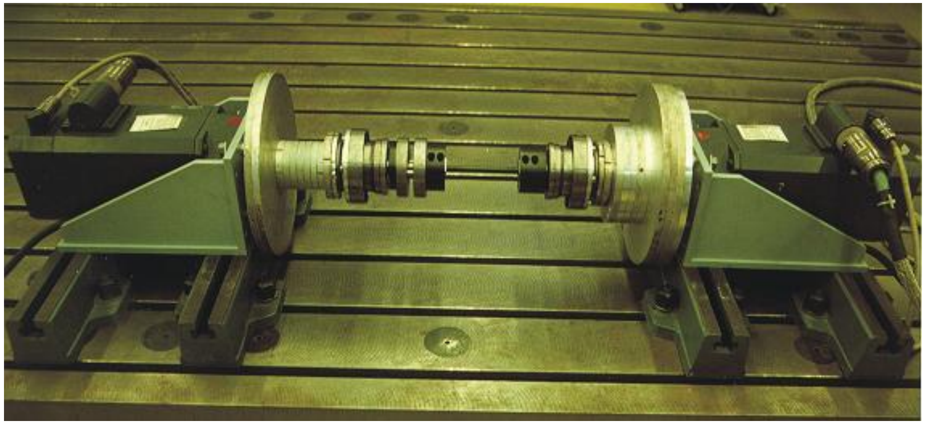
Funnel-Control with Integrating Prefilter for Nonlinear Time-varying Two-Mass Flexible Servo Systems

C.M. Hackl, H. Schuster, C. Westermaier and D. Schröder



Overview

1. Funnel-Control (FC)
   - preliminaries, basic idea -

2. Nonlinear Two Mass System
   - block diagram, nonlinearities, auxiliary system -

3. Integrating Prefilter
   - design, stability -

4. Simulation
   - results, comparison with state feedback (SF) -

5. Conclusion & Outlook

Funnel-Control (FC) - Preliminaries

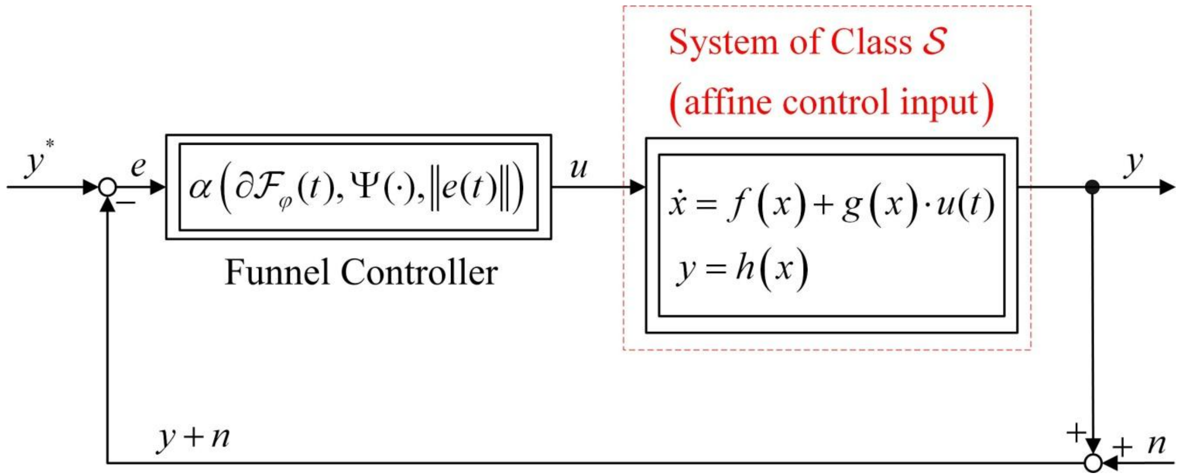


Fig. 1: Block diagramm - System controlled with FC

Controller properties:

- proportional controller
- adaptive time-varying gain
- control law

$$u(t)=\alpha\left(\partial\mathcal{F}_\varphi(t),\Psi(t),\|e(t)\|\right)\cdot e(t) \quad (1)$$

System of Class $\mathcal{S}$ - prerequisites:

