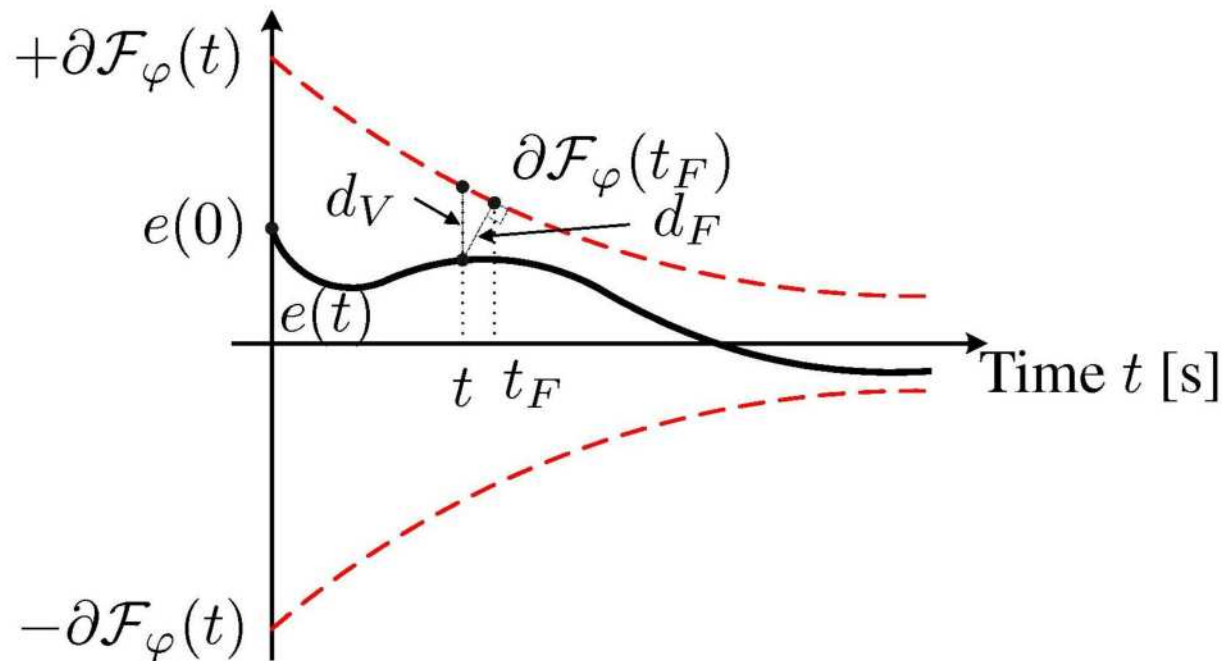


Enhanced Funnel-Control with Improved Performance

C.M. Hackl, Y. Ji and D. Schröder



Funnel
Control

Scaling
Function

Future
Distance

Active
Damping

Simulation

Conclusion
& Outlook

Overview

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- Prerequisites, Basic Idea -
2. Scaling Function
3. Online Foresight: Minimal (Future) Distance (MD)
4. Active Damping
5. Simulation
6. Conclusion & Outlook

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Funnel-Control (FC) – Prerequisites

(developed by Ilchmann et al.)

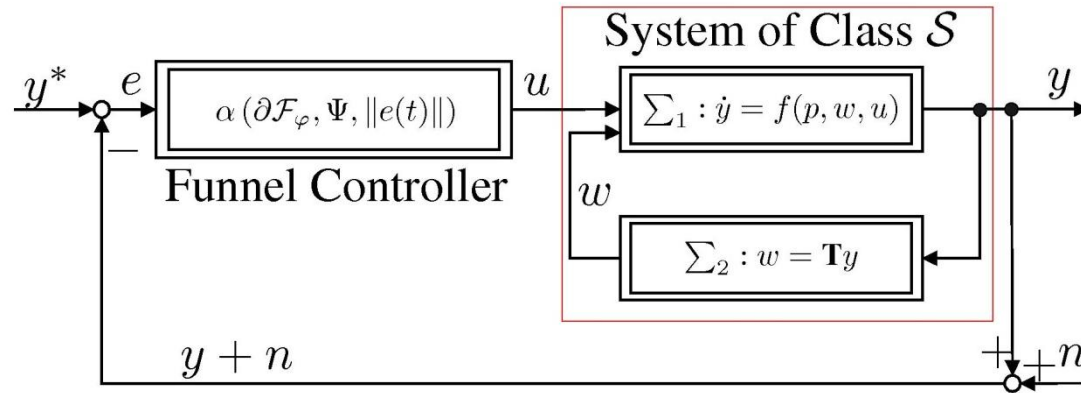


Fig. 1: Block diagramm – System of Class $\mathcal{S}$ controlled with FC

Controller properties:

