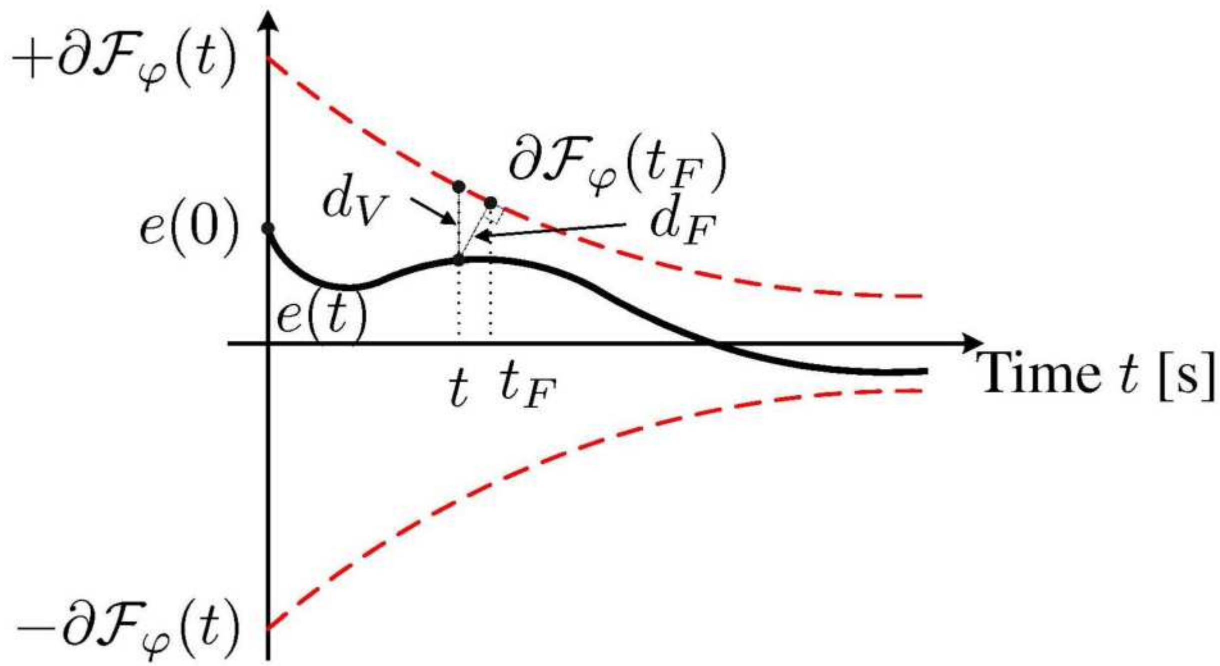
Funnel-Control with Online Foresight

Christoph M. Hackl and Dierk Schröder



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Overview

1. Funnel-Control (FC)
   - Prerequisites, Basic Idea -

2. Scaling Function

3. Online Foresight: Minimal (Future) Distance (MD)

4. Simulation

5. Conclusion & Outlook

Funnel-Control (FC) – Prerequisites

(developed by *Ilchmann et al.*)

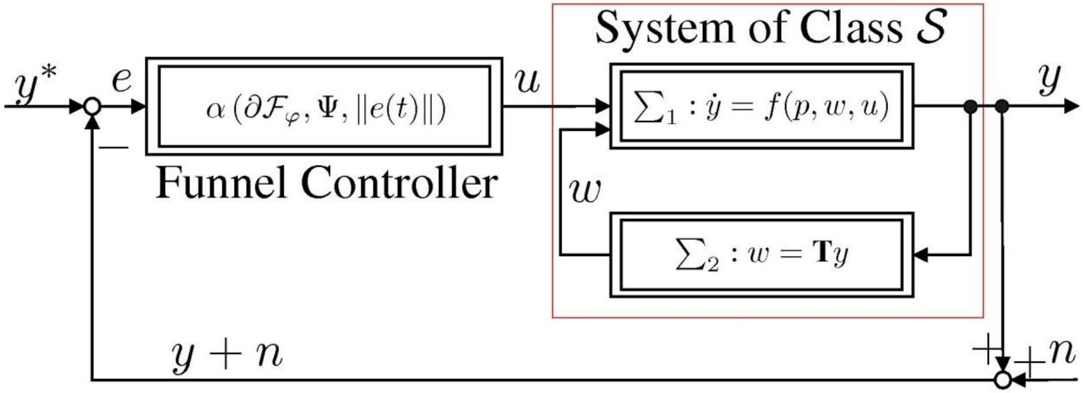

Fig. 1: Block diagramm – System of Class S controlled with FC

Controller properties:

- Proportional Controller
- Adaptive Time-Varying Gain
- Control Law:

$$u(t) = \alpha\left(\partial\mathcal{F}_\varphi(t), \Psi(t), \|e(t)\|\right) \cdot e(t)$$

System of Class $\mathcal{S}$ - prerequisites:

- Relative Degree $r = 1$
- Minimum-Phase (stable zero-dynamics)
- Known High-Frequency Gain

Funnel-Control (FC) with Vertical Distance (VD) - Basic Idea

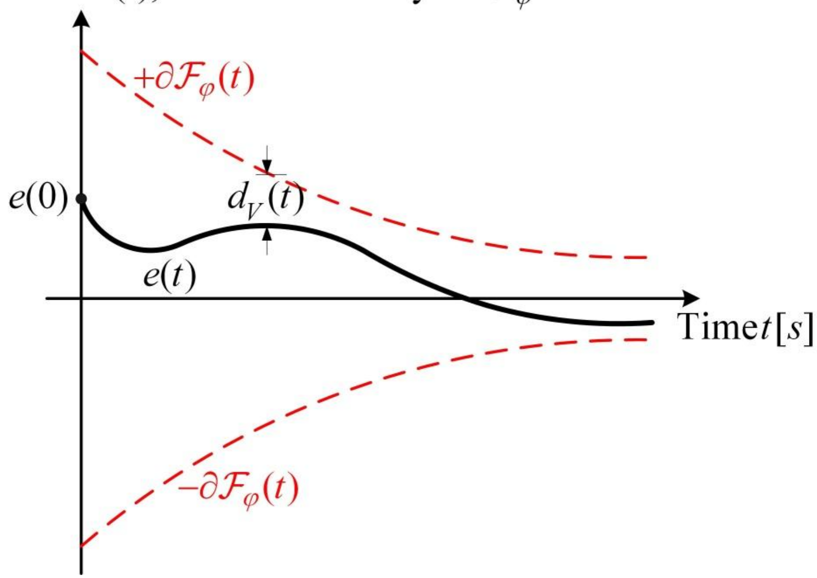

Fig. 2: Error evolution within Funnel Boundary

Adaption of gain depending on (vertical) distance:

$$\boxed{\alpha_D(t) = \frac{1}{d_V(t)}}$$

- Short distance => high gain (more aggressive control)
- Great distance => small gain (more relaxed control)

Time-varying (adaptive) gain $\alpha(\cdot)$ $\Rightarrow$ Boundedness of control error (for sufficiently large but finite control inputs!)

$$\boxed{\alpha\left(\partial\mathcal{F}_\varphi(t), \Psi(t), \|e(t)\|\right) = \Psi(t)\cdot\alpha_D(t) = \frac{\Psi(t)}{d_V(t)} = \frac{\Psi(t)}{\underbrace{\partial\mathcal{F}_\varphi(t) - \|e(t)\|}_{0<\cdot\le\partial\mathcal{F}_\varphi(t)}}}$$

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Scaling Function

- any continuous, bounded and positive function is allowed: $\Psi(t) > 0 \text{ for all } t \ge t_0$

- to choose e.g. minimal gain („memory effect"): $\alpha(t) = \dfrac{\Psi(t)}{\underbrace{\partial\mathcal{F}_\varphi(t) - \|e(t)\|}_{0 < \square \le \partial\mathcal{F}_\varphi(t)}} \ge \dfrac{\Psi(t)}{\partial\mathcal{F}_\varphi(t)}$