- relative degree $r=1$

  $$\dot{y}=L_{\mathrm{f}}h(x)+\underbrace{L_{\mathrm{g}}h(x)}_{\neq 0}\cdot u(t)$$
- minimum-phase (stable zero-dynamics)
- known high-frequency gain

  $$hfg=\mathrm{sgn}\left(L_{\mathrm{g}}h(x)\right)>0$$

Funnel-Control (FC) - Basic Idea

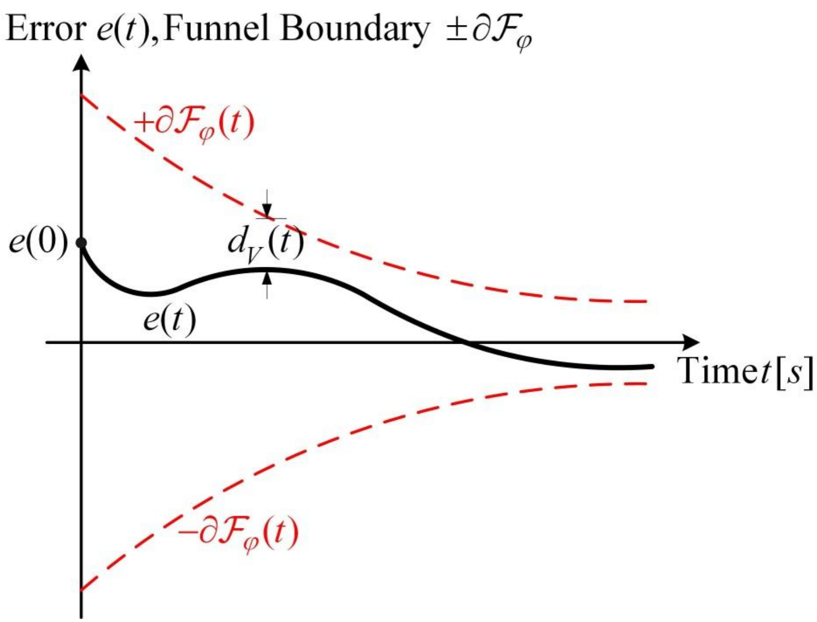


Fig. 2: Error evolution within Funnel Boundary

Adaption of gain depending on (vertical) distance:

- Short distance => high gain (more aggressive control)
- Great distance => small gain (more relaxed control)

Time-varying (adaptive) gain $\alpha(\cdot)$ $\Rightarrow$ Boundedness of control error (for sufficiently large but finite control inputs!)

$$\alpha\left(\partial\mathcal{F}_\varphi(t),\Psi(t),\|e(t)\|\right)=\frac{\Psi(t)}{d_V(t)}=\frac{\Psi(t)}{\underbrace{\partial\mathcal{F}_\varphi(t)-\|e(t)\|}_{0\le\cdot<1}}\ge\frac{\Psi(t)}{\partial\mathcal{F}_\varphi(t)}\quad\forall t\ge 0\quad(2)$$

Scaling factor $\Psi(t)$ to choose minimum gain value in advance ("memory effect").