- Proportional Controller
- Adaptive Time-Varying Gain
- Control Law:

$$u(t) = \alpha\left(\partial\mathcal{F}_\varphi(t), \Psi(t), \|e(t)\|\right) \cdot e(t)$$

System of Class $\mathcal{S}$ - prerequisites:

- Relative Degree $r = 1$
- Minimum-Phase
(stable zero-dynamics)
- Known High-Frequency Gain

Funnel-Control (FC) with Vertical Distance (VD) - Basic Idea

Error $e(t)$, Funnel Boundary $\pm \partial \mathcal{F}_\varphi$

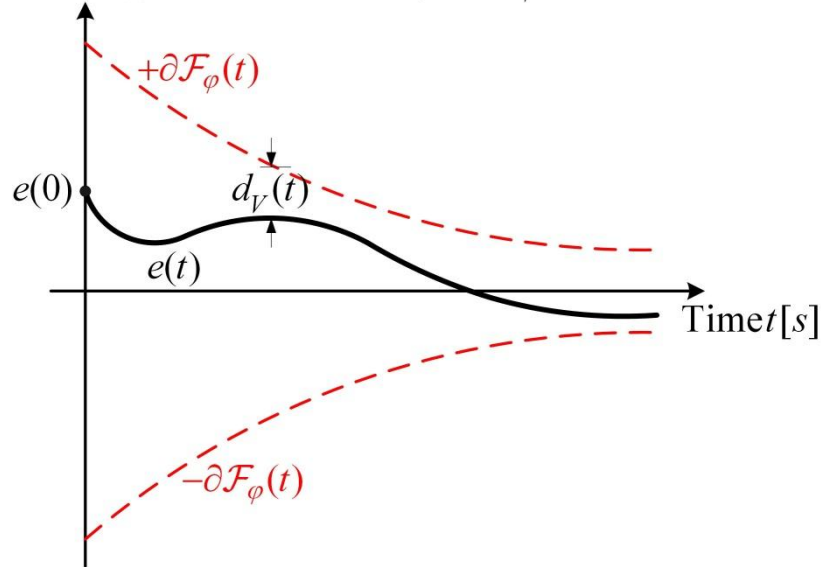


Fig. 2: Error evolution within Funnel Boundary

Adaption of gain depending on (vertical) distance:

$$\alpha_D(t) = \frac{1}{d_V(t)}$$

- Short distance $\Rightarrow$ high gain (more aggressive control)
- Great distance $\Rightarrow$ small gain (more relaxed control)

Time-varying (adaptive) gain $\alpha(\cdot)$ $\Rightarrow$ Boundedness of control error (for sufficiently large but finite control inputs!)

$$\alpha(\partial \mathcal{F}_\varphi(t), \Psi(t), \|e(t)\|) = \Psi(t) \cdot \alpha_D(t) = \frac{\Psi(t)}{d_V(t)} = \frac{\Psi(t)}{\underbrace{\partial \mathcal{F}_\varphi(t) - \|e(t)\|}_{0 < \leq \partial \mathcal{F}_\varphi(t)}}$$

Scaling Function

- any continuous, bounded and positive function is allowed: $\Psi(t) > 0$ for all $t \geq t_0$
- to choose e.g. minimal gain („memory effect“): $\alpha(t) = \frac{\Psi(t)}{\underbrace{\partial \mathcal{F}_\varphi(t) - \|e(t)\|}_{0 < \leq \partial \mathcal{F}_\varphi(t)}} \geq \frac{\Psi(t)}{\partial \mathcal{F}_\varphi(t)}$
- only to increase** the distance gain (acceleration effect):

$$\begin{aligned}\alpha(t) &= \Psi(t) \cdot \alpha_D(t) \geq \alpha_D(t) \\ \Rightarrow \Psi(t) &\geq 1 \text{ for all } t \geq t_0\end{aligned}$$

(otherwise the principle of Funnel Control may be deteriorated)

- if Funnel Boundary itself is used for scaling:

$$\Psi(t) = \partial \mathcal{F}_\varphi(t) + (1 - \mu) \text{ if } \lim_{t \rightarrow \infty} \partial \mathcal{F}_\varphi(t) = \mu < 1$$

