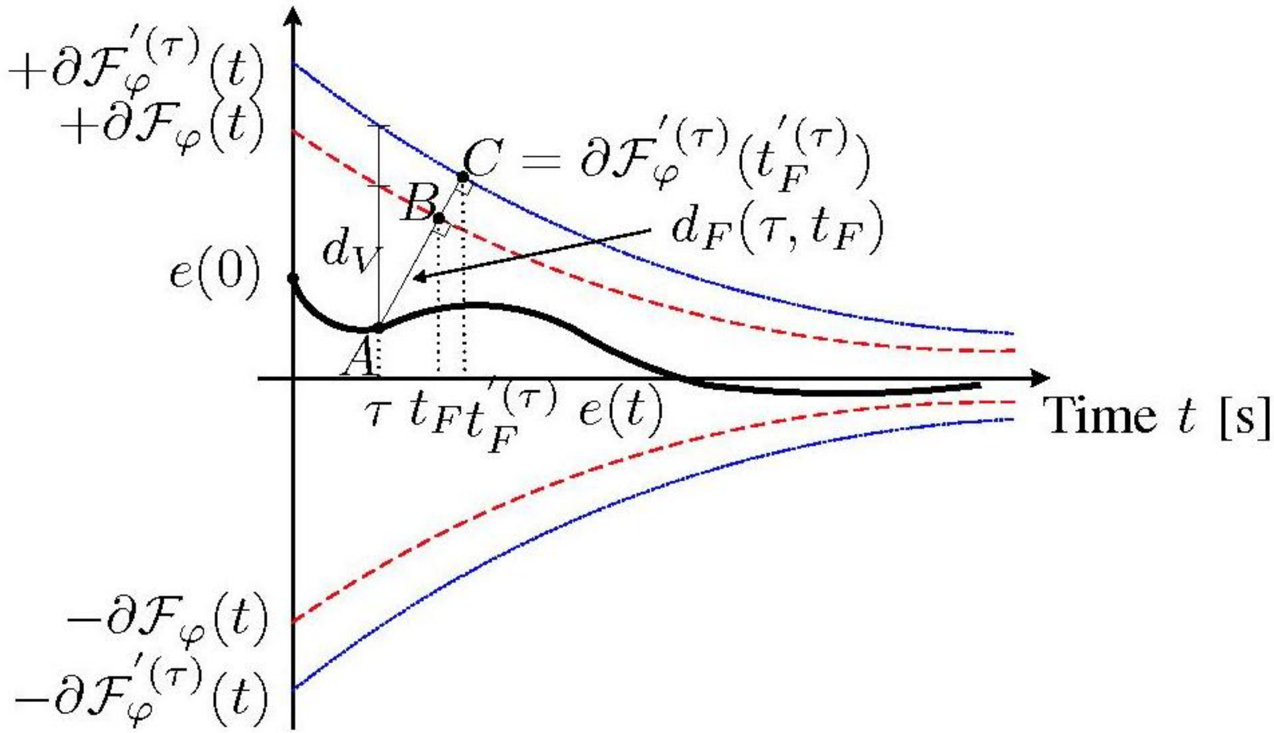
Nonidentifier-based Adaptive Control with Saturated Control Input Compensation

C.M. Hackl, Y. Ji and D. Schröder



Institute for Electrical Drive Systems - Technical University of Munich
Arcisstr. 21, D-80333 Munich - christoph.hackl@tum.de

Overview

1. Funnel-Control (FC)
   - Prerequisites, Basic Idea –

2. Motivation

3. Saturated Control Input Compensation (SIC)

4. Simulation
   - Vertical Distance
   - Minimal (Future) Distance

5. Conclusion & Outlook

Funnel-Control (FC) – Prerequisites

(developed by *Ilchmann et al.*)