Nonlinear Two-Mass Flexible Servo System (TMS) - System Model

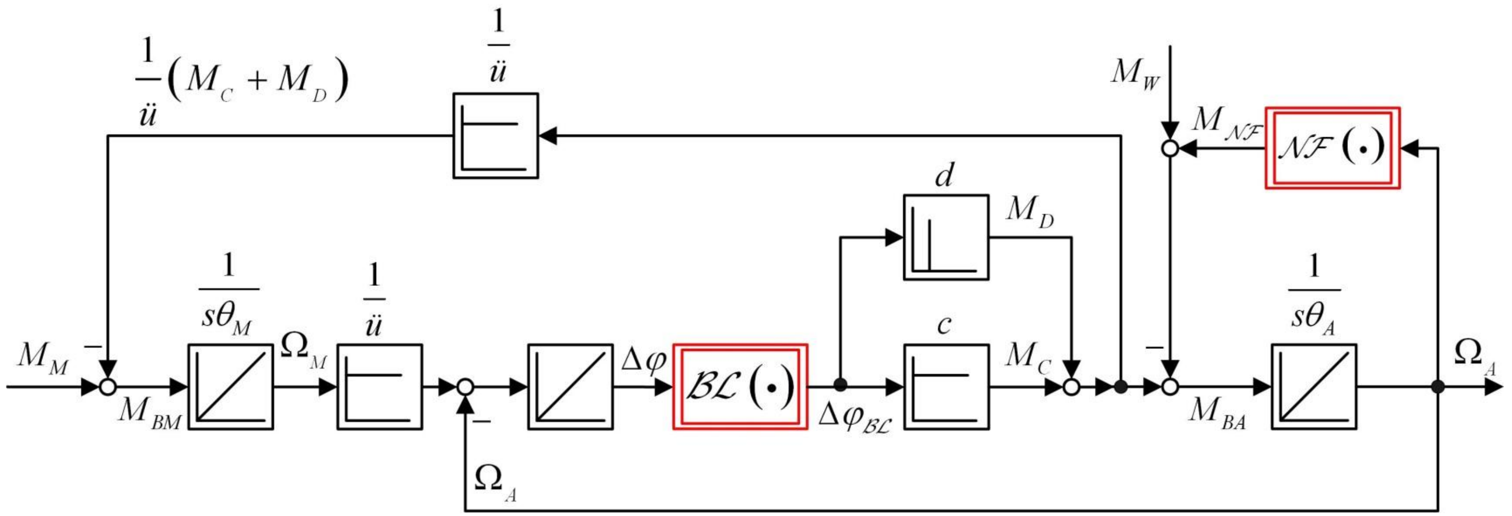


Fig. 3: Block diagramm - nonlinear Two-Mass-System (converter neglected)

ODE:

$$\dot{x} = \begin{bmatrix} \dot{\Omega}_M \\ \Delta\dot{\varphi} \\ \dot{\Omega}_A \end{bmatrix} = \begin{bmatrix} \frac{1}{\Theta_M}\left(M_M - \frac{d}{\ddot{u}} \cdot \frac{\partial \mathcal{BL}(\Delta\varphi)}{\partial t} - \frac{c}{\ddot{u}} \mathcal{BL}(\Delta\varphi) \right) \\ \frac{1}{\ddot{u}}\Omega_M - \Omega_A \\ \frac{1}{\Theta_A}\left(-M_W - \mathcal{NL}(\Omega_A) + d \cdot \frac{\partial \mathcal{BL}(\Delta\varphi)}{\partial t} + c \cdot \mathcal{BL}(\Delta\varphi) \right) \end{bmatrix}$$