Online Foresight: Minimal (Future) Distance (MD)

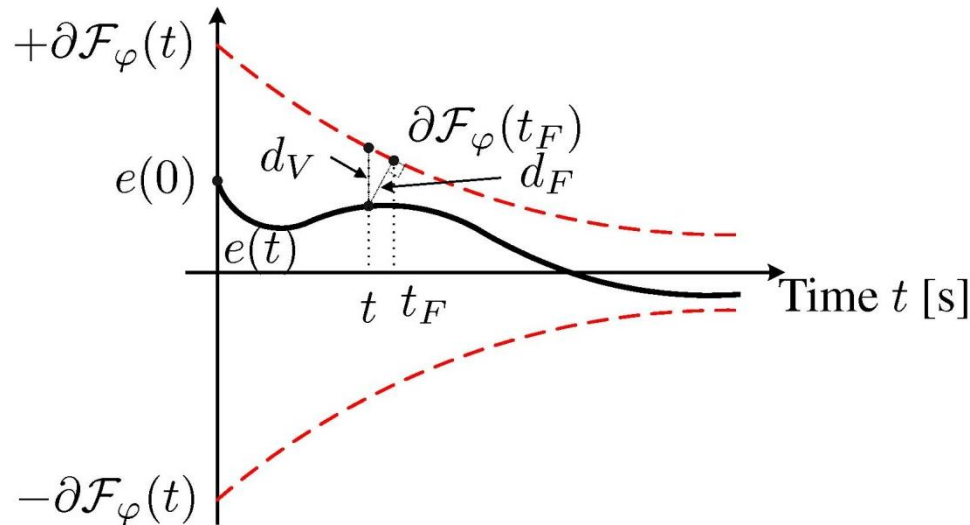


Fig. 3: Vertical (VD) & Minimal (Future) Distance (MD)

- Minimal (Future) Distance (MD) exists with $t_F \geq t$ and:

$$d_F(t, t_F) = \min_{t_F \geq t} \sqrt{(\partial \mathcal{F}_\varphi(t_F) - \|e(t)\|)^2 + (t_F - t)^2} \leq d_V(t)$$

- Funnel Control more effective with (especially for initial error $e(t_0)$) as:

$$u_F(t) = \alpha_F(t) \cdot e(t) \geq \alpha_V(t) \cdot e(t) = u_V(t)$$

Active Damping: Asymmetric Funnel Boundaries (Intuitive Approach)

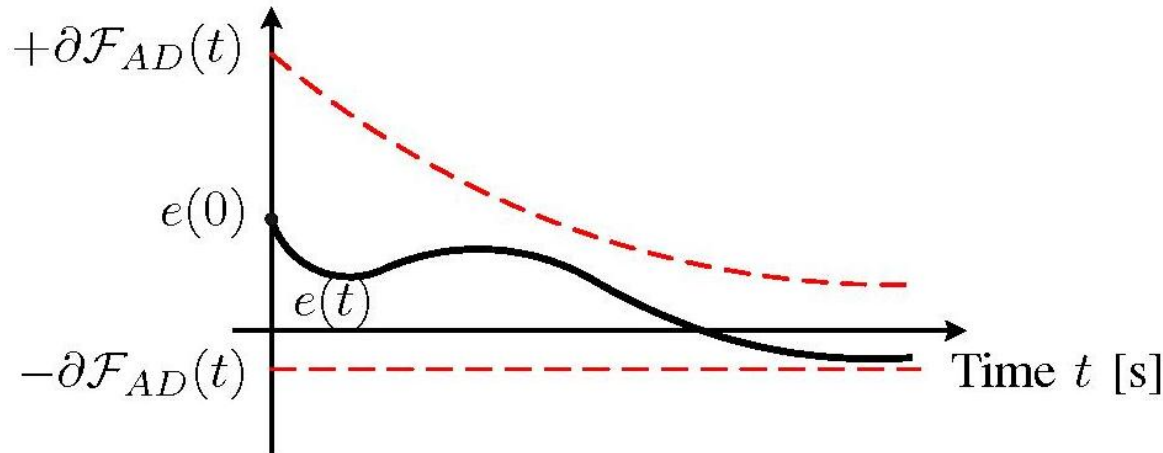


Fig. 6: Asymmetric Funnel Boundary

- Asymmetric choice of the Funnel Boundary with maximal overshoot limit $\ddot{u} > 0$:

$$\mathcal{F}_{AD} = \left\{ e(t) \left| \begin{array}{ll} -\ddot{u} \cdot y^*(t) < e(t) < \partial\mathcal{F}_\varphi(t) & \forall y^*(t) \geq 0 \\ -\partial\mathcal{F}_\varphi(t) < e(t) < -\ddot{u} \cdot y^*(t) & \forall y^*(t) < 0 \end{array} \right. \right\}$$