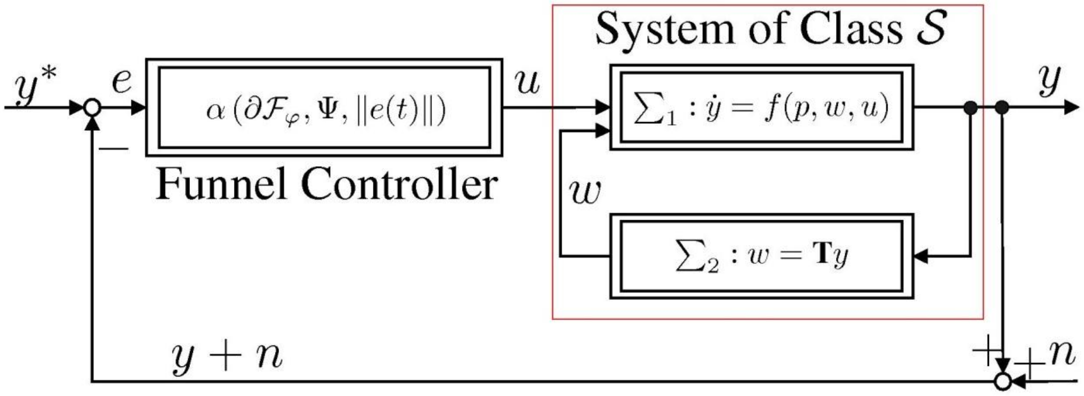


Fig. 1: Block diagramm – System of Class S controlled with FC

Controller properties:

- Proportional Controller
  (simple and robust)
- Adaptive Time-Varying Gain
- Control Law:

$$u(t) = \alpha\left(\partial\mathcal{F}_\varphi(t), \Psi(t), \|e(t)\|\right) \cdot e(t)$$

System of Class $\mathcal{S}$ - prerequisites:

- Relative Degree $r = 1$
- Minimum-Phase
  (stable zero-dynamics)
- Known High-Frequency Gain

Funnel-Control (FC) - Basic Idea

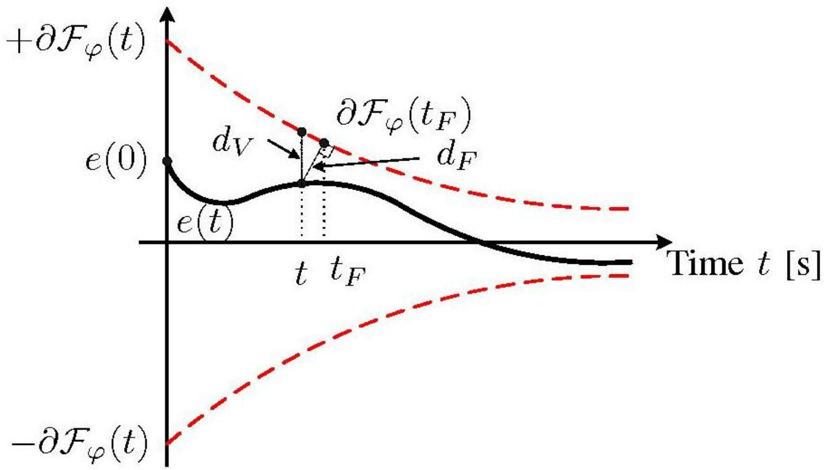


Fig. 2: Error evolution within Funnel Boundary
[Predefinition of Transient Behavior possible!]

Adaption of gain depending on (vertical or minimal) distance:

$$\alpha_D(t) = \frac{1}{\mathrm{dist}\left(\partial\mathcal{F}(\cdot), \|e(t)\|\right)}$$

- Short distance => high gain (more aggressive control)
- Great distance => small gain (more relaxed control)

Time-varying (adaptive) gain $\alpha(\square)$ $\Rightarrow$ Boundedness of control error (for sufficiently large but finite control inputs!)

$$\alpha\left(\partial\mathcal{F}_\varphi(t), \Psi(t), \|e(t)\|\right) = \Psi(t)\cdot\alpha_D(t)$$