- ***only to increase*** the distance gain (acceleration effect):

$$\boxed{\begin{aligned} &\alpha(t) = \Psi(t)\cdot\alpha_D(t) \ge \alpha_D(t) \\ &\Rightarrow\ \Psi(t) \ge 1 \text{ for all } t \ge t_0 \end{aligned}}$$

(otherwise the principle of Funnel Control may be deteriotated)

- if Funnel Boundary itself is used for scaling:

$$\boxed{\Psi(t) = \partial\mathcal{F}_\varphi(t) + (1-\mu) \text{ if } \lim_{t\to\infty} \partial\mathcal{F}_\varphi(t) = \mu < 1}$$

Online Foresight: Minimal (Future) Distance (MD)

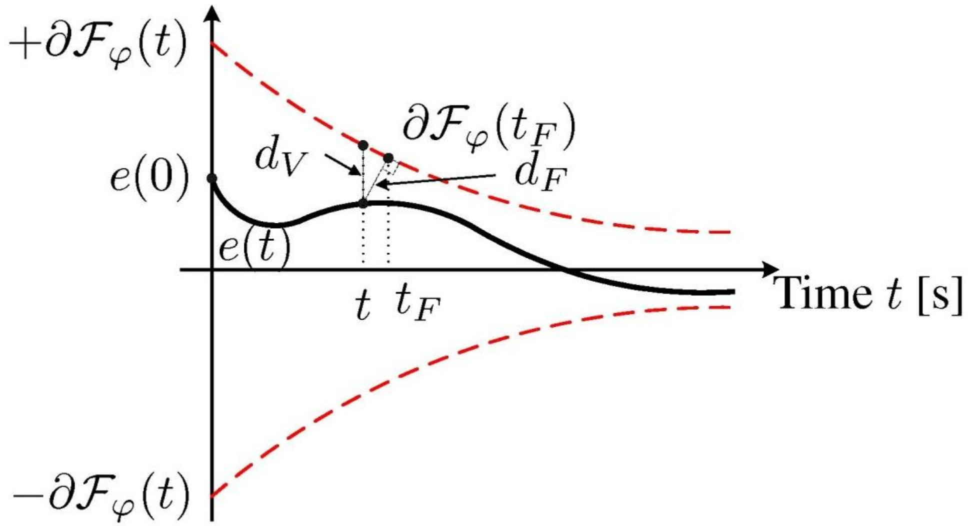


Fig. 3: Vertical (VD) & Minimal (Future) Distance (MD)

- Minimal (Future) Distance (MD) exists with $t_F \geq t$ and:

$$\boxed{d_F(t, t_F) = \min_{t_F \geq t} \sqrt{\left(\partial\mathcal{F}_\varphi(t_F) - \|e(t)\|\right)^2 + (t_F - t)^2} \leq d_V(t)}$$

- Funnel Control more effective with (especially for initial error $e(t_0)$) as:

$$\boxed{u_F(t) = \alpha_F(t) \cdot e(t) \geq \alpha_V(t) \cdot e(t) = u_V(t)}$$

Simulation Results (Analytic Approach)

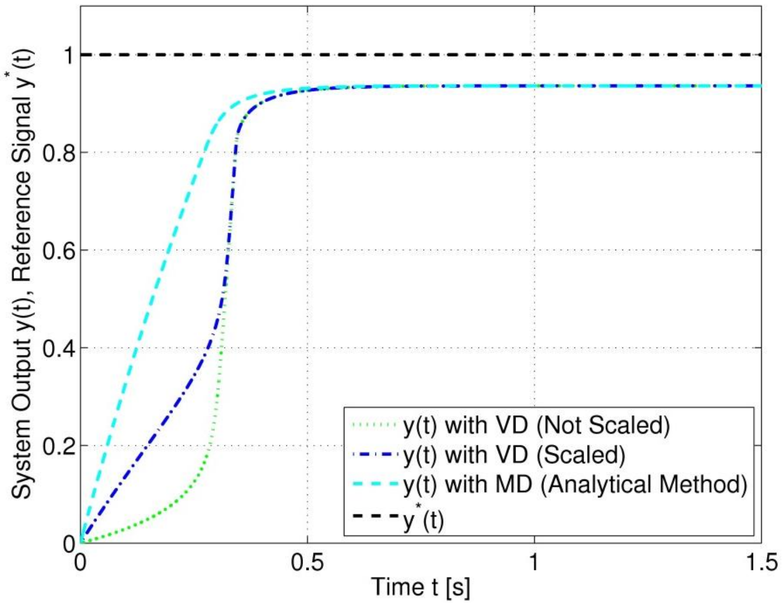


Fig. 7a: System Output y(t)