- Generation of control input for asymmetric boundaries:

$$u(t) = \begin{cases} \Psi(t) \cdot \alpha_D^+(t) \cdot e(t) & , e(t) > 0 \\ \Psi(t) \cdot \alpha_D^-(t) \cdot e(t) & , e(t) \leq 0 \end{cases}$$

Simulation Results (Analytic Approach)

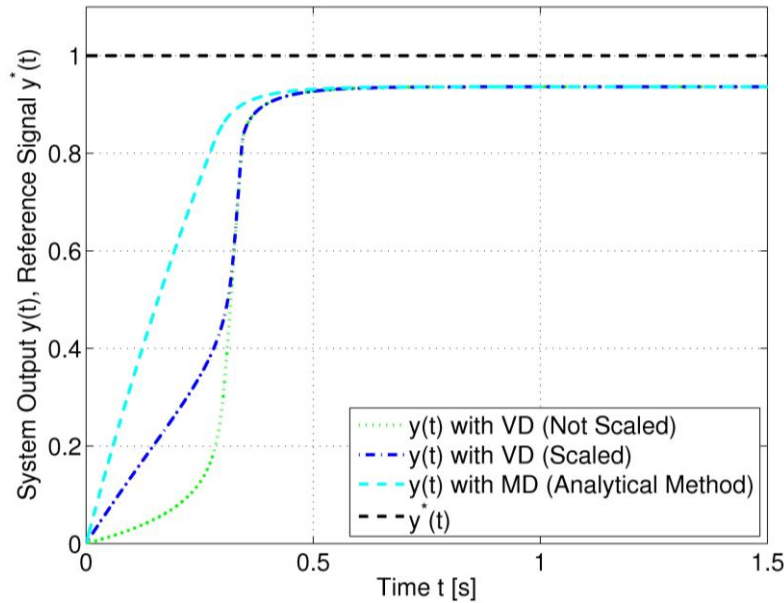


Fig. 7a: System Output $y(t)$

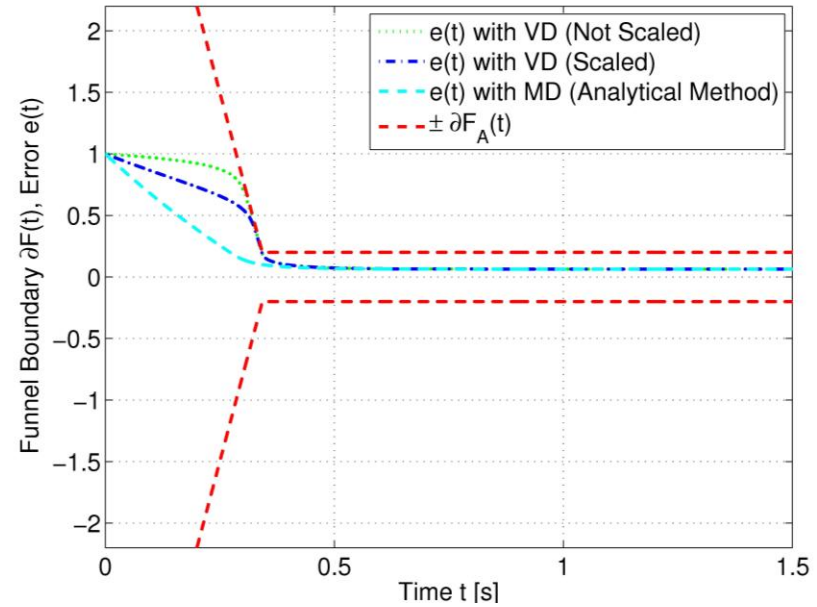


Fig. 7b: Error $e(t)$, Funnel Boundary $pF(t)$

Example System:

$$F_{PT_1}(s) = \frac{2}{1+s2}$$

Funnel Boundary and Scaling Function:

$$\partial \mathcal{F}_A(t) = \begin{cases} -\frac{\varphi_0 - \varphi_\infty}{\tau_\mu} \cdot t + \varphi_0 & t < \tau_\mu \\ \varphi_\infty & t \geq \tau_\mu \end{cases}$$

$$\Psi(t) = \partial \mathcal{F}_A(t) + (1 - \varphi_\infty) \text{ for VD}$$

Simulation Results (Analytic Approach)