Motivation: Simulation Results with Noise (Comparison w/o Saturation)

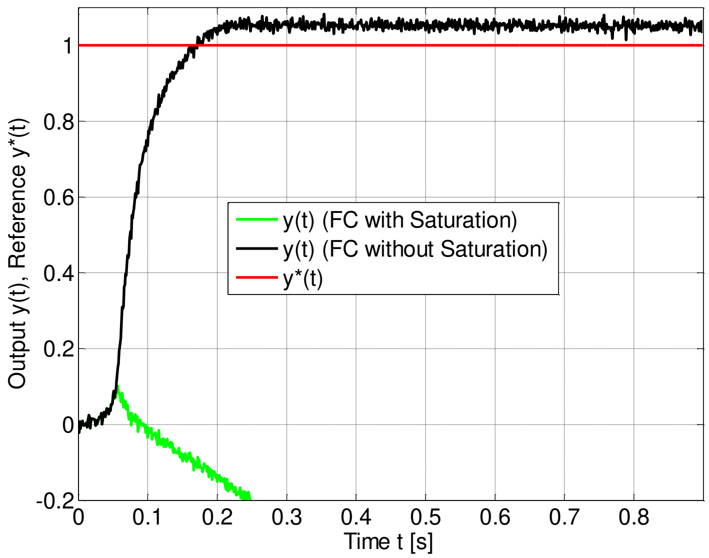


Fig. 3a: System Output y(t)

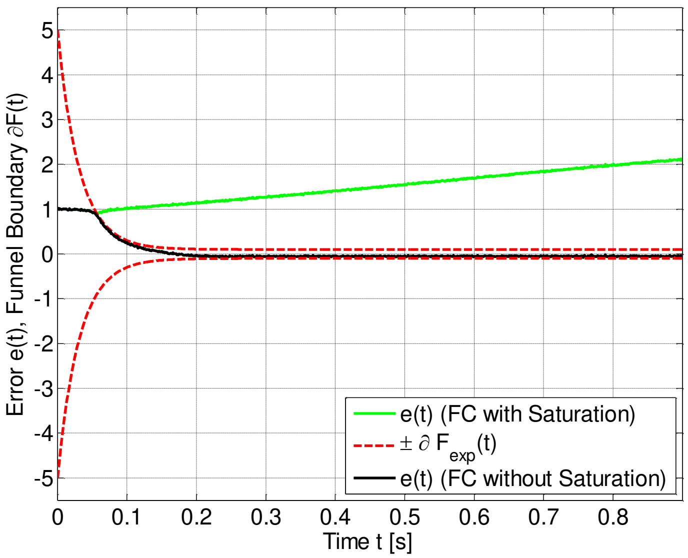


Fig. 3b: Error e(t), Funnel Boundary pF(t)

Example System:

$$\dot{z}_1 = \tanh(z_1) + z_1^2 + z_2^3 + \sqrt{z_1^2 + |z_2| + 1} \cdot u(t)$$

$$\dot{z}_2 = -\sin(z_1) - z_2 \cdot \exp(z_2)$$

$$y = z_1$$

Funnel Boundary, Scaling and Saturation:

$$\partial\mathcal{F}_{\exp}(t) = \varphi_{\exp,0} \cdot \exp(-\frac{t}{T_{\exp}}) + \varphi_{\exp,\infty}$$

$$\Psi(t) = 1$$

$$|u_{\max}| = 5$$

Motivation: Simulation Results with Noise (Comparison w/o Saturation)