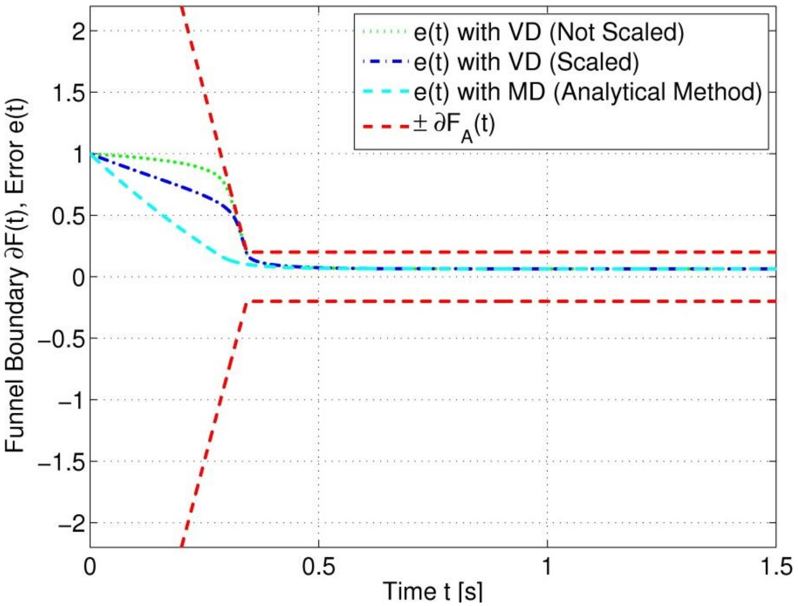


Fig. 7b: Error e(t), Funnel Boundary pF(t)

Example System:

$$F_{PT_1}(s) = \frac{2}{1+s2}$$

Funnel Boundary and Scaling Function:

$$\partial\mathcal{F}_A(t) = \begin{cases} -\dfrac{\varphi_0 - \varphi_\infty}{\tau_\mu} \cdot t + \varphi_0 & t < \tau_\mu \\ \varphi_\infty & t \geq \tau_\mu \end{cases}$$

$$\Psi(t) = \partial\mathcal{F}_A(t) + (1 - \varphi_\infty) \text{ for VD}$$

Simulation Results (Analytic Approach)

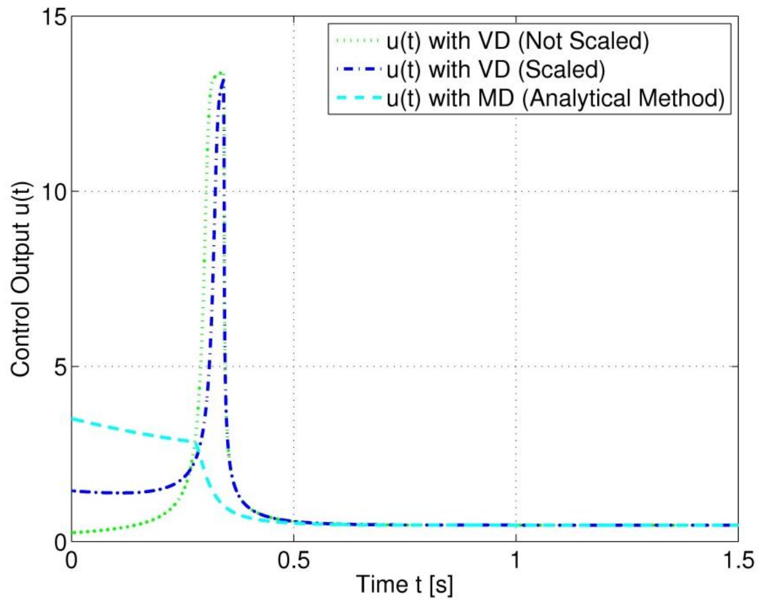


Fig. 7c: Control Output u(t)