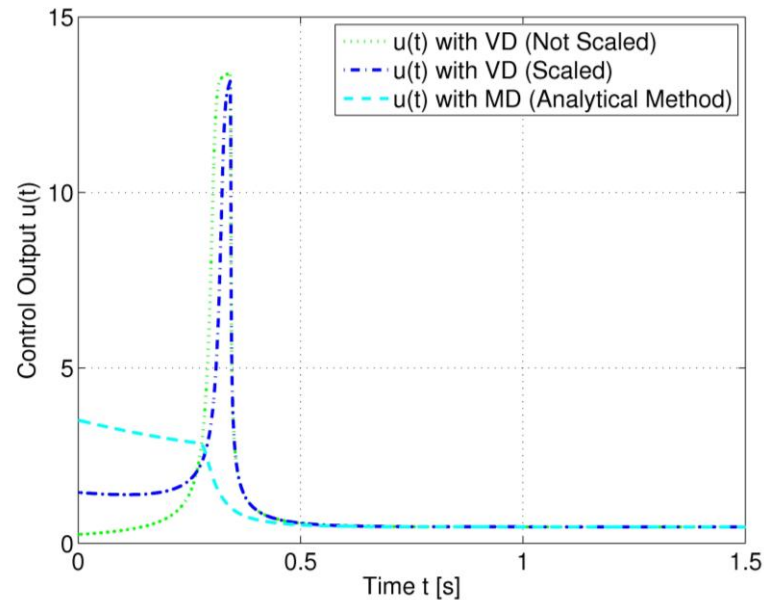


Fig. 7c: Control Output $u(t)$

Example System:

$$F_{PT_1}(s) = \frac{2}{1+s^2}$$

Funnel Boundary and Scaling Function:

$$\partial \mathcal{F}_A(t) = \begin{cases} -\frac{\varphi_0 - \varphi_\infty}{\tau_\mu} \cdot t + \varphi_0 & t < \tau_\mu \\ \varphi_\infty & t \geq \tau_\mu \end{cases}$$

$$\Psi(t) = \partial \mathcal{F}_A(t) + (1 - \varphi_\infty) \text{ for VD}$$

Simulation Results with Noise (Analytic Approach)