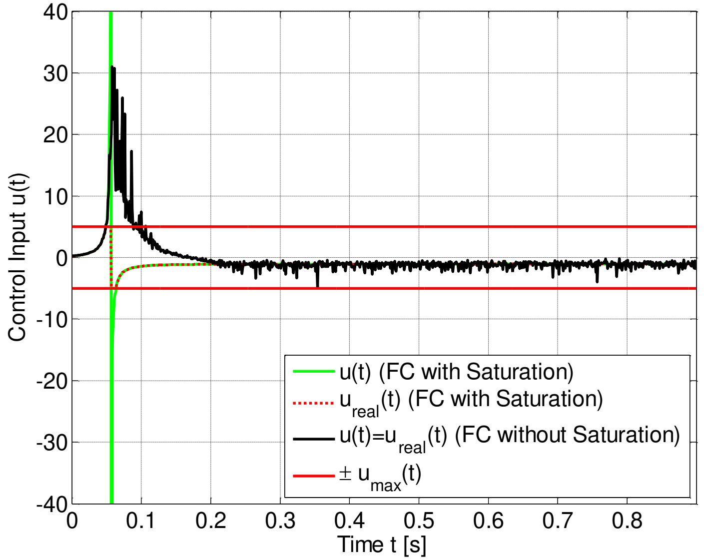


Fig. 3c: Control Output u(t)

Example System:

$$\dot{z}_1 = \tanh(z_1) + z_1^2 + z_2^3 + \sqrt{z_1^2 + |z_2| + 1} \cdot u(t)$$

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Basic Idea of Saturated Control Input Compensation (SIC)

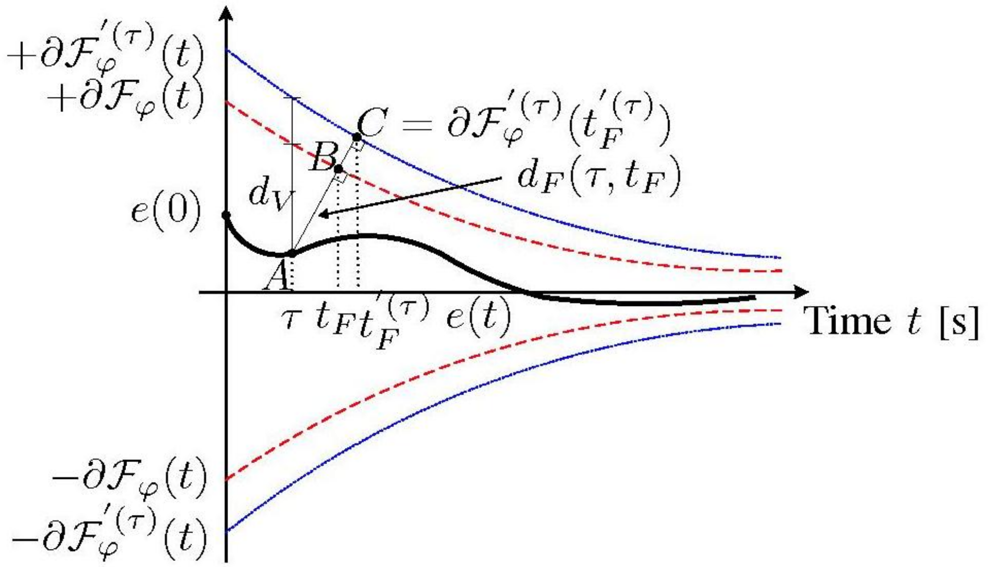

Fig. 4: Saturated Control Input Compensation (SIC):
Funnel Boundary (red) and Funnel Prototype (blue) for Adjustment

Leaving the valid range of the control input:

$$|u(\tau_i)| \geq u_{\max} \quad \tau_i > \tau_{i-1} \geq t_0 \geq 0$$

Re-adjustment of the Funnel Boundary:

$$\partial\mathcal{F}_\varphi^{'(\tau_i)}(T) \ \ \forall T \geq \tau_i \Rightarrow \left|u^{'(\tau_i)}(\tau_i)\right| = \frac{\Psi(\tau_i)}{d^{'(\tau_i)}(\tau_i)} \cdot |e(\tau_i)| \overset{!}{=} u_{\max}$$

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Simulation Results with Noise (SIC for Vertical Distance)