Output: $y = \Omega_A$

Nonlinear Two-Mass Flexible Servo System (TMS) - Control Loop

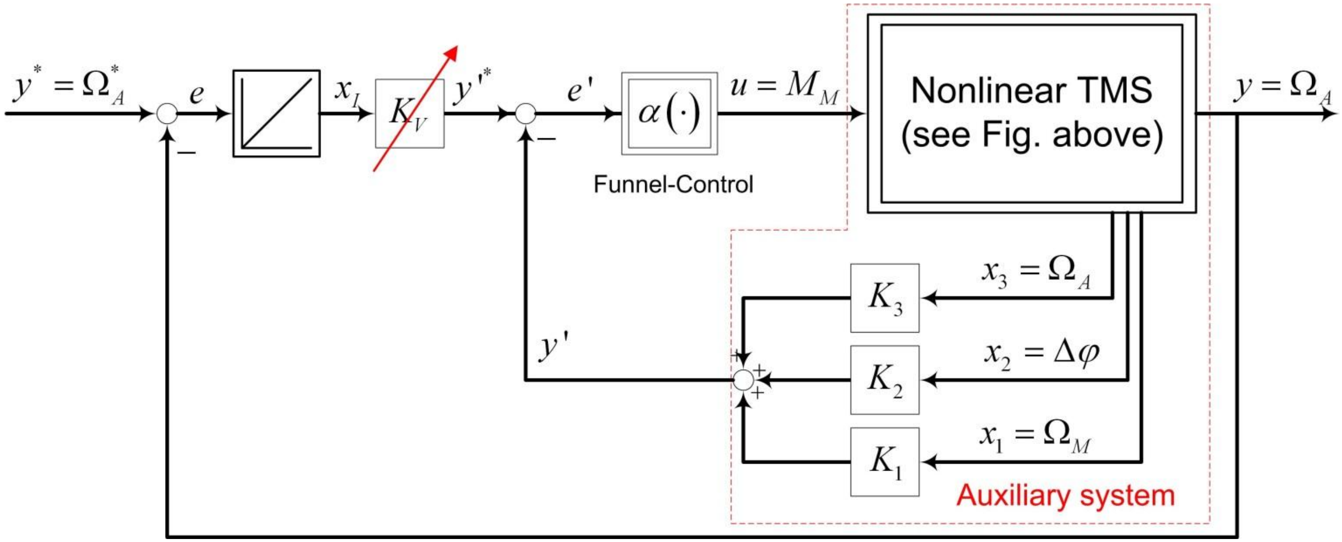


Fig. 4: Block diagramm - Control-Loop with prefilter and Funnel-Control

Introduction of auxiliary system ($u \rightarrow y'$):

- Reduction of relative degree
- Secure minimum-phase property
- Maintain influence of motor torque ($hfg > 0$)

$\Rightarrow$ Free choice of feedback coefficients (design parameters)

$$K_1 > 0$$

$$K_2 > 0 > -\frac{d}{\Theta_A}\left(K_1 + K_3\right) \quad (3)$$

$$0 > K_3 > -K_1$$

Prefilter - Design

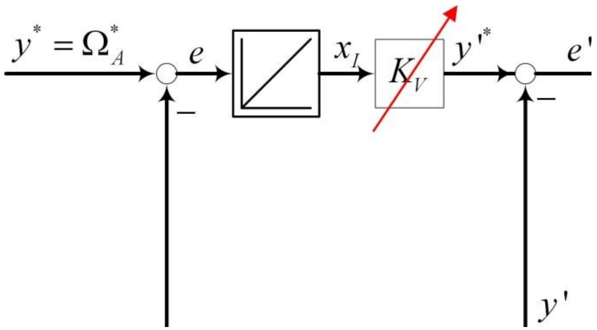


Fig. 5: Cascaded Loop - Integrating prefilter with variable gain

Generation of auxiliary error:

$$\boxed{e'(t) = y'^*(t) - y'(t) = K_V(t)\underbrace{\int_{t_0}^{t} e(\tau)d\tau}_{x_I} - y'(t) \quad (4)}$$

where $K_V(t) > 0$

Auxiliary error starts at: $e'(t_0) = 0 \;\; \forall y'(t_0) = 0 \wedge x_I(t_0) = 0$

Choice of *constant Tolerance Funnel Boundary*: $\partial\mathcal{F}_\varphi(t) \square \partial\mathcal{F}_{const} > 0$

(Scaling factor $\Psi(t) \square \partial\mathcal{F}_{const}$ used to guarantee minimum gain of $\alpha(\square) \geq 1$)

Prefilter - Stability

Saturation of control input:

$$|u(t)| = \alpha(\square) \cdot |e'(t)| \le u^{\max} \quad (5)$$

$\Rightarrow$ Endanger stability (auxiliary error might leave Funnel Boundary)

Ability to adjust auxiliary error by prefilter gain $K_v(t)$:

$$|e'(t)| = \left| \underbrace{K_V(t)\int_{t_0}^{t} e(\tau)d\tau}_{y'^*(t)} - y'(t) \right| \le \frac{u^{\max} \cdot \partial\mathcal{F}_{const}}{u^{\max} + \partial\mathcal{F}_{const}}$$

$\Rightarrow$ **Preservation of stability** (considering saturation of control input!)