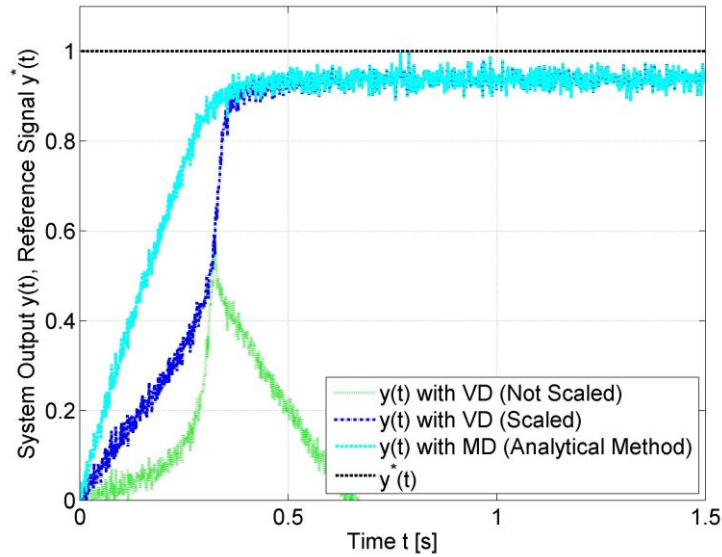


Fig. 8a: System Output $y(t)$

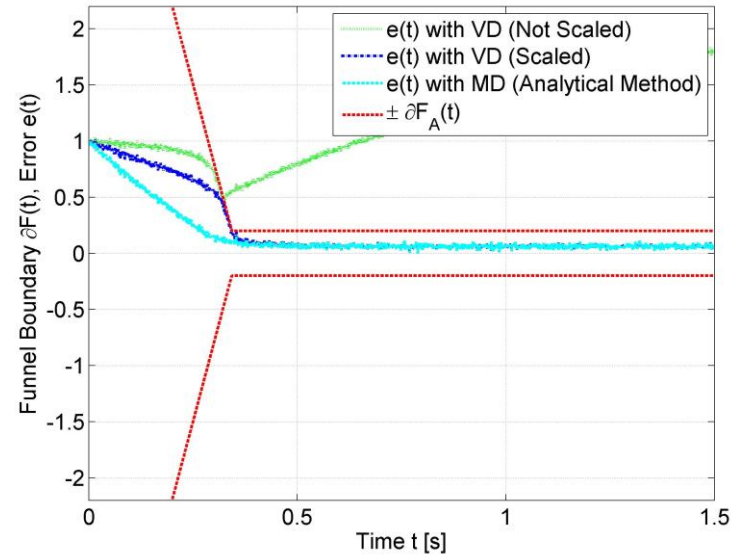


Fig. 8b: Error $e(t)$, Funnel Boundary $pF(t)$

Example System:

$$F_{PT_1}(s) = \frac{2}{1+s^2}$$

Funnel Boundary and Scaling Function:

$$\partial \mathcal{F}_A(t) = \begin{cases} -\frac{\varphi_0 - \varphi_\infty}{\tau_\mu} \cdot t + \varphi_0 & t < \tau_\mu \\ \varphi_\infty & t \geq \tau_\mu \end{cases}$$

$$\Psi(t) = \partial \mathcal{F}_A(t) + (1 - \varphi_\infty) \text{ for VD}$$

Simulation Results with Noise (Numeric & Derivative Approach)

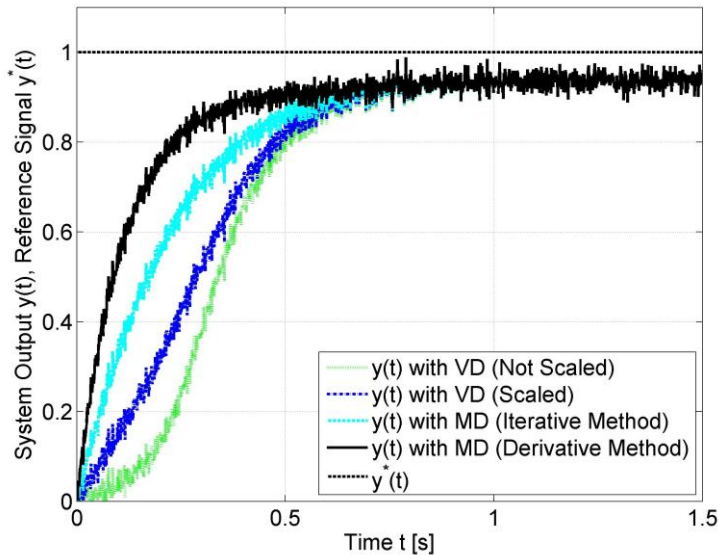


Fig. 9a: System Output $y(t)$

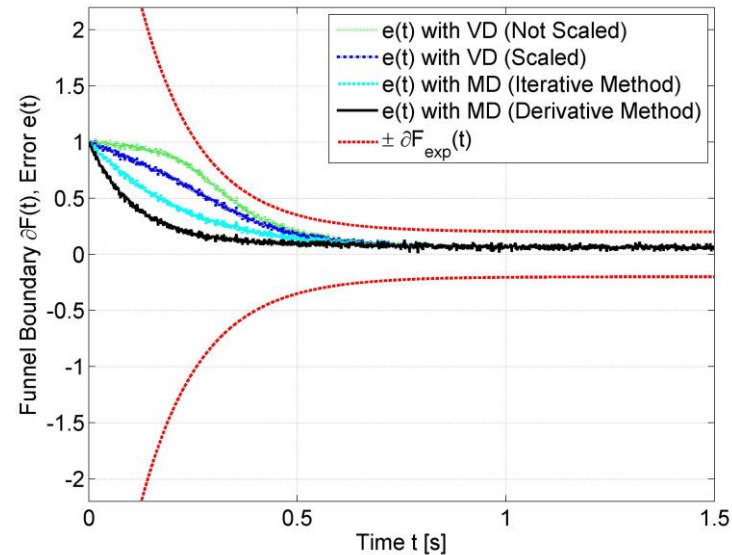


Fig. 9b: Error $e(t)$, Funnel Boundary $pF(t)$

Example System:

$$F_{PT_1}(s) = \frac{2}{1 + s^2}$$

Funnel Boundary and Scaling Function:

$$\partial \mathcal{F}_{\text{exp}}(t) = \varphi_{\text{exp},0} \cdot \exp\left(-\frac{t}{T_{\text{exp}}}\right) + \varphi_{\text{exp},\infty}$$

$$\Psi(t) = \partial \mathcal{F}_{\text{exp}}(t) + (1 - \varphi_{\text{exp},\infty}) \text{ for VD}$$