Example System:

$$F_{PT_1}(s) = \frac{2}{1+s2}$$

Funnel Boundary and Scaling Function:

$$\partial\mathcal{F}_A(t) = \begin{cases} -\dfrac{\varphi_0 - \varphi_\infty}{\tau_\mu} \cdot t + \varphi_0 & t < \tau_\mu \\ \varphi_\infty & t \geq \tau_\mu \end{cases}$$

$$\Psi(t) = \partial\mathcal{F}_A(t) + (1 - \varphi_\infty) \text{ for VD}$$

Simulation Results with Noise (Analytic Approach)

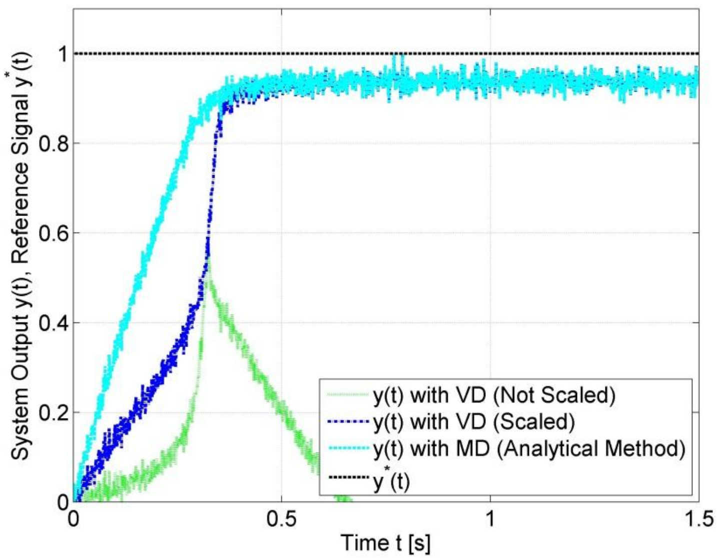


Fig. 8a: System Output y(t)

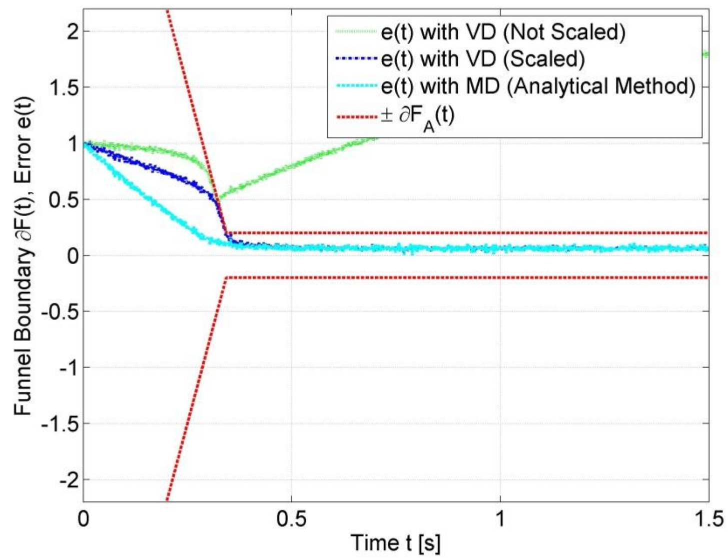


Fig. 8b: Error e(t), Funnel Boundary pF(t)

Example System:

$$F_{PT_1}(s) = \frac{2}{1+s2}$$

Funnel Boundary and Scaling Function:

$$\partial\mathcal{F}_A(t) = \begin{cases} -\dfrac{\varphi_0 - \varphi_\infty}{\tau_\mu} \cdot t + \varphi_0 & t < \tau_\mu \\ \varphi_\infty & t \geq \tau_\mu \end{cases}$$

$$\Psi(t) = \partial\mathcal{F}_A(t) + (1 - \varphi_\infty) \text{ for VD}$$

Simulation Results with Noise (Numeric & Derivative Approach)