Initial prefilter gain:

$$K_V^0 = \frac{u^{\max} \cdot \partial\mathcal{F}_{const}}{u^{\max} + \partial\mathcal{F}_{const}} \quad (6)$$

Violation of Eq. (5) at time t^*

$\Rightarrow$ **online reduction** (recursively)

$$K_V^0(t^*) = \frac{K_V^0}{2} \quad (7)$$

Simulation Results - Disturbance Load

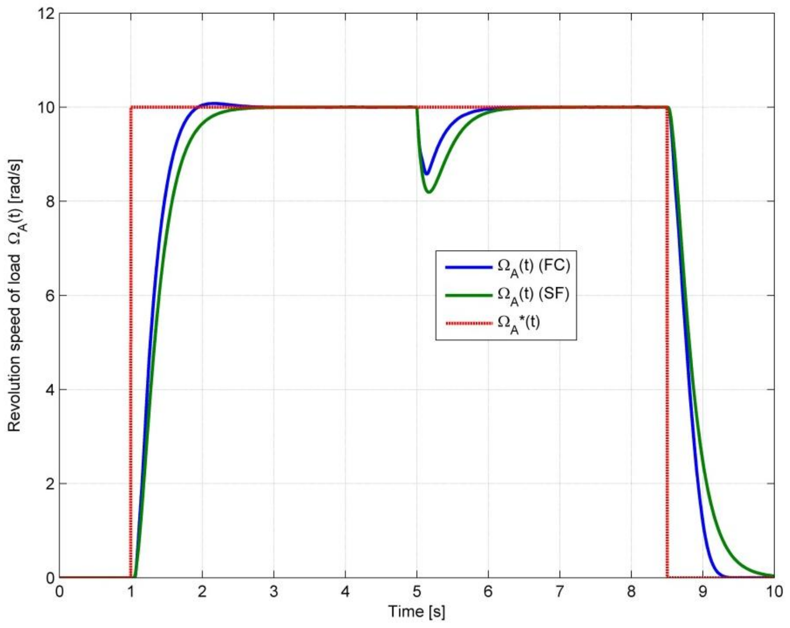


Fig. 5a: Load revolution speed for FC & SF

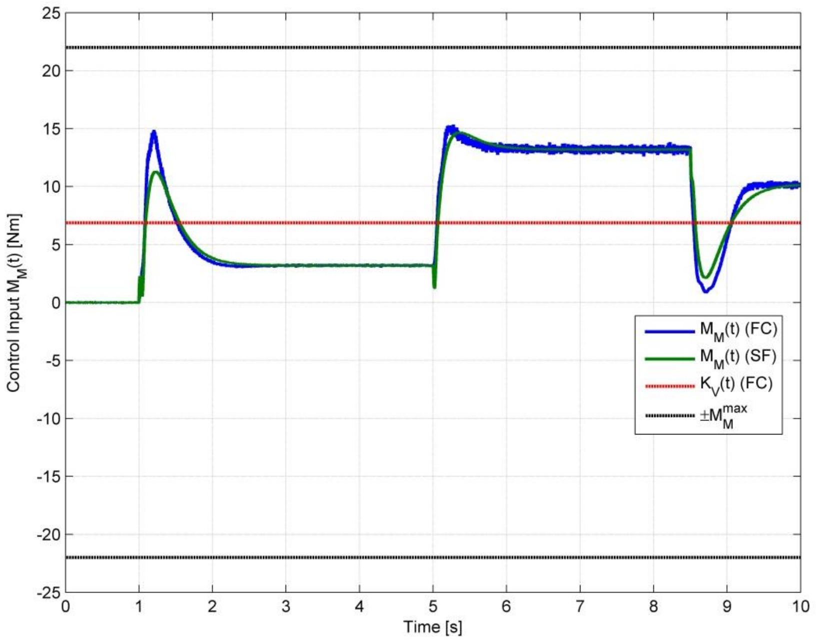


Fig. 5b: Control input (motor torque) for FC & SF

Reference Load Speed:

$$\Omega_A^*(t) = \begin{cases} 10\dfrac{rad}{s} & ;t \in [1;8,5s] \\ 0\dfrac{rad}{s} & ;\forall t \in [0;10s] \end{cases}$$