Simulation Results with Noise (Numeric & Derivative Approach)

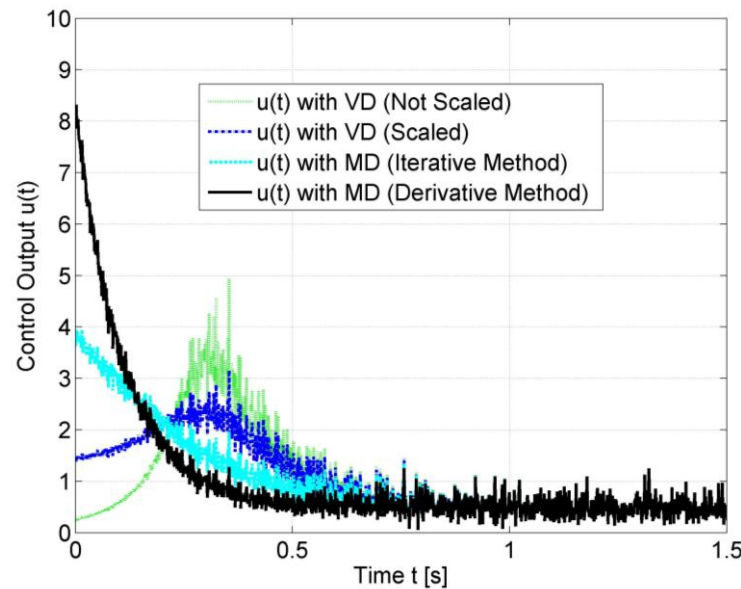


Fig. 9c: Control Output $u(t)$

Example System:

$$F_{PT_1}(s) = \frac{2}{1 + s2}$$

Funnel Boundary and Scaling Function:

$$\partial \mathcal{F}_{\text{exp}}(t) = \varphi_{\text{exp},0} \cdot \exp\left(-\frac{t}{T_{\text{exp}}}\right) + \varphi_{\text{exp},\infty}$$

$$\Psi(t) = \partial \mathcal{F}_{\text{exp}}(t) + (1 - \varphi_{\text{exp},\infty}) \text{ for VD}$$

Simulation Results with Noise (Active Damping)