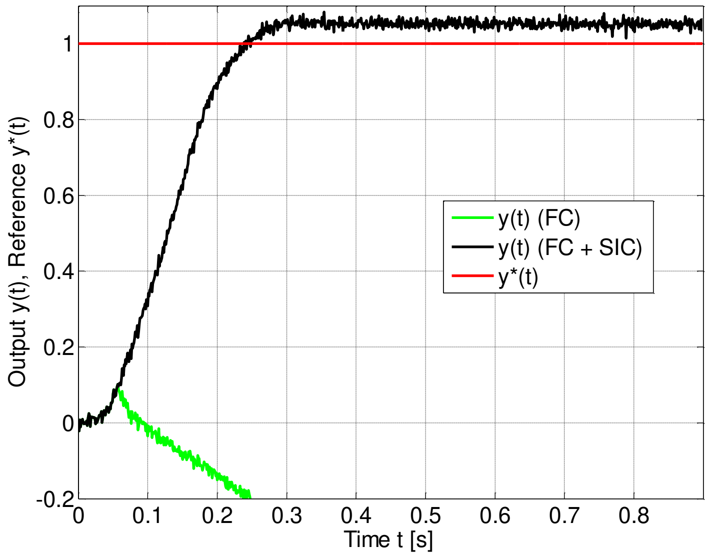

Fig. 5a: System Output y(t)

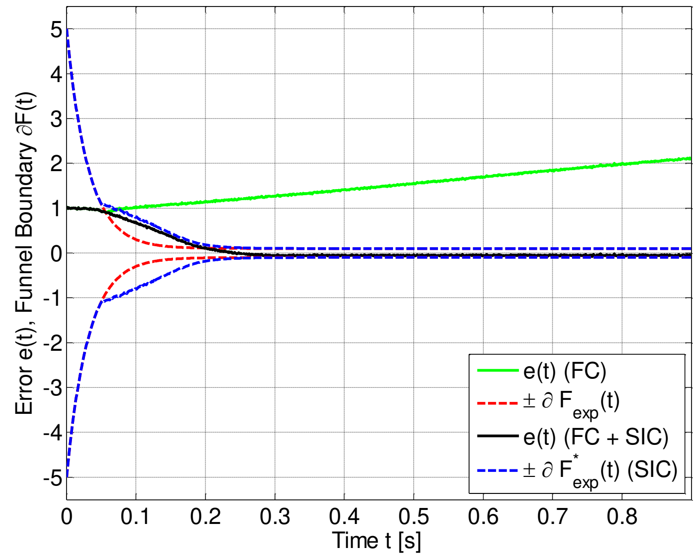

Fig. 5b: Error e(t), Funnel Boundary pF(t)

Example System:

$$\dot{z}_1 = \tanh(z_1) + z_1^2 + z_2^3 + \sqrt{z_1^2 + |z_2| + 1} \cdot u(t)$$

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Simulation Results with Noise (SIC for Vertical Distance)

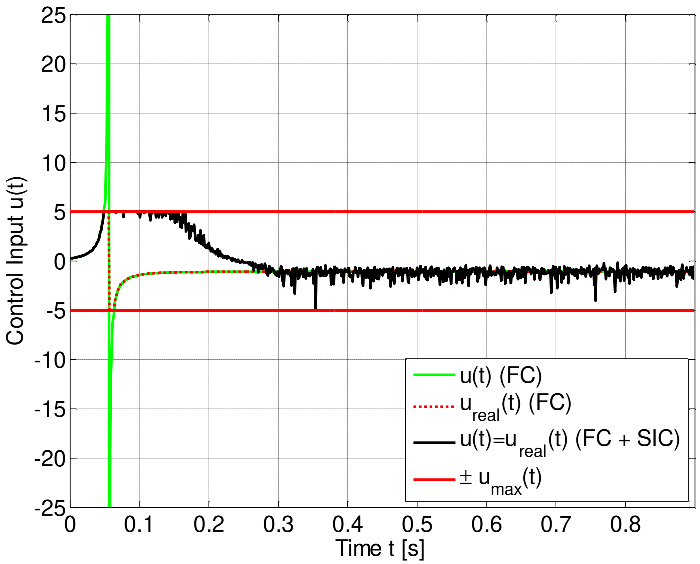

Fig. 5c: Control Output u(t)