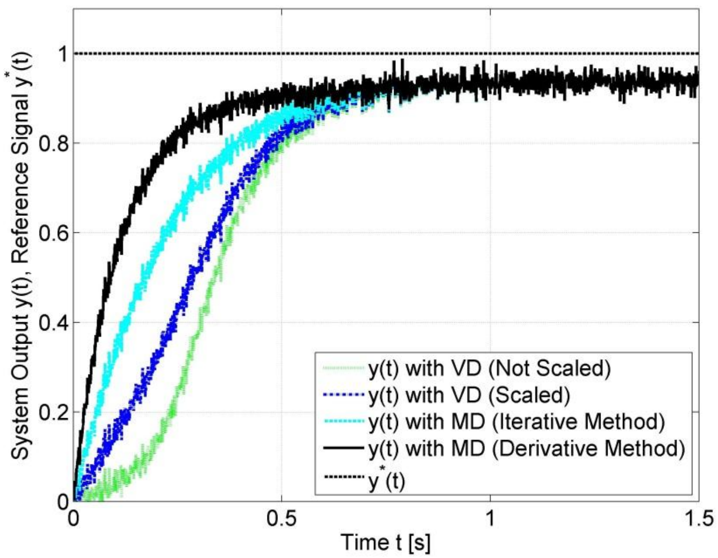


Fig. 9a: System Output y(t)

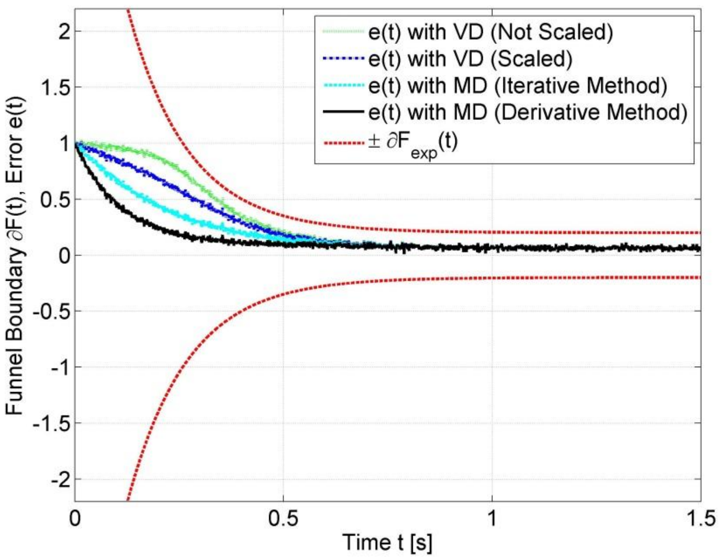


Fig. 9b: Error e(t), Funnel Boundary pF(t)

Example System:

$$F_{PT_1}(s) = \frac{2}{1+s2}$$

Funnel Boundary and Scaling Function:

$$\partial\mathcal{F}_{\exp}(t) = \varphi_{\exp,0} \cdot \exp\left(-\frac{t}{T_{\exp}}\right) + \varphi_{\exp,\infty}$$

$$\Psi(t) = \partial\mathcal{F}_{\exp}(t) + (1 - \varphi_{\exp,\infty}) \text{ for VD}$$

Simulation Results with Noise (Numeric & Derivative Approach)