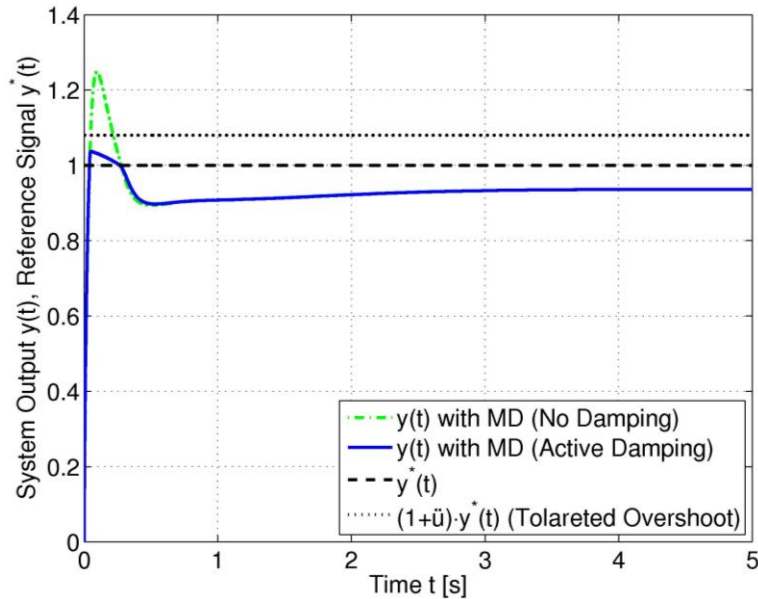


Fig. 10a: System Output $y(t)$

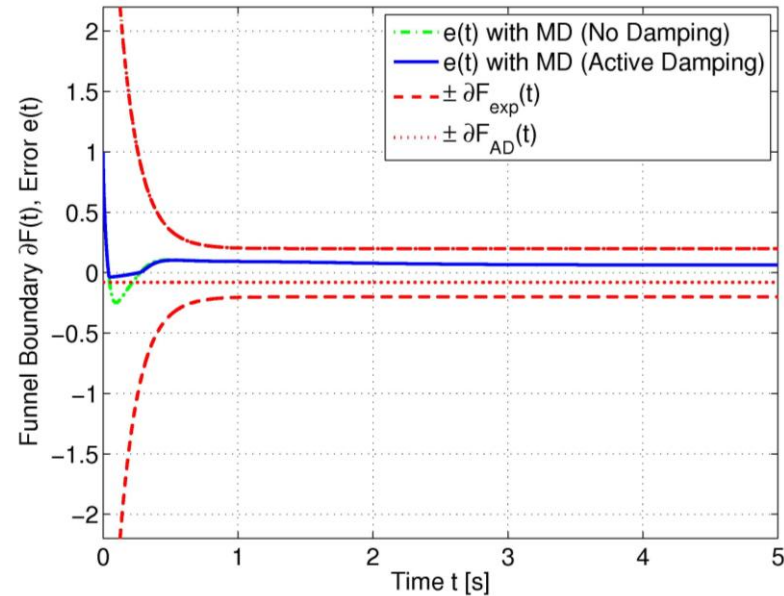


Fig. 10b: Error $e(t)$, Funnel Boundary $pF(t)$

Example System:

$$F_{PD_2T_3}(s) = \frac{2(1 + s + s^2 0.25)}{(1 + s2)(1 + s0.033 + s^2 0.028)}$$

Funnel Boundary and Scaling Function:

$$\partial \mathcal{F}_{AD+}(t) = \varphi_{\exp,0} \cdot \exp\left(-\frac{t}{T_{\exp}}\right) + \varphi_{\exp,\infty}$$

$$\partial \mathcal{F}_{AD-}(t) = -\ddot{u} \cdot y^*$$

Conclusion:

- Simple, robust and adaptive control structure
- Pre-definition of desired transient behavior possible
- Measurement noise is admissible
- **Improved performance** for:
 1. ***well chosen scaling function*** (Acceleration)
 2. ***implemented minimal (future) distance*** (Acceleration)
(derivative method shows best performance with simple realization and free choice of adequate Funnel Boundary)
 3. ***asymmetric Funnel boundaries (Active Damping)***

Outlook:

- Consideration of constrained (saturated) control inputs (MED'07: T01-015)
- Further experimental results in laboratory (e.g. Flexible Three-Mass-System or Robot Manipulator Control)

Funnel-Control – Achieved Goals

- Extension of the usable system class (reduction of the relative degree)
 - ✓ *H. Schuster, D. Schröder „High-Gain Control of Systems with Arbitrary Relative Degree: Speed Control for a Two Mass Flexible Servo System“. 8th IEEE International Conference on Intelligent Engineering Systems 2004 (INES 2004), Cluj-Napoca, Romania, September 19-21, 2004, p. 486-491.*
- Robustness analysis (parameter variations)
 - ✓ *H.Schuster, C.M. Hackl, C. Westermaier and D. Schröder, "Funnel-Control for Electrical Drives with Uncertain Parameters", 7th International Power Engineering Conference - IPEC 2005 Singapore, Nov. 29 - Dec. 2, 2005. (published in IPEC Proceedings)*
 - ✓ *C.M. Hackl, D. Schröder, "Funnel-Control for Nonlinear Multi-Mass Flexible Systems", IECON 2006 - Paris, November 7 - 10, 2006, IECON'06 Proceedings*
- Steady state accuracy (vanishing control error)
 - ✓ *C.M. Hackl, H.Schuster, C. Westermaier and D. Schröder, "Funnel-Control with Integrating Prefilter for Nonlinear, Time-varying Two-Mass Flexible Servo Systems", The 9th International Workshop on Advanced Motion Control, AMC 2006 - Istanbul, March 27-29, 2006*
 - ✓ *C.M. Hackl, D. Schröder, "Extension of High-Gain Controllable Systems for Improved Accuracy", Joint CCA, CACSD, and ISIC, 2006 - Munich, October 4-6, 2006, Joint CCA, CACSD, and ISIC Proceedings, pp. 2231-2236*
- Saturated Control Input Compensation
 - ✓ *C.M. Hackl, Y. Ji and D. Schröder, „Nonidentifier-based Adaptive Control with Saturated Control Input Compensation", MED 2007 - Athens, Greece June 27-29, 2007, T01-015*

Thank you for your attention