Disturbance Load Torque:

$$M_W(t) = 10\,Nm \cdot \sigma(t-5s)$$

Simulation Results - Time-varying Stiffness

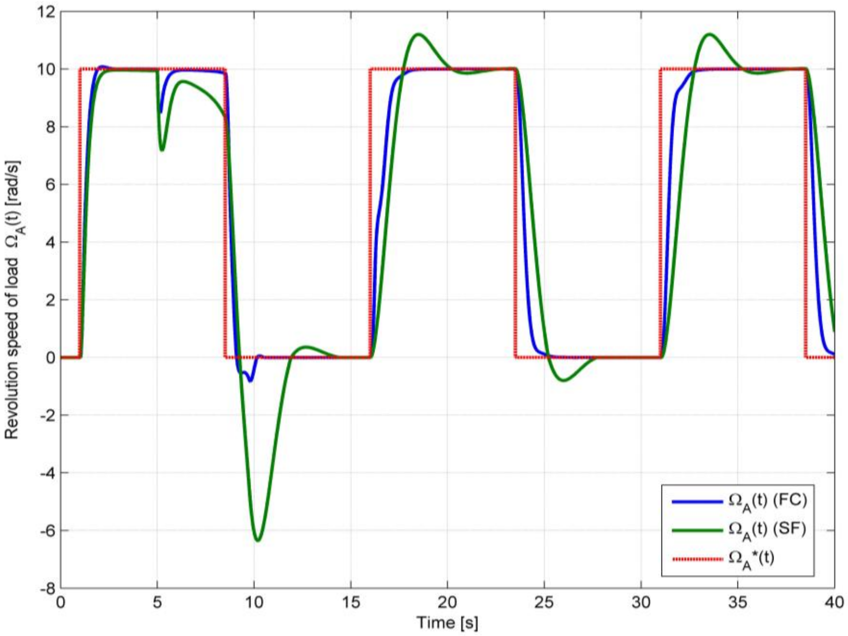


Fig. 6a: Load revolution speed for FC & SF

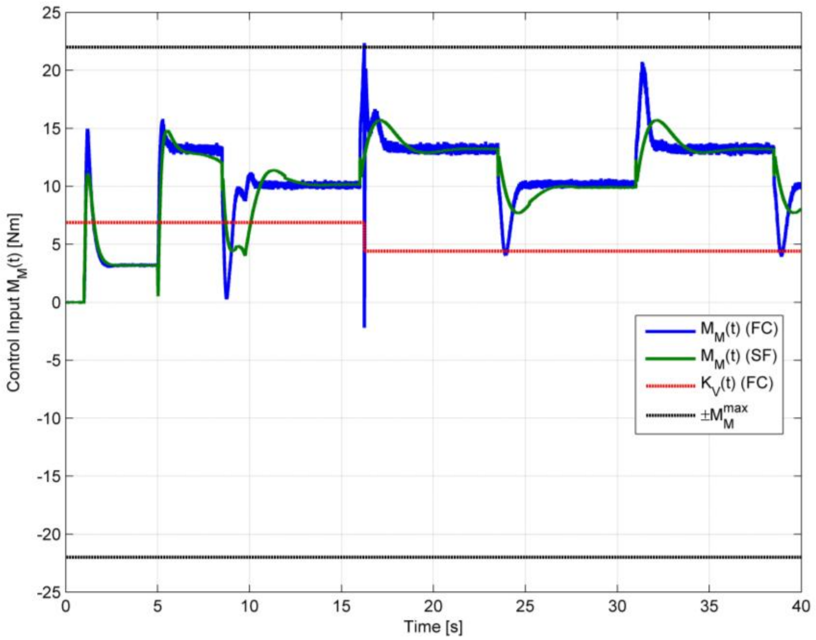


Fig. 6b: Control input (motor torque) for FC & SF

Time-varying stiffness:
(negativ ramp)

$$c(t) = \begin{cases} c_0 - 40t & ;0 \le t < 9.5s \\ 30\dfrac{Nm}{rad} & ;t \ge 9.5s \end{cases} \quad with\ c_0 = 410\frac{Nm}{rad}$$