Example System:

$$\dot{z}_1 = \tanh(z_1) + z_1^2 + z_2^3 + \sqrt{z_1^2 + |z_2| + 1} \cdot u(t)$$

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$$\partial\mathcal{F}_{\exp}(t) = \varphi_{\exp,0} \cdot \exp(-\frac{t}{T_{\exp}}) + \varphi_{\exp,\infty}$$

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Simulation Results with Noise (SIC for Minimal Future Distance)

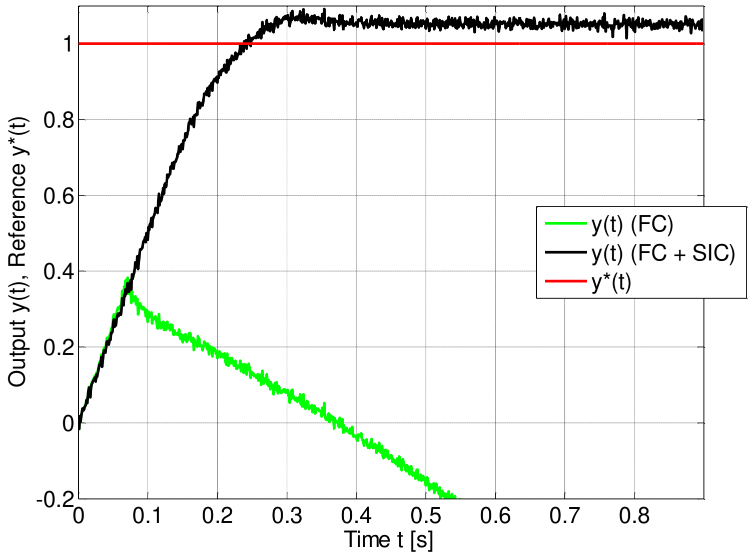

Fig. 6a: System Output y(t)

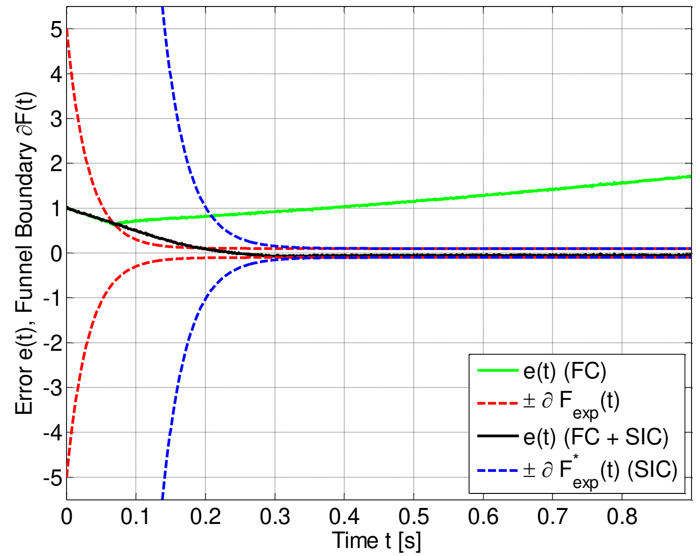

Fig. 6b: Error e(t), Funnel Boundary pF(t)

Example System:

$$\dot{z}_1 = \tanh(z_1) + z_1^2 + z_2^3 + \sqrt{z_1^2 + |z_2| + 1} \cdot u(t)$$

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Simulation Results with Noise (SIC for Minimal Future Distance)