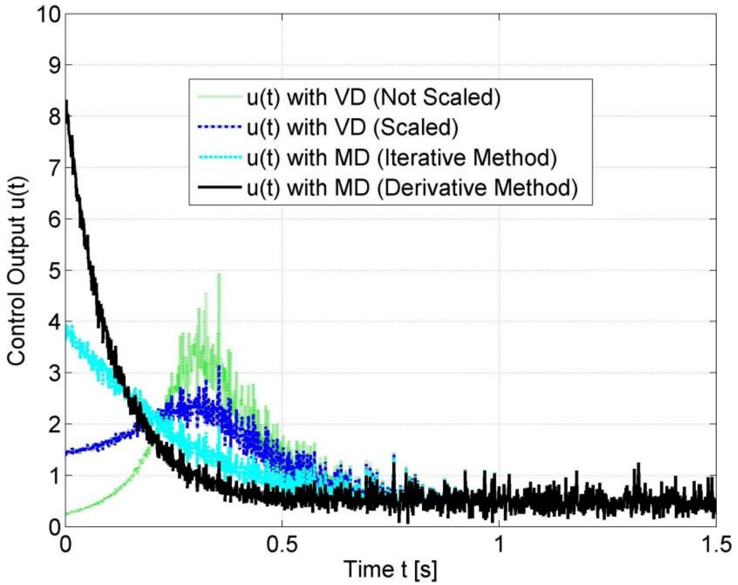


Fig. 9c: Control Output u(t)

Example System:

$$F_{PT_1}(s) = \frac{2}{1+s2}$$

Funnel Boundary and Scaling Function:

$$\partial\mathcal{F}_{\exp}(t) = \varphi_{\exp,0} \cdot \exp(-\frac{t}{T_{\exp}}) + \varphi_{\exp,\infty}$$

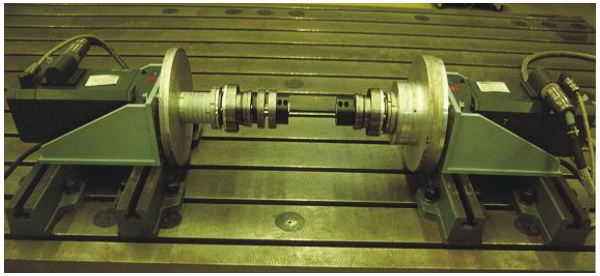
$$\Psi(t) = \partial\mathcal{F}_{\exp}(t) + (1-\varphi_{\exp,\infty}) \text{ for VD}$$

Conclusion:

- Simple, robust and adaptive control structure
- Pre-definition of desired transient behavior possible
- Measurement noise is admissible
- **Improved performance** (Acceleration) for:
  1. ***well chosen scaling function***
  2. ***implemented minimal (future) distance***
     (derivative method shows best performance with simple realization and free choice of adequate Funnel Boundary)

Outlook:

- Consideration of constrained (saturated) control inputs
- Active Damping
- Further experimental results in laboratory



Funnel-Control – Achieved Goals

- Extension of the usable system class (reduction of the relative degree)

  ✓ *H. Schuster, D. Schröder „High-Gain Control of Systems with Arbitrary Relative Degree: Speed Control for a Two Mass Flexible Servo System“. 8th IEEE International Conference on Intelligent Engineering Systems 2004 (INES 2004), Cluj-Napoca, Romania, September 19-21, 2004, p. 486-491.*

- Robustness analysis (parameter variations)

  ✓ *H.Schuster, C.M. Hackl, C. Westermaier and D. Schröder, "Funnel-Control for Electrical Drives with Uncertain Parameters", 7th International Power Engineering Conference - IPEC 2005 Singapore, Nov. 29 - Dec. 2, 2005. (published in IPEC Proceedings)*

  ✓ *C.M. Hackl, D. Schröder, "Funnel-Control for Nonlinear Multi-Mass Flexible Systems", IECON 2006 - Paris, November 7 - 10, 2006, IECON'06 Proceedings*

- Steady state accuracy (vanishing control error)

  ✓ *C.M. Hackl, H.Schuster, C. Westermaier and D. Schröder, "Funnel-Control with Integrating Prefilter for Nonlinear, Time-varying Two-Mass Flexible Servo Systems", The 9th International Workshop on Advanced Motion Control, AMC 2006 - Istanbul, March 27-29, 2006*

  ✓ *C.M. Hackl, D. Schröder, "Extension of High-Gain Controllable Systems for Improved Accuracy", Joint CCA, CACSD, and ISIC, 2006 - Munich, October 4-6, 2006, Joint CCA, CACSD, and ISIC Proceedings, pp. 2231-2236*

Thank you for your attention