Disturbance Load Torque:

$$\mathrm{M_W}(t) = 10\,Nm \cdot \sigma(t - 5s)$$

Simulation Results - Increased Backlash

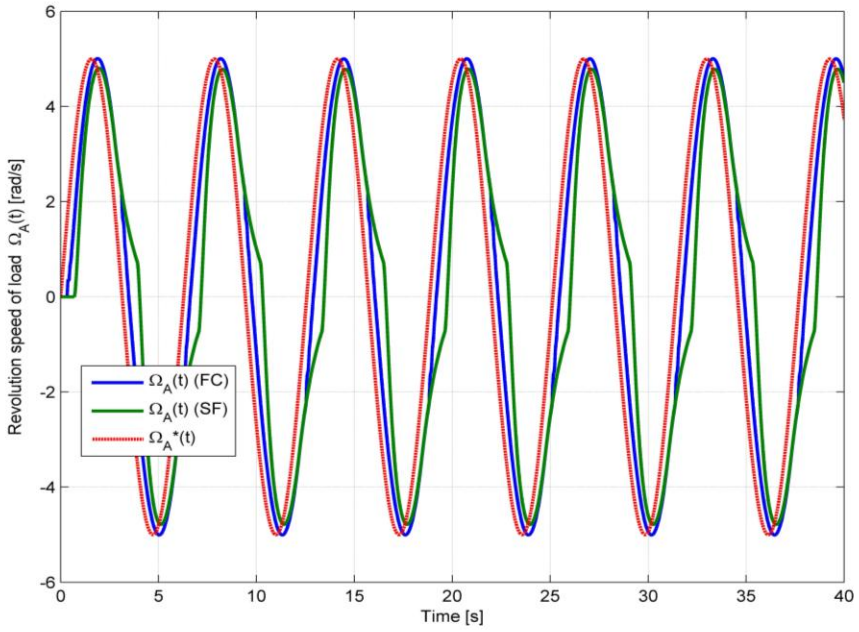


Fig. 7a: Load revolution speed for FC & SF

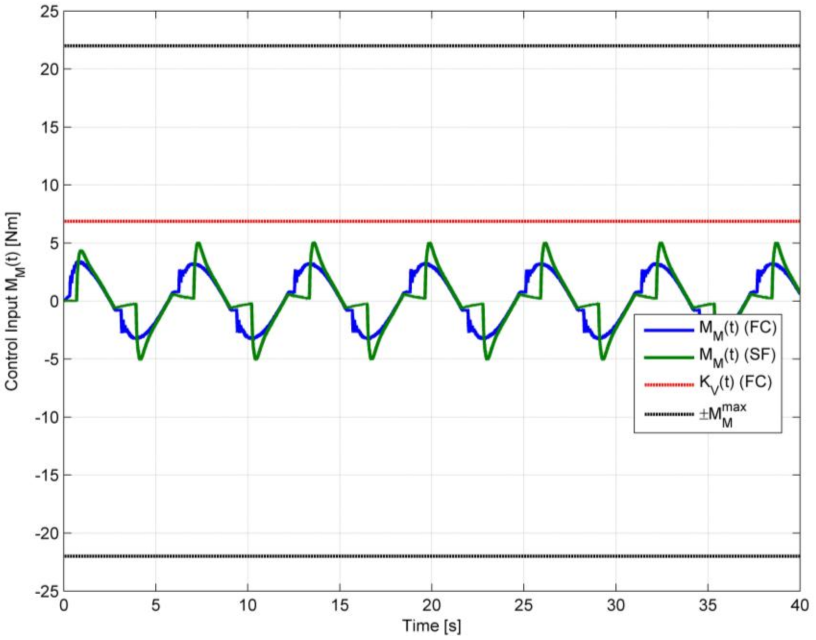


Fig. 7b: Control input (motor torque) for FC & SF

Reference Load Speed: $$\Omega_A^*(t) = 5 \cdot \sin\left(1 \frac{rad}{s} \cdot t\right)$$

Increased Backlash: $$a_{\mathcal{BL}} = 5°$$

Conclusion (achieved goals):

- Stationary accuracy (despite unknown plant!)
- Robustness due parameter deviations or time-varying behavior
- Nonlinearities do *not* affect control performance
- Measurement noise is admissible
- Efficient usage of control input due to adequate weighting of control error (adaption of control gain)
- No Identification/observation and compensation of nonlinearities necessary

Outlook (next steps):

- Expansion of experimental results (laboratory tests)
- Adaptive observers for state estimation