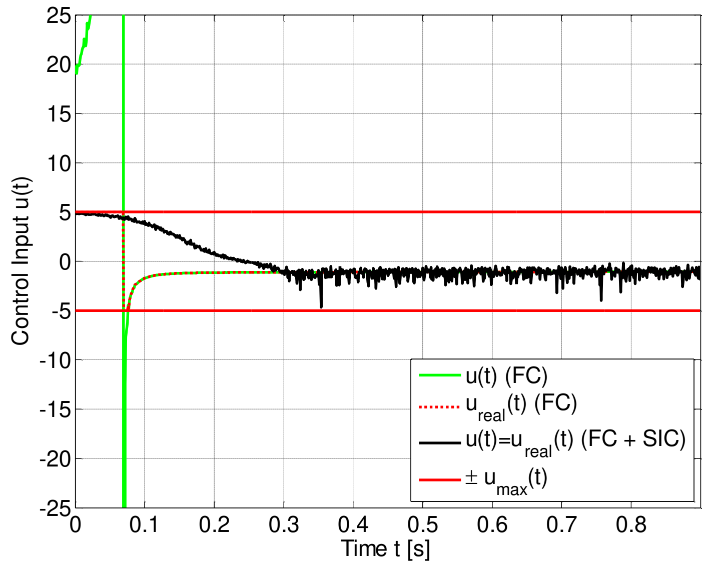

Fig. 6c: Control Output u(t)

Example System:

$$\dot{z}_1 = \tanh(z_1) + z_1^2 + z_2^3 + \sqrt{z_1^2 + |z_2| + 1} \cdot u(t)$$

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$$y = z_1$$

Funnel Boundary, Scaling and Saturation:

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Conclusion:

- Simple, robust and adaptive control structure
- Pre-definition of desired transient behavior possible
- Measurement noise is admissible
- **Saturated Control Input Compensation** (online Adjustment) for:
  1. *Vertical Distance*
  2. *Minimal (Future) Distance*

Outlook:

- Acceleration & Active Damping (MED'07: T01-016)
- Further experimental results in laboratory (e.g. Flexible Three-Mass-System or Robot Manipulator Control)

Funnel-Control – Achieved Goals

- Extension of the usable system class (reduction of the relative degree)
  - ✓ *H. Schuster, D. Schröder „High-Gain Control of Systems with Arbitrary Relative Degree: Speed Control for a Two Mass Flexible Servo System". 8th IEEE International Conference on Intelligent Engineering Systems 2004 (INES 2004), Cluj-Napoca, Romania, September 19-21, 2004, p. 486-491.*
- Robustness analysis (parameter variations)
  - ✓ *H.Schuster, C.M. Hackl, C. Westermaier and D. Schröder, "Funnel-Control for Electrical Drives with Uncertain Parameters", 7th International Power Engineering Conference - IPEC 2005 Singapore, Nov. 29 - Dec. 2, 2005. (published in IPEC Proceedings)*
  - ✓ *C.M. Hackl, D. Schröder, "Funnel-Control for Nonlinear Multi-Mass Flexible Systems", IECON 2006 - Paris, November 7 - 10, 2006, IECON'06 Proceedings*
- Steady state accuracy (vanishing control error)
  - ✓ *C.M. Hackl, H.Schuster, C. Westermaier and D. Schröder, "Funnel-Control with Integrating Prefilter for Nonlinear, Time-varying Two-Mass Flexible Servo Systems", The 9th International Workshop on Advanced Motion Control, AMC 2006 - Istanbul, March 27-29, 2006*
  - ✓ *C.M. Hackl, D. Schröder, "Extension of High-Gain Controllable Systems for Improved Accuracy", Joint CCA, CACSD, and ISIC, 2006 - Munich, October 4-6, 2006, Joint CCA, CACSD, and ISIC Proceedings, pp. 2231-2236*
- Acceleration & Active Damping
  - ✓ *C.M. Hackl, D. Y. Ji and D. Schröder, „Enhanced Funnel-Control with Improved Performance", MED 2007 - Athens, Greece June 27-29, 2007, T01-016*

Thank you for